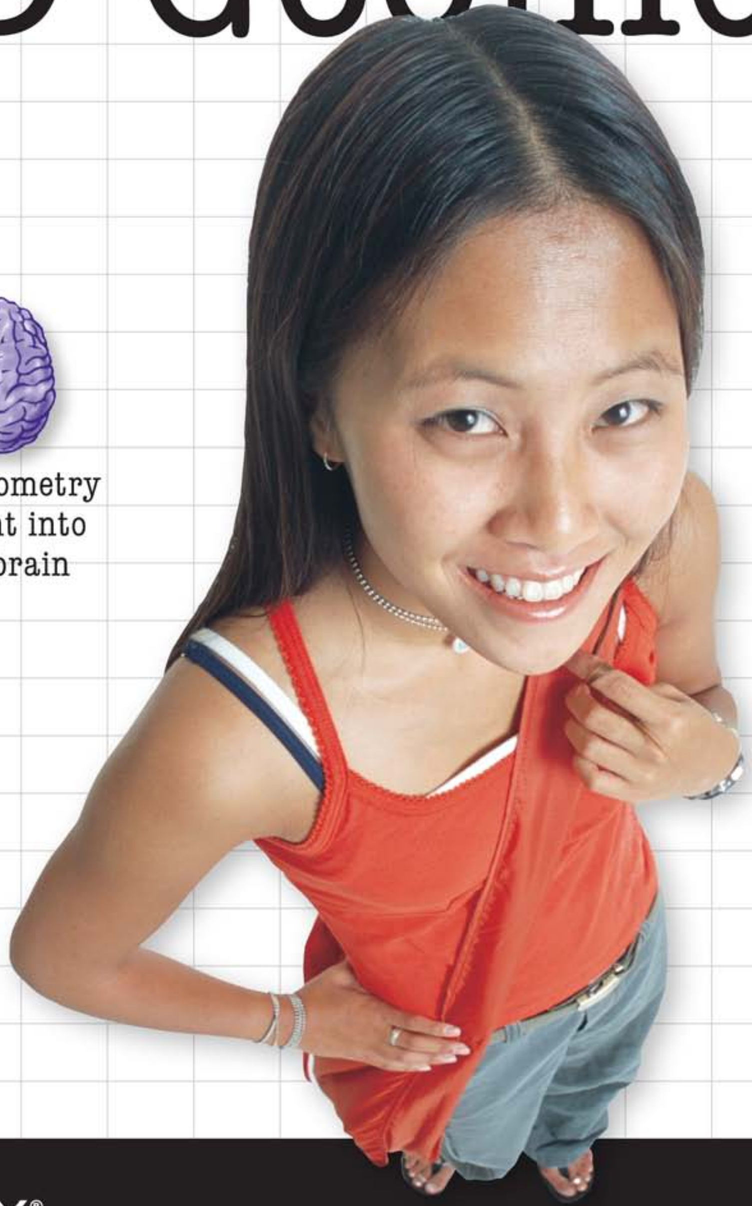
A Brain-Friendly Guide

Head First 2D Geometry





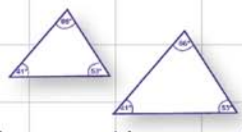
Load geometry straight into your brain



Plan a rockin' concert while showing off your geometry chops



Use angles to crack a mysterious crime



Discover time-saving secrets like congruence and similarity

Lindsey Fallow
& Dawn Griffiths

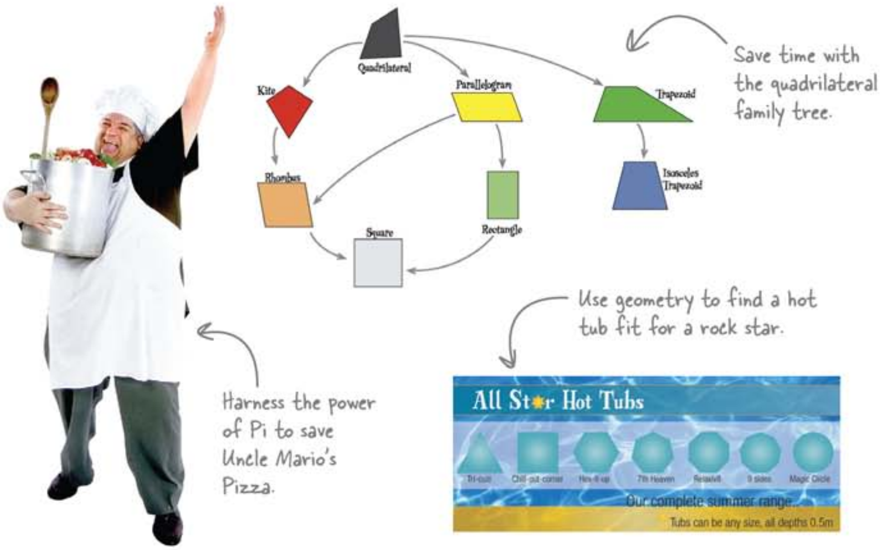
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Head First 2D Geometry

Geometry/Study Guide

What will you learn from this book?

Having trouble with geometry? Do Pi, the Pythagorean Theorem, and angle calculations just make your head spin? Relax. With *Head First 2D Geometry*, you'll master everything from triangles, quads, and polygons to the time-saving secrets of similar and congruent angles—and it'll be quick, painless, and fun.



Why does this book look so different?

We think your time is too valuable to waste struggling with new concepts. Using the latest research in cognitive science and learning theory to craft a multi-sensory learning experience, *Head First 2D Geometry* uses a visually rich format designed for the way your brain works, not a text-heavy approach that puts you to sleep.

"*Head First 2D Geometry* lets you see that geometry is not only in the classroom, it is all around you and a part of your everyday life."

—*Herbert Tracey, Instructor of Mathematical Sciences at Loyola University*

"The way the information is presented and organized makes learning cohesive and easy...This book helps you understand the basics and build on them. I wish I had [it] when I was taking Geometry!"

—*Amanda Borcky, College student*

"*Head First 2D Geometry* is clear and readable while other textbooks drag students through a thicket of academic jargon. Head First has interesting examples, fun design, and a conversational style that the textbook industry would do well to emulate."

—*Dan Meyer, High school math teacher*

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— Herbert Tracey
Instructor of mathematical sciences at Loyola University Maryland and former department chair of mathematics at Hereford High School

"*Head First 2D Geometry* is clear and readable, while other textbooks drag students through a thicket of academic jargon. Head First has interesting examples, fun design, and a conversational style that the textbook industry would do well to emulate."

— Dan Meyer
High school math teacher and recipient of Cable in the Classroom's Leader in Learning award

"*Head First 2D Geometry* grabs your attention with inventive and clever applied problems. It pursues thorough solutions with persistence and energy. There is one character who appears throughout the book and delights me—a serious, seemingly humorless girl who suspects the authors are trying to get away with inconsistency and poor logic. They always praise her questions and give in to her demands that they level with her."

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Retired college and high school math teacher

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College student

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— Dawn Griffiths
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— Marion Long
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"Dawn Griffiths has split some very complicated concepts into much smaller, less frightening bits of stuff that real-life people will find very easy to digest. *Head First Statistics* has lots of graphics and photos that make the material very approachable, and I have developed quite a crush on the attractive lady model who is asking about gumballs on page 458."

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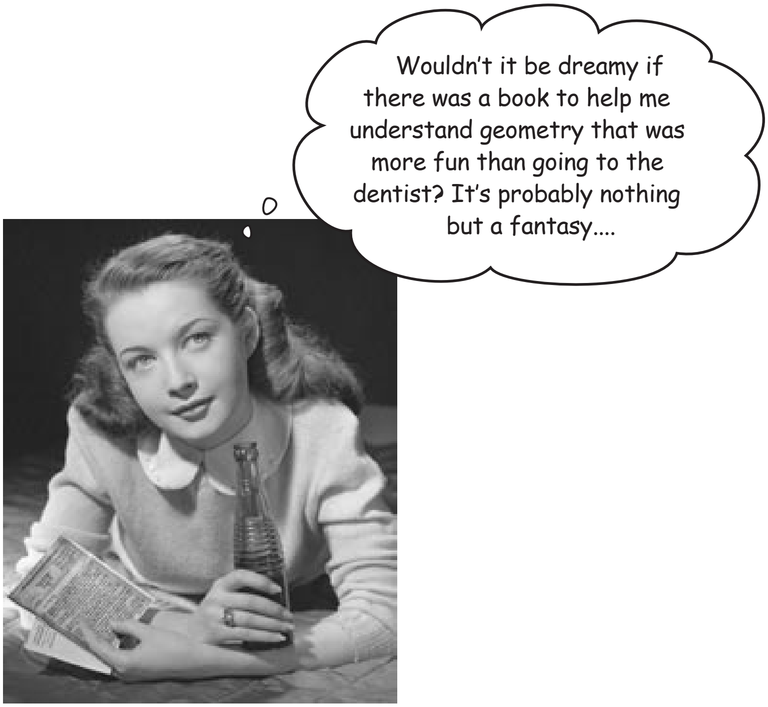
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Lindsey Fallow
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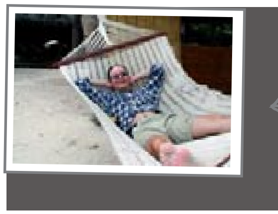
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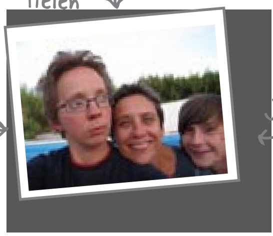
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No sheep or triangles were harmed in the making of this book.

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[M]

To Mum and Dad for buying me construction kits. To my fantastic Yorkshire family for endless support, humour, and psychotherapy—I love you even more than I love triangles. And to triangles and sheep, for making the world a fascinating place to be.

—Lindsey

To David, Mum, Dad, and Carl for their ongoing love and support. Also in loving memory of Peter Lancaster Walker, an unsung hero who made so many things possible.

—Dawn





Lindsey Fallow is a self-confessed geek who has spent the past decade exploring science and technology as a writer, software developer, and TV presenter.

After earning her undergraduate degree in manufacturing engineering, she fronted a science show for 8–12-year-olds on Disney, and went on to become a reporter and associate producer for *Tomorrow's World* (the BBC's #1 prime-time UK science and technology show) from 1998–2002.

She's stood on the top of the Golden Gate bridge, fed sharks, filmed brain surgery, flown in military planes, and been bitten by a baby tiger, but is the most excited by far when her 14-year-old stepson "gets" his math homework.

She is an avid fan of the Head First series and can't quite believe she's actually written one.

Lindz claims that if she were a superhero, her superpower would be tesselating. When she's not working, she likes to spend time with her super-lovely partner Helen, dabble in sheep farming, play with her boxer dog, Ruby, rock the drums on *Guitar Hero*, and walk in the wilderness.

Dawn Griffiths started life as a mathematician at a top UK university where she was awarded a first-class honours degree in mathematics. She went on to pursue a career in software development, and she currently combines IT consultancy with writing, editing, and mathematics.

Dawn is the author of *Head First Statistics*, and has also worked on a host of other books in the series, from Networking to Programming.

When Dawn's not working on Head First books, you'll find her honing her Tai Chi skills, making bobbin lace, or cooking. She hasn't yet mastered the art of doing all three at the same time. She also enjoys traveling, and spending time with her wonderful husband, David.

Dawn has a theory that *Head First Bobbin Lacemaking* might prove to be a big cult hit, but she suspects that Brett might disagree.

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Table of Contents (the real thing)

Intro

Your brain on Geometry. Here *you* are trying to *learn* something, while here your *brain* is doing you a favor by making sure the learning doesn't *stick*. Your brain's thinking, "Better leave room for more important things, like which wild animals to avoid and whether naked snowboarding is a bad idea." So, how *do* you trick your brain into thinking that your life depends on knowing about triangles and circles and the Pythagorean Theorem?

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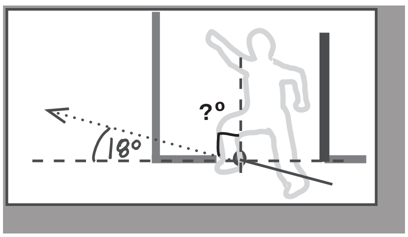
finding missing angles

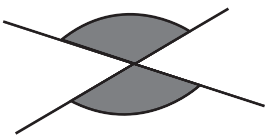
1 Reading between the lines

Ever get the feeling there's something they're not telling you?

If you want to master the **real world**, you need to get geometry. It's a set of **tools** for turning a little bit of information into a ***complete picture***. Whether you want to design something, build something or find out how a situation really went down, geometry can make sure you've always got **the lowdown**. So if you want to keep in the loop, grab your hat, pack your pencil, and join us on the bus to Geometryville.









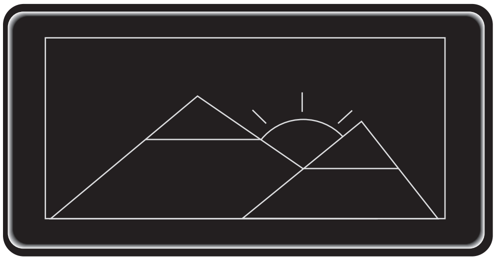
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2 similarity and congruence

Shrink to fit

Sometimes, size does matter.

Ever drawn or built something and then found out it's the **wrong size**? Or made something just perfect and wanted to ***recreate it exactly?*** You need *Similarity* and *Congruence*: the **time-saving techniques for duplicating** your designs smaller, bigger, or exactly the same size. Nobody likes doing the same work over—and with similarity and congruence, you'll ***never have to repeat an angle calculation again.***







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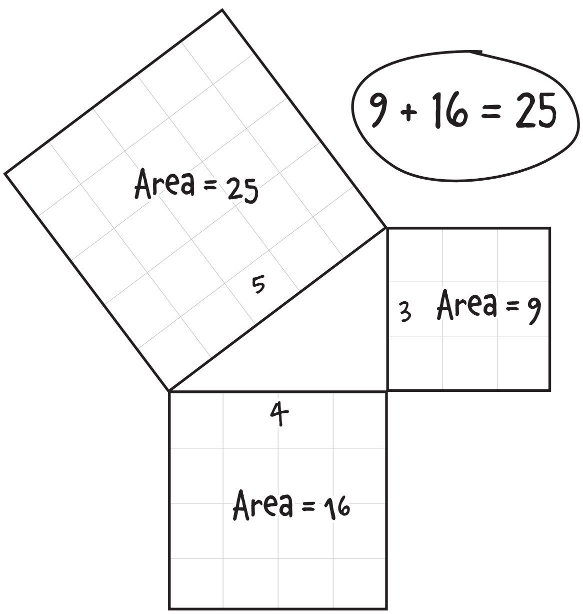
All the right angles

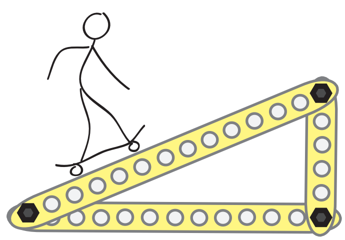
Sometimes, you really need to get things straight.

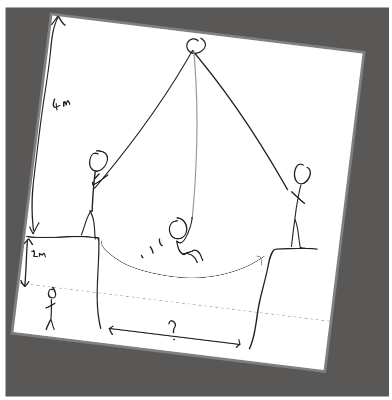
Ever tried to eat at a wobbly table? Annoying, isn't it? There is an alternative to shoving screwed-up paper under the table leg though: use the Pythagorean Theorem to make sure your designs are **dead straight** and not just *quite straight*. Once you know this pattern, you'll be able to **spot and create right angles** that are **perfect every time.** Nobody likes to pick spagetti out of their lap, and with the **Pythagorean Theorem**, you don't have to.



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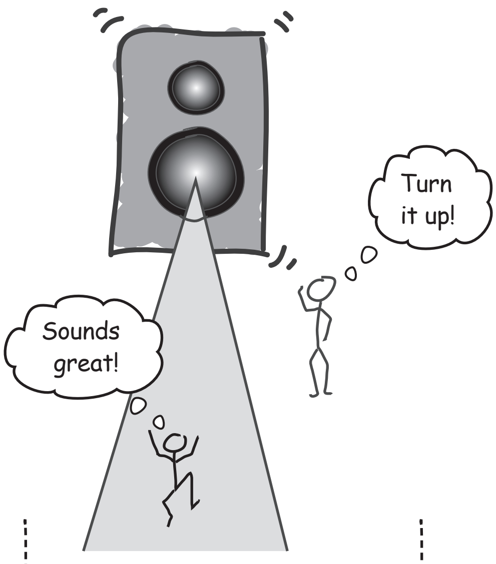
Between a rock show and a triangular place

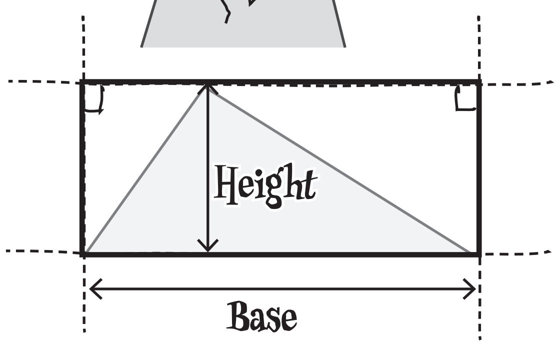
Ever had that sinking feeling that you've made a bad decision?

In the real world, *choices can be complex*, and wrong decisions can cost you **money** and **time**. Many solutions aren't always straightforward: even in geometry, bigger doesn't always mean better—it might not even mean longer. ***So what should you do?*** The good news is that you can *combine your triangle tools* to **make great decisions** even when it seems like you don't have the right information to answer the question.









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5 circles

Going round and round

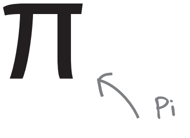
OK, life doesn't have to be so straight after all!

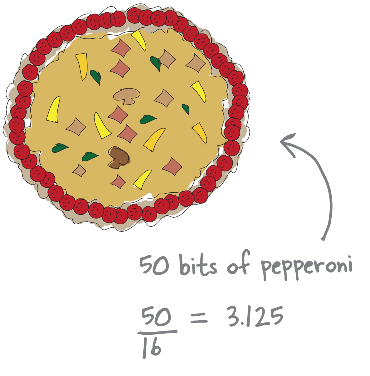
There's no need to reinvent the wheel, but aren't you glad you're able to use it? From cars to rollercoasters, many of the **most important solutions** to life's problems rely on ***circles*** to get the job done. Free yourself from straight edges and pointy corners—there's no end to the curvy possibilities once you master **circumference**, **arcs**, and **sectors**.





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6 quadrilaterals

It's hip to be square

Maybe three isn't the (only) magic number.

The world isn't just made up of triangles and circles. Wherever you look, you'll see **quadrilaterals**, shapes with four straight sides. Knowing your way 'round the quad family can save you a lot of time and effort. Whether it's **area**, **perimeter**, or **angles** you're after, there are ***shortcuts galore*** that you can **use to your advantage**. Keep reading, and we'll give you the lowdown.







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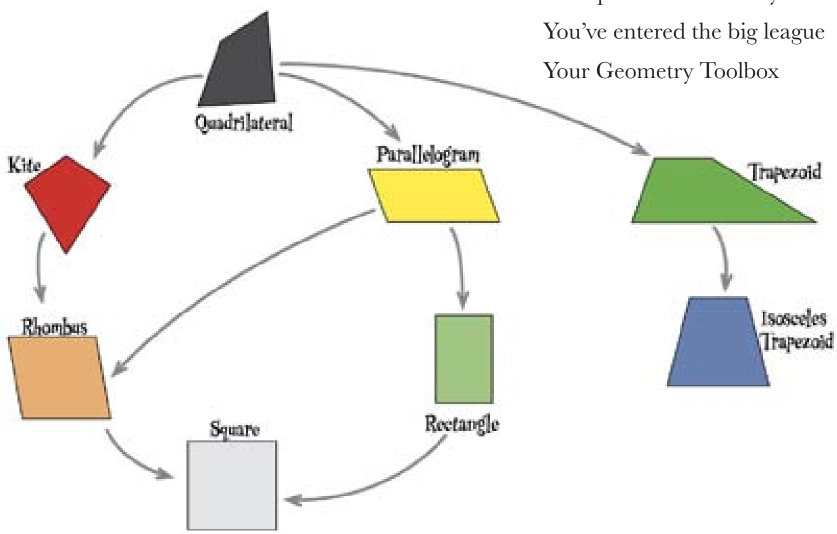
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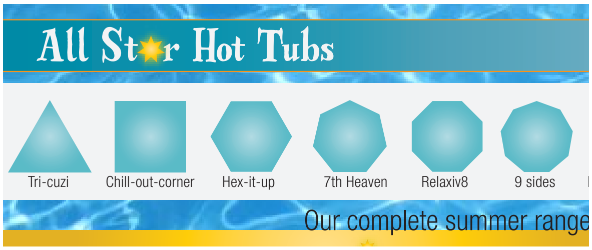




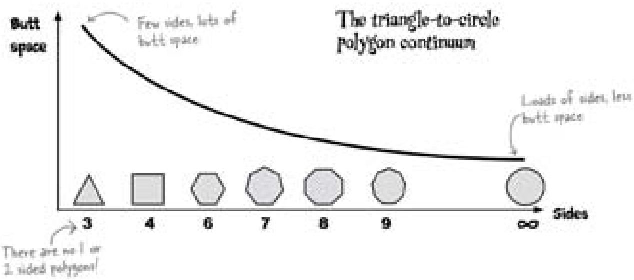
7 regular polygons

It's all shaping up

Want to have it your way? Life's full of compromises, but you don't have to be restricted to triangles, squares, and circles. **Regular polygons** give you the **flexibility** to demand exactly the shape you need. But don't think that means learning a list of new formulas: *you can treat 6-, 16-, and 60-sided shapes the same*. So, whether it's for your own creative project, some required homework that's due tomorrow, or the demands of an important client, you'll have the tools to **deliver exactly what you want**.



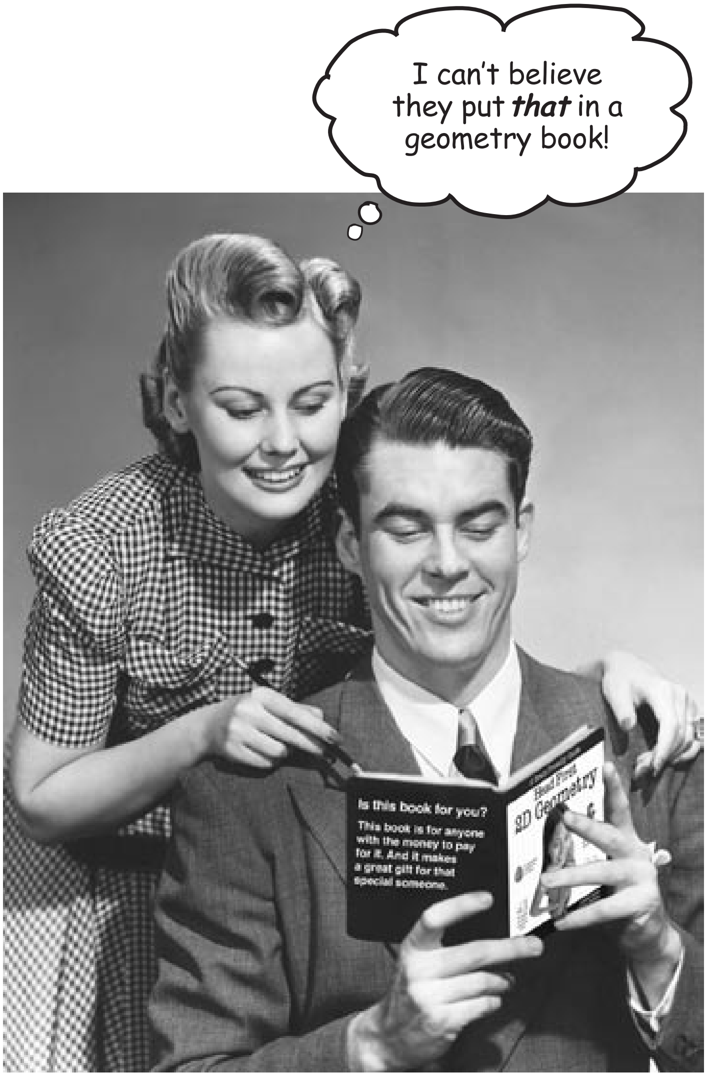




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how to use this book

Intro



In this section, we answer the burning question: "So why DID they put that in a geometry book?"

Who is this book for?

If you can answer "yes" to all of these…

1. Are you already pretty comfortable with algebra?

If not, check out Head First Algebra first!

2. Do you want to **learn**, **understand**, **remember**, and ***apply*** geometry concepts, and not just memorize formulas?

3. Do you prefer **fun, casual conversation** to **dry**, **dull**, **school lectures**?

…this book is for you.

Who should probably back away from this book?

If you can answer "yes" to any of these…

1. Are you **still struggling** with solving for unknowns in algebra?

If you can solve $3(x + 4) = 21$, you'll be fine

2. Are you afraid of sketching, drawing, and using your hands to figure things out?

3. Are you someone who'd rather just plug stuff into calculators or have someone give you the answers? Do you believe that a math book can't be serious if there's a rock concert in it?



…this book is not for you.

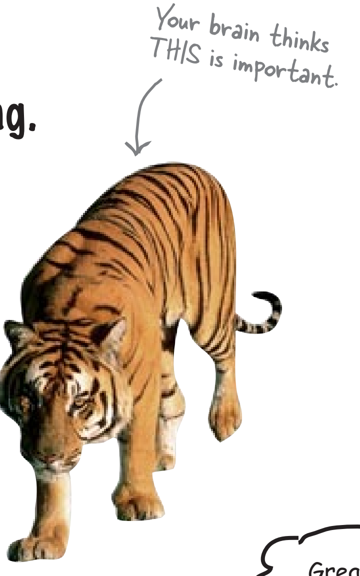
[Note from marketing: this book is for anyone with a credit card. Or cash. Cash is nice, too. —Ed]

We know what you're thinking.

"How can *this* be a serious geometry book?"

"What's with all the graphics?"

"Can I actually *learn* it this way?"



And we know what your *brain* is thinking.

Your brain craves novelty. It's always searching, scanning, *waiting* for something unusual. It was built that way, and it helps you stay alive.

So what does your brain do with all the routine, ordinary, normal things you encounter? Everything it *can* to stop them from interfering with the brain's *real* job—recording things that *matter*. It doesn't bother saving the boring things; they never make it past the "this is obviously not important" filter.

How does your brain *know* what's important? Suppose you're out for a day hike and a tiger jumps in front of you, what happens inside your head and body?

Neurons fire. Emotions crank up. *Chemicals surge.*

And that's how your brain knows....

This must be important! Don't forget it!

But imagine you're at home, or in a library. It's a safe, warm, tiger-free zone. You're studying. Getting ready for an exam. Or trying to learn some tough math thing that your teacher is going to test you on tomorrow.

Just one problem. Your brain's trying to do you a big favor. It's trying to make sure that this *obviously* non-important content doesn't clutter up scarce resources. Resources that are better spent storing the really *big* things. Like tigers. Like the danger of fire. Like how you should never again snowboard in shorts.

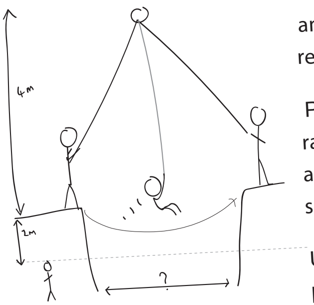
And there's no simple way to tell your brain, "Hey brain, thank you very much, but no matter how dull this book is, and how little I'm registering on the emotional Richter scale right now, I really *do* want you to keep this stuff around."



We think of a "Head First" reader as a learner.

So what does it take to *learn* something? First, you have to *get* it, then make sure you don't *forget* it. It's not about pushing facts into your head. Based on the latest research in cognitive science, neurobiology, and educational psychology, *learning* takes a lot more than text on a page. We know what turns your brain on.

Some of the Head First learning principles:



Make it visual. Images are far more memorable than words alone, and make learning much more effective (up to 89% improvement in recall and transfer studies). It also makes things more understandable.

Put the words within or near the graphics they relate to, rather than on the bottom or on another page, and learners will be up to *twice* as likely to solve problems related to the content.



Use a conversational and personalized style. In recent studies, students performed up to 40% better on post-learning tests if the content spoke directly to the reader, using a first-person, conversational style rather than taking a formal tone. Tell stories instead of lecturing. Use casual language. Don't take yourself too seriously. Which would *you* pay more attention to: a stimulating dinner party companion, or a lecture?

Get the learner to think more deeply. In other words, unless you actively flex your neurons, nothing much happens in your head. A reader has to be motivated, engaged, curious, and inspired to solve problems, draw conclusions, and generate new knowledge. And for that, you need challenges, exercises, and thought-provoking questions, and activities that involve both sides of the brain and multiple senses.

Get—and keep—the reader's attention. We've all had the "I really want to learn this but I can't stay awake past page one" experience. Your brain pays attention to things that are out of the ordinary, interesting, strange, eye-catching, unexpected. Learning a new, tough, technical topic doesn't have to be boring. Your brain will learn much more quickly if it's not.



Touch their emotions. We now know that your ability to remember something is largely dependent on its emotional content. You remember what you care about. You remember when you *feel* something. No, we're not talking heart-wrenching stories about a boy and his dog. We're talking emotions like surprise, curiosity, fun, "what the...?" , and the feeling of "I Rule!" that comes when you solve a puzzle, learn something everybody else thinks is hard, or realize you know something that "I'm more technical than thou" Bob from engineering *doesn't*.

Metacognition: thinking about thinking

If you really want to learn, and you want to learn more quickly and more deeply, pay attention to how you pay attention. Think about how you think. Learn how you learn.



Most of us did not take courses on metacognition or learning theory when we were growing up. We were *expected* to learn, but rarely *taught* to learn.

But we assume that if you're holding this book, you really want (or need) to learn about geometry. And you probably don't want to spend a lot of time. And since you're going to have to use this stuff in the future, you need to *remember* what you read. And for that, you've got to *understand* it. To get the most from this book, or *any* book or learning experience, take responsibility for your brain.
Your brain on *geometry*.

The trick is to get your brain to see the new material you're learning as Really Important. Crucial to your well-being. As important as a tiger. Otherwise, you're in for a constant battle, with your brain doing its best to keep the new content from sticking.

So just how *DO* you get your brain to think that geometry is a hungry tiger?

There's the slow, tedious way, or the faster, more effective way. The slow way is about sheer repetition. You obviously know that you *are* able to learn and remember even the dullest of topics if you keep pounding the same thing into your brain. With enough repetition, your brain says, "This doesn't *feel* important to him, but he keeps looking at the same thing *over* and *over* and *over*, so I suppose it must be."

The faster way is to do ***anything that increases brain activity,*** especially different *types* of brain activity. The things on the previous page are a big part of the solution, and they're all things that have been proven to help your brain work in your favor. For example, studies show that putting words *within* the pictures they describe (as opposed to somewhere else in the page, like a caption or in the body text) causes your brain to try to makes sense of how the words and picture relate, and this causes more neurons to fire. More neurons firing = more chances for your brain to *get* that this is something worth paying attention to, and possibly recording.

A conversational style helps because people tend to pay more attention when they perceive that they're in a conversation, since they're expected to follow along and hold up their end. The amazing thing is, your brain doesn't necessarily *care* that the "conversation" is between you and a book! On the other hand, if the writing style is formal and dry, your brain perceives it the same way you experience being lectured to while sitting in a roomful of passive attendees. No need to stay awake.

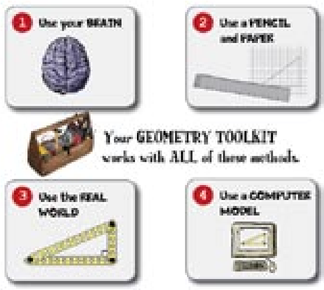
But pictures and conversational style are just the beginning.

Here's what WE did:



We used ***pictures***, because your brain is tuned for visuals, not text. As far as your brain's concerned, a picture really *is* worth a thousand words. And when text and pictures work together, we embedded the text *in* the pictures because your brain works more effectively when the text is *within* the thing the text refers to, as opposed to in a caption or buried in the text somewhere.

We used ***redundancy***, saying the same thing in *different* ways and with different media types, and *multiple senses*, to increase the chance that the content gets coded into more than one area of your brain.



We used concepts and pictures in ***unexpected*** ways because your brain is tuned for novelty, and we used pictures and ideas with at least *some* ***emotional*** *content*, because your brain is tuned to pay attention to the biochemistry of emotions. That which causes you to *feel* something is more likely to be remembered, even if that feeling is nothing more than a little ***humor***, ***surprise***, or ***interest.***

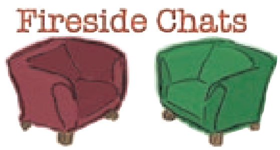
We used a personalized, ***conversational style***, because your brain is tuned to pay more attention when it believes you're in a conversation than if it thinks you're passively listening to a presentation. Your brain does this even when you're *reading*.

We included loads of ***activities***, because your brain is tuned to learn and remember more when you ***do*** things than when you *read* about things. And we made the exercises challenging-yet-do-able, because that's what most people prefer.



We used ***multiple learning styles***, because *you* might prefer step-by-step procedures, while someone else wants to understand the big picture first, and someone else just wants to see an example. But regardless of your own learning preference, *everyone* benefits from seeing the same content represented in multiple ways.

We include content for ***both sides of your brain***, because the more of your brain you engage, the more likely you are to learn and remember, and the longer you can stay focused. Since working one side of the brain often means giving the other side a chance to rest, you can be more productive at learning for a longer period of time.



And we included ***stories*** and exercises that present ***more than one point of view,*** because your brain is tuned to learn more deeply when it's forced to make evaluations and judgments.

We included ***challenges***, with exercises, and by asking ***questions*** that don't always have a straight answer, because your brain is tuned to learn and remember when it has to *work* at something. Think about it—you can't get your *body* in shape just by *watching* people at the gym. But we did our best to make sure that when you're working hard, it's on the *right* things. That ***you're not spending one extra dendrite*** processing a hard-to-understand example, or parsing difficult, jargon-laden, or overly terse text.



We used ***people***. In stories, examples, pictures, etc., because, well, because *you're* a person. And your brain pays more attention to *people* than it does to *things*.



Here's what YOU can do to bend your brain into submission

So, we did our part. The rest is up to you. These tips are a starting point; listen to your brain and figure out what works for you and what doesn't. Try new things.

Cut this out and stick it on your refrigerator.

---

1. **Slow down. The more you understand, the less you have to memorize.**

   Don't just *read*. Stop and think. When the book asks you a question, don't just skip to the answer. Imagine that someone really *is* asking the question. The more deeply you force your brain to think, the better chance you have of learning and remembering.

2. **Do the exercises. Write your own notes.**

   We put them in, but if we did them for you, that would be like having someone else do your workouts for you. And don't just *look* at the exercises. **Use a pencil.** There's plenty of evidence that physical activity *while* learning can increase the learning.

3. **Read the "There are No Dumb Questions"**

   That means all of them. They're not optional sidebars—***they're part of the core content!*** Don't skip them.

4. **Make this the last thing you read before bed. Or at least the last challenging thing.**

   Part of the learning (especially the transfer to long-term memory) happens *after* you put the book down. Your brain needs time on its own, to do more processing. If you put in something new during that processing time, some of what you just learned will be lost.

5. **Drink water. Lots of it.**

   Your brain works best in a nice bath of fluid. Dehydration (which can happen before you ever feel thirsty) decreases cognitive function.

6. **Talk about it. Out loud.**

   Speaking activates a different part of the brain. If you're trying to understand something, or increase your chance of remembering it later, say it out loud. Better still, try to explain it out loud to someone else. You'll learn more quickly, and you might uncover ideas you hadn't known were there when you were reading about it.

7. **Listen to your brain.**

   Pay attention to whether your brain is getting overloaded. If you find yourself starting to skim the surface or forget what you just read, it's time for a break. Once you go past a certain point, you won't learn faster by trying to shove more in, and you might even hurt the process.

8. **Feel something!**

   Your brain needs to know that this *matters*. Get involved with the stories. Make up your own captions for the photos. Groaning over a bad joke is *still* better than feeling nothing at all.

9. **Create something!**

   Pick up a model kit or some wood and tools and make something really cool! Or work out something you will build one day when you have the time and money. All you need is a pencil and a problem to solve…a problem that might benefit from using the tools and techniques you're studying to get geometry.

Read Me

This is a learning experience, not a reference book. We deliberately stripped out everything that might get in the way of learning whatever it is we're working on at that point in the book. And the first time through, you need to begin at the beginning, because the book makes assumptions about what you've already seen and learned.

We don't follow a regular school syllabus.

We couldn't cover every single element of the syllabus so we paid attention to what questions our own brains were asking, asked students what they found tricky, and we included extra things which allow you to find patterns that link the learning together because ***your brain loves patterns.***

So, if you're going to need to pass a test, then you'll also need a reference book that covers the syllabus for that test, but don't worry. We've picked out the trickiest and most interesting parts in this book, and we've emphasized *understanding* geometry so you should be in great shape to slot those extra details into place quickly.

We don't drag you through formal proofs of new concepts.

If you're doing high school geometry you'll probably be familiar with—and possibly terrified of—geometry proofs. ***There are no formal proofs in this book.*** We believe that, for most people, proofs make learning geometry harder than it needs to be. Instead, we've used visual exercises to explore patterns and general rules in ways that we are confident that you'll remember and even be able to show other people.

We're working on another book in this mini-series that will handle all that formal logic and proof stuff, but for now you're in great shape if you understand geometry in the real world first.

This is just about two-dimensional (2D) geometry.

We promise it's not just so we can sell you another book called *Head First 3D Geometry* soon! We've covered many of the most important techniques you'll use when working in two dimensions, so you're all set for exploring further dimensions at the end of this book. In fact, we've even snuck in a couple of 3D problems that you can work in 2D, because geometry is about *solving interesting problems in the real world,* not just on paper.

We use plain English and not geometry jargon.

We believe your brain needs to see what something is, and figure out ***why you would even care about it***, before you can give it an unfamiliar label. We do use the geometry jargon you'll need to know for tests from time to time, but not until we're sure you'll know what we're talking about. We encourage you to *use real words to describe patterns* and not sweat the official formulas too much.

We don't consider this to be the end of our conversation with you.

Come and talk to us at *www.headfirstlabs.com/geometry*. If there's something we didn't cover that's really puzzling you, then throw us a question and we'll see if we can help you with a Head First style way of figuring it out. Of course we won't do the work for you, which is why...

...the activities are NOT optional.

The exercises and activities are not add-ons; they're part of the core content of the book. Some of them are to help with memory, some are for understanding, and some will help you apply what you've learned. ***Don't skip the exercises.*** The geometry investigations are particularly important; they'll help you discover how your brain likes to figure stuff out—known as logic in formal geometry proofs that come much later (not in this book). They also give your brain a chance to hook in to the geometry that is all around you in the real world.

The redundancy is intentional and important.

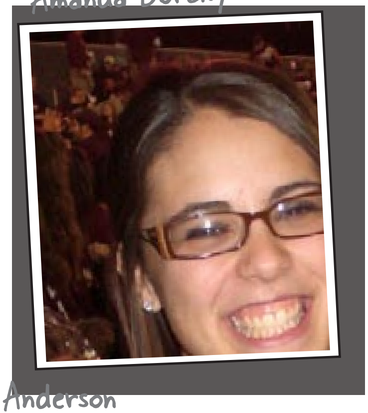
One distinct difference in a Head First book is that we want you to *really* get it. And we want you to finish the book remembering what you've learned. Most reference books don't have retention and recall as a goal, but this book is about *learning*, so you'll see some of the same concepts come up more than once.

The Brain Power exercises don't have answers.

For some of them, there is no right answer, and for others, part of the learning experience of the Brain Power activities is for you to decide if and when your answers are right. In some of the Brain Power exercises, you will find hints to point you in the right direction.

The technical review team

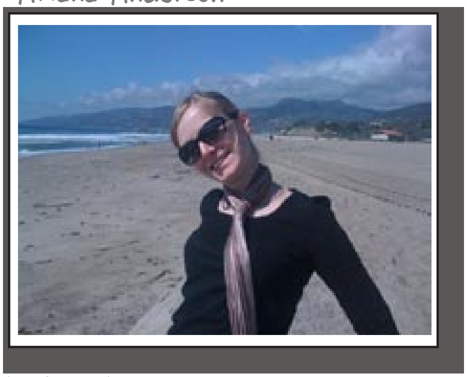
Amanda Borcky

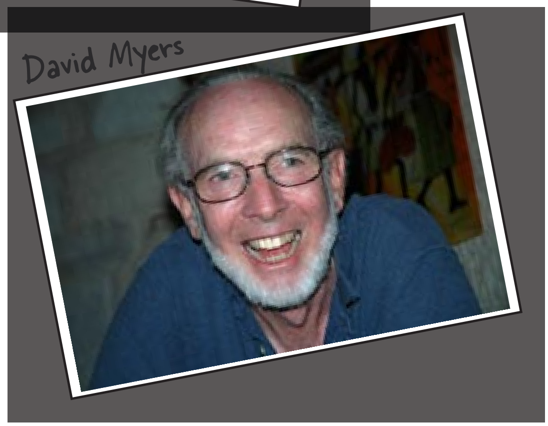


Herbert Tracey



Ariana Anderson





Technical Reviewers:

For this book we had an amazing group, many of whom have reviewed other Head First books in the past. They did a fantastic job, and we're really grateful that they keep coming back for more!

Amanda Borcky is a student at Virginia Tech in Blacksburg, Virginia. She is studying nutrition with plans of getting a second degree in nursing. This is her second time reviewing for the Head First series.

David Myers taught college and high school math for 36 years. Mostly for fun, he collaborated on several math and programming textbooks in the '80s and '90s. Since retiring in 2006 from a long tenure at The Winsor School in Boston, MA, he has been delighted to start a new completely-for-fun career as a volunteer at his Quaker Meeting and in prison-related activities.

Ariana Anderson is a statistician working on "reading" brain scans at the Center for Cognitive Neuroscience at UCLA. She got her PhD from UCLA and her bachelor's from UCLA, but was forced to go to high school elsewhere.

Herbert Tracey received his BS from Towson University and a MS from Johns Hopkins University. Currently, he is an instructor of mathematical sciences at Loyola University Maryland and served as department chair of mathematics (retired) at Hereford High School.

Jonathan Schofield graduated in civil engineering from University of Bradford in the UK, where he works "with water." He provided the essential final pass to quadruple-check the numbers in the book.

Acknowledgments

Our editors:

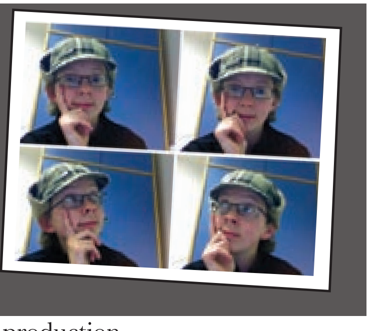
Thanks to **Courtney Nash**, who supported our efforts to think beyond the conventional curriciulum and took time to wonder with us about what the learner's brain wanted to know next. She consistently pushed us toward the bigger picture—and it's a much better book for that. Extra props to her for googling our odd British phrases and sayings to find U.S.-friendly translations!





And to **Brett McLaughlin**, who started us off on this book, and provided some really kick-ass training on the Head First way and why it rules the world. Also to the folk who rocked training in Boston 2008 and added so much to the experience: Lou Barr, Elizabeth Robson, John Guenin, Edward Ocampo-Gooding, Aaron Glimme, Dave Sussman, Les Hardin, David Flatley and Tracey Pilone. (We heard everybody named in person buys a copy….)

Our artworker and brain-fuzz-detector:



To **Badger** (Jennie Routley), for the initial "you should write one of those…I could do the pictures!", through to many hours of genius graphics work. Also for never holding back on telling us that some algebra was "disgusting" when we'd crossed the line.

The O'Reilly team:

To **Karen Shaner**, who handled the tech-review process, and provided a pep talk when the comments first started coming. To our production editor, **Rachel Monaghan**, for being patient about the fact that we mistyped "Length" eight times in one chapter. To **Lou Barr**, for her genius Head First template. And to **Scott DeLugan** and **Sanders Kleinfeld**, for once again going above and beyond to get the book out.

Lindsey's friends and family:

To **Helen**, for understanding that when I say, "I'll be there in a minute…I'm almost done…", it's an expression of hope and not reality. And for accommodating a year of lost weekends and working-on-vacation, and never, ever being intolerant of yet another conversation about triangles. And to **Joe**—my brilliant, dyspraxic stepson—for being our "learning differences" guinea pig.

Dawn's friends and family:

Work on this book would have been lot harder without my amazing support network of family and friends. Special thanks go to David, Mum and Dad, Carl, Steve, Gill, and Paul. I've truly appreciated all your support and encouragement.

And finally: To **Bert Bates** and **Kathy Sierra**, for creating the series that changed our lives.

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1 finding missing angles

Reading between the lines



Ever get the feeling there's something they're not telling you?

If you want to master the **real world**, you need to get geometry. It's a set of **tools** for turning a little bit of information into a ***complete picture***. Whether you want to design something, build something or find out how a situation really went down, geometry can make sure you've always got **the lowdown**. So if you want to keep in the loop, grab your hat, pack your pencil, and join us on the bus to Geometryville.

There's been a homicide

And the number one suspect, Benny, is firmly behind bars. To the officers it looked like an open and shut case but Benny is still claiming he's innocent.





Crime scene photo

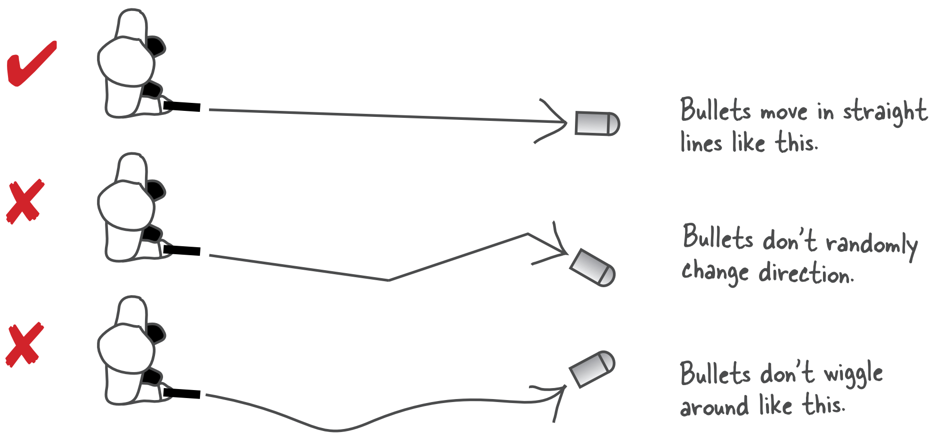
There are very few clues to go on, so the CSIs are relying on you to work up the only solid clues they've got: the ballistics evidence.

In the ballistics lab you've got to cover all the angles

As the ballistics investigator, your mission is to work out what happened to the bullet between when it was fired from the gun and when it stopped moving, and whether that ties up with what the investigators suspect—that Benny shot Micky.

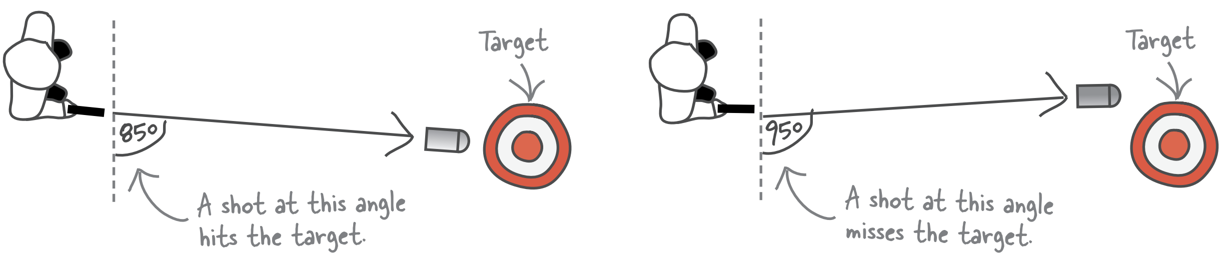
Bullets travel in straight lines

The whole basis of your investigation is something that all bullets have in common: **they travel in straight lines**.



Angles matter

Angles are formed where straight lines meet. Because bullets move in straight lines, *taking aim* really means deciding what the **angle** is between you and the target. The angle of the shot determines whether the bullet hits or misses the target.

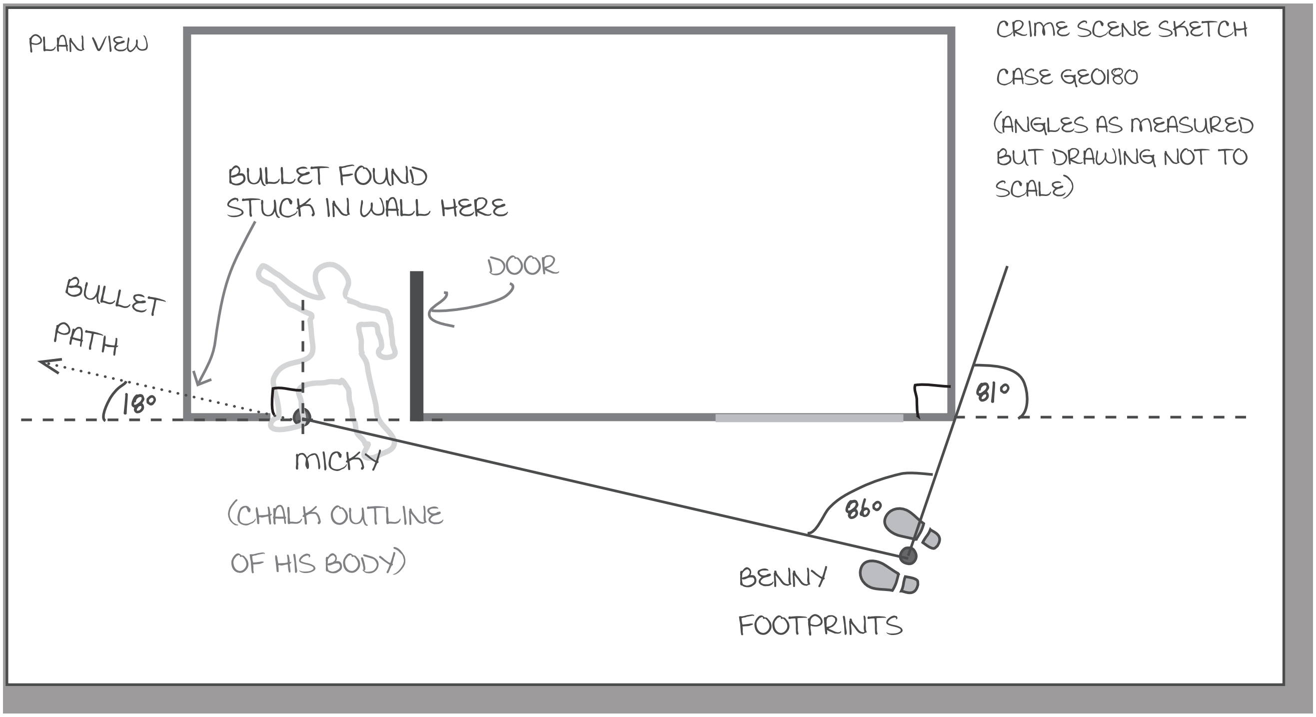


The case rests upon you finding the answer to just one question...

Do the angles between Benny, Micky, and the bullet match up?

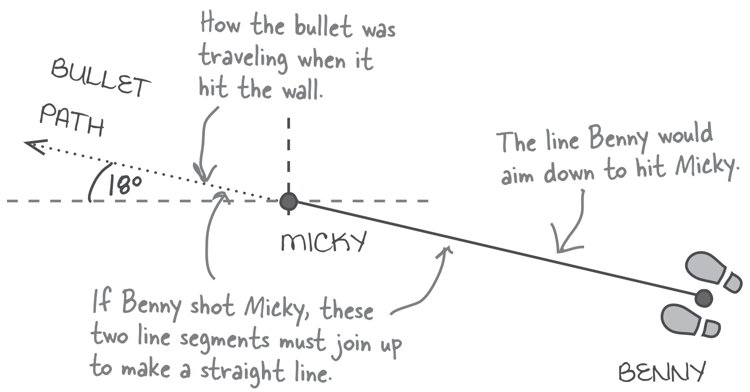
At the crime scene, the investigators took some measurements and sketched the positions of Benny, Micky's body, and the point where they found the bullet in the wall, all relative to the building where the crime took place.

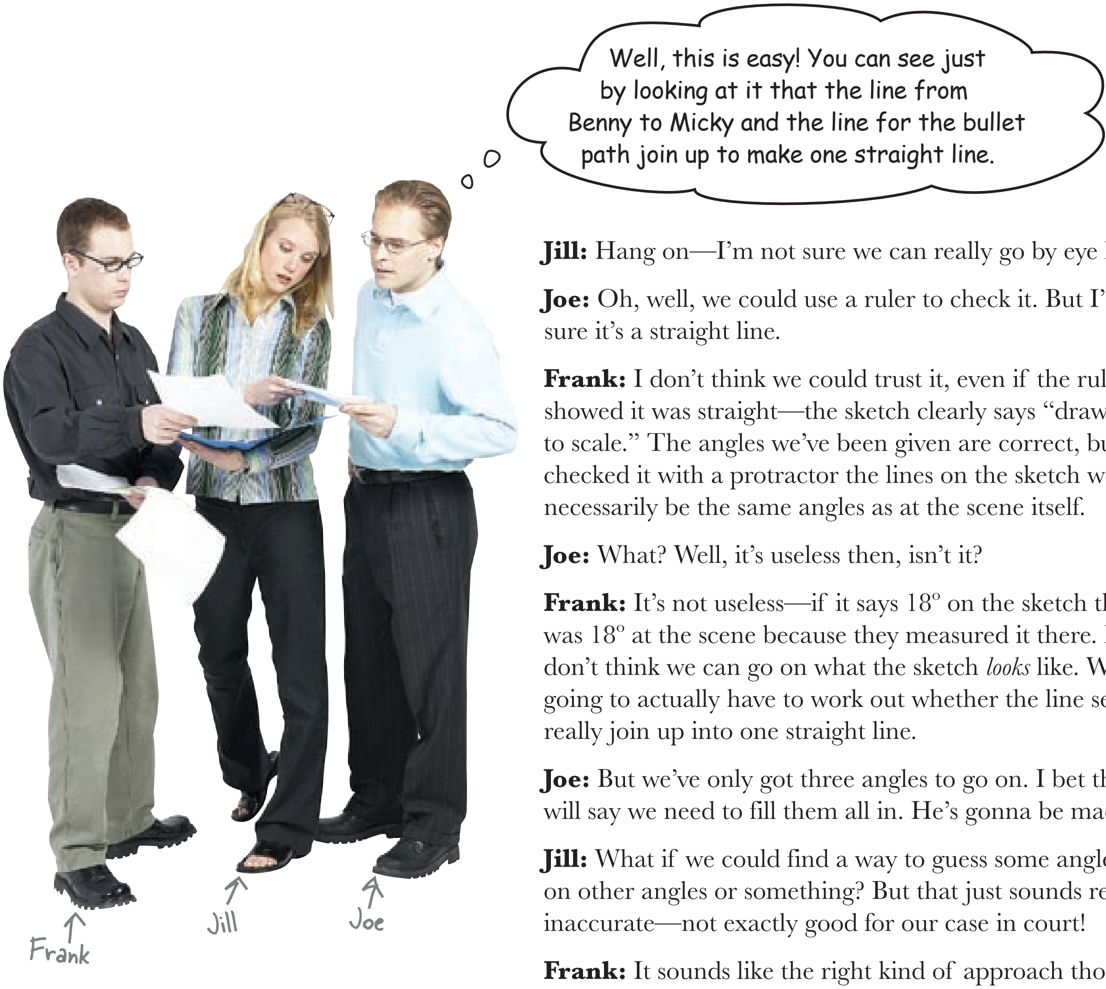
Top-down view of the crime scene



They must have been busy that day, because they didn't measure everything! But, they found one important detail: the bullet entered the wall at an angle of 18º through the front wall.

So, to solve the crime, all you have to do is prove that from where Benny was standing to the target (Micky), the bullet would have traveled in a straight line that joins up perfectly with the bullet path.





Jill: Hang on—I'm not sure we can really go by eye like that!

Joe: Oh, well, we could use a ruler to check it. But I'm pretty sure it's a straight line.

Frank: I don't think we could trust it, even if the ruler showed it was straight—the sketch clearly says "drawing not to scale." The angles we've been given are correct, but if you checked it with a protractor the lines on the sketch wouldn't necessarily be the same angles as at the scene itself.

Joe: What? Well, it's useless then, isn't it?

Frank: It's not useless—if it says 18° on the sketch then it was 18° at the scene because they measured it there. But I don't think we can go on what the sketch *looks* like. We're going to actually have to work out whether the line segments really join up into one straight line.

Joe: But we've only got three angles to go on. I bet the chief will say we need to fill them all in. He's gonna be mad.

Jill: What if we could find a way to guess some angles based on other angles or something? But that just sounds really inaccurate—not exactly good for our case in court!

Frank: It sounds like the right kind of approach though. And anyway, I think they've measured five angles, not just three.…

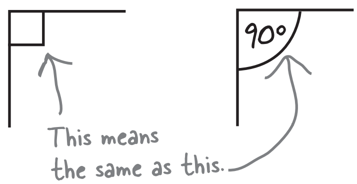
BRAIN POWER

Does the sketch tell you five angles, or just three?

Is there a way you could start to find some of the angles you haven't been given on the sketch?

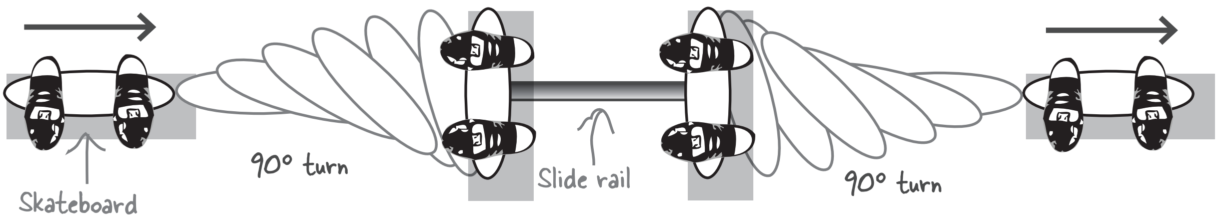
Right angles aren't always marked with numbers

A right angle is an internal angle (the smallest angle) between two lines equal to 90º. Instead of drawing a curve, like we do for most angles, we mark it with a square corner.



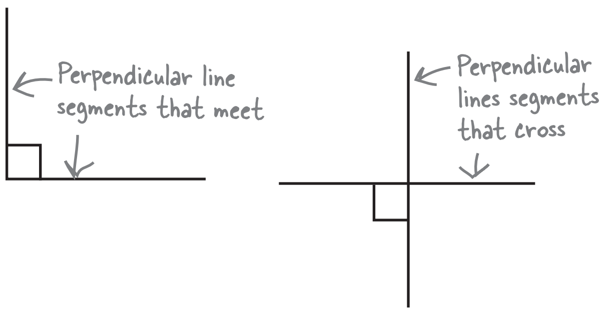
A right angle is a quarter turn—like the angle between the hands on the clock at 3 o'clock or the amount you have to turn a skateboard to get into and out of a boardslide without landing on your face.

It is still possible to land on your face even if you make the 90° turn.



Lines that meet at a right angle are perpendicular

Whether they actually cross each other or only meet at a point, two lines that form a right angle are called **perpendicular** lines. We could also say that one of the lines is **perpendicular to** the other.



So it turns out that we do have five angles on the sketch after all.

Although they don't say "90º" on them, the little square angle marks tell us that some of the angles are right angles.

But what about all those other angles that aren't marked on the sketch?

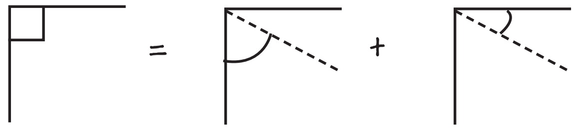


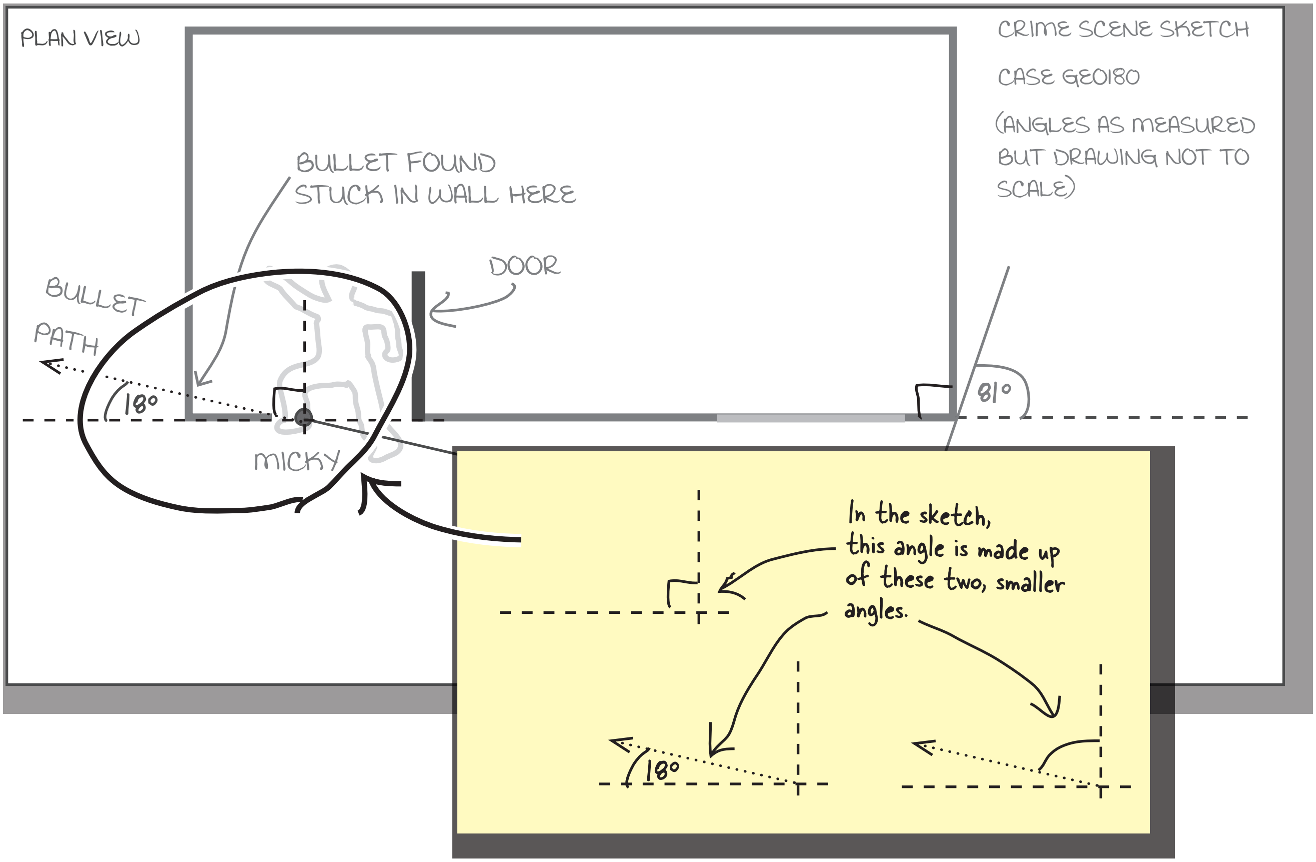
Line segments are parts of lines

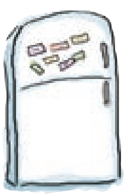
In geometry jargon, a ***line*** goes on forever in both directions. A ***line segment*** is just a bit of a line with a start and an end.

Angles can be made up of other, smaller angles

If you cut up an angle into pieces, the smaller angles add up to the original angle.

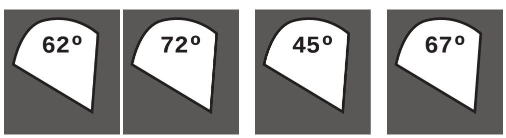


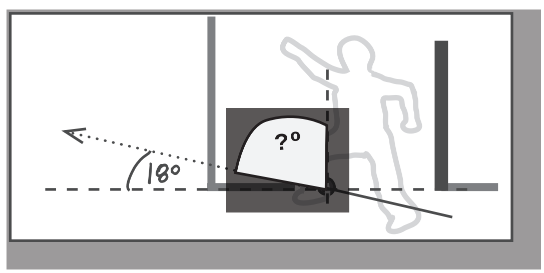




Angle Magnets

Which of these angles matches the mystery angle?

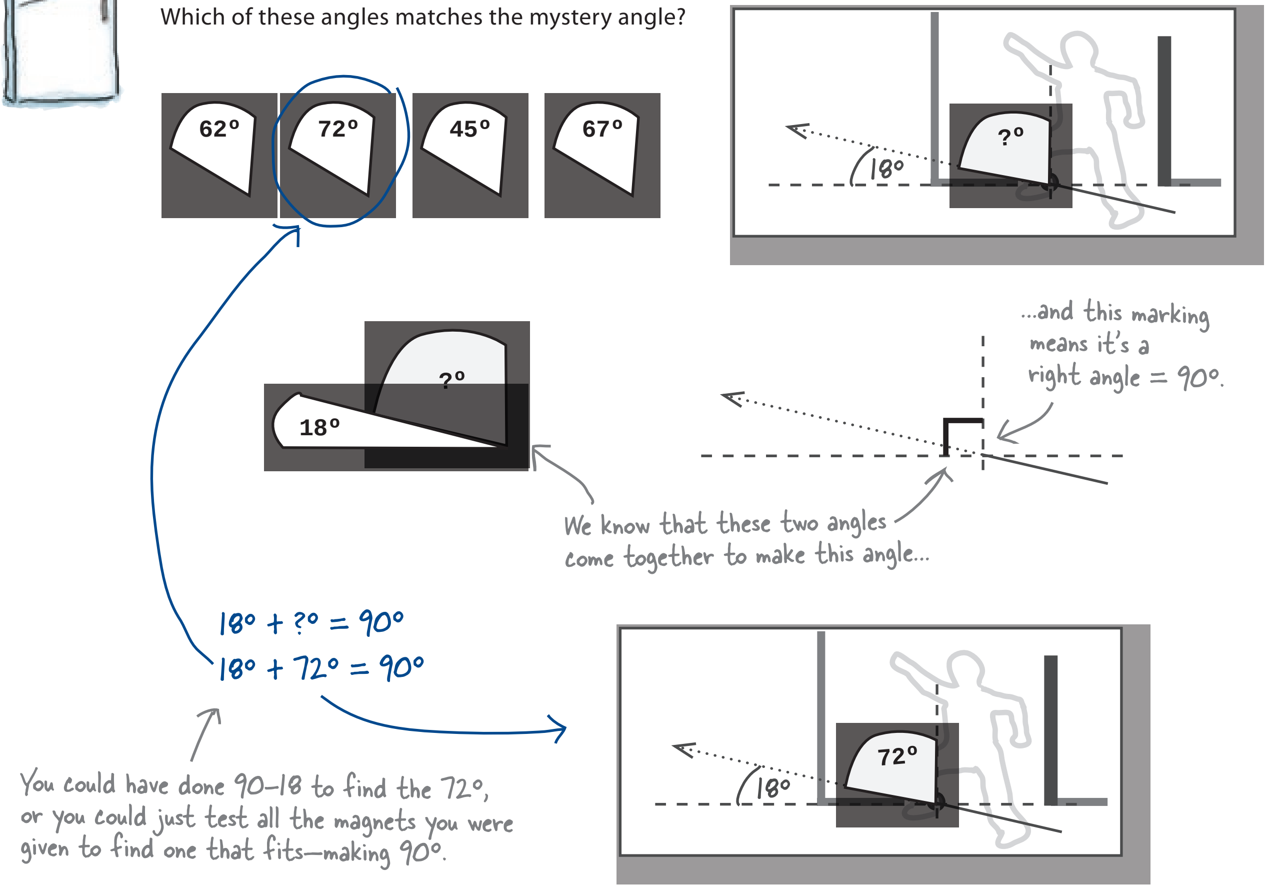






Angle Magnets Solution

Which of these angles matches the mystery angle?



there are no Dumb Questions

Q: I checked that mystery angle with a protractor and it wasn't 72°—so how come you're saying it is?

A: In geometry, unless you're specifically told to measure an angle, assume that the *drawing* isn't accurate, but that the *numbers* on the sketch are. We **calculate** missing angles rather than **measure** them.

Q: What about all the other lines on the diagram? Do we know if they're straight?

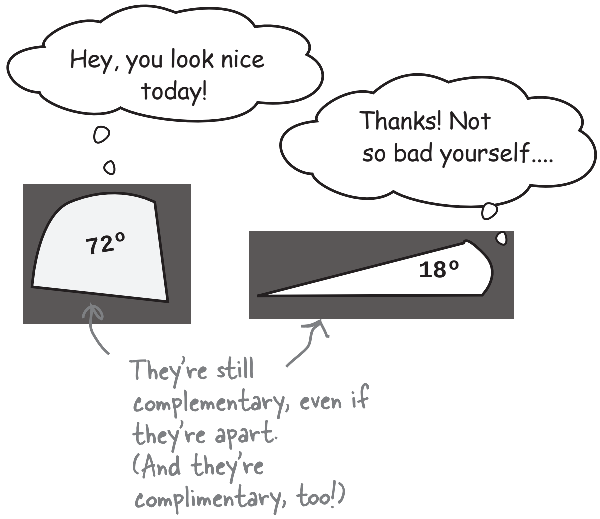
A: The other line segments on the sketch represent things like walls or the path between two points. They're all definitely straight. In fact, the two line segments we're interested in—the bullet path and the path from Benny to Micky—are also straight. What we need to find out is whether they join up to form one single straight line.

Complementary angles always add up to a right angle (90°)

If two angles combine to creat a right angle we call them *complementary angles*. Usually, complementary angles are adjacent (next to each other), but they can be any two angles anywhere that add up to 90 degrees.

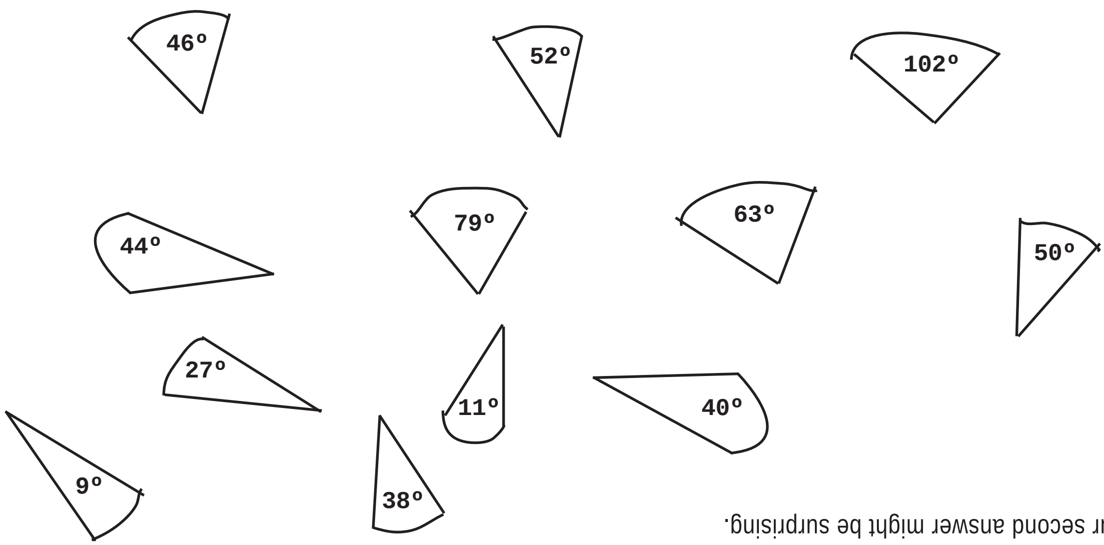
We say that the angles **complement** each other.







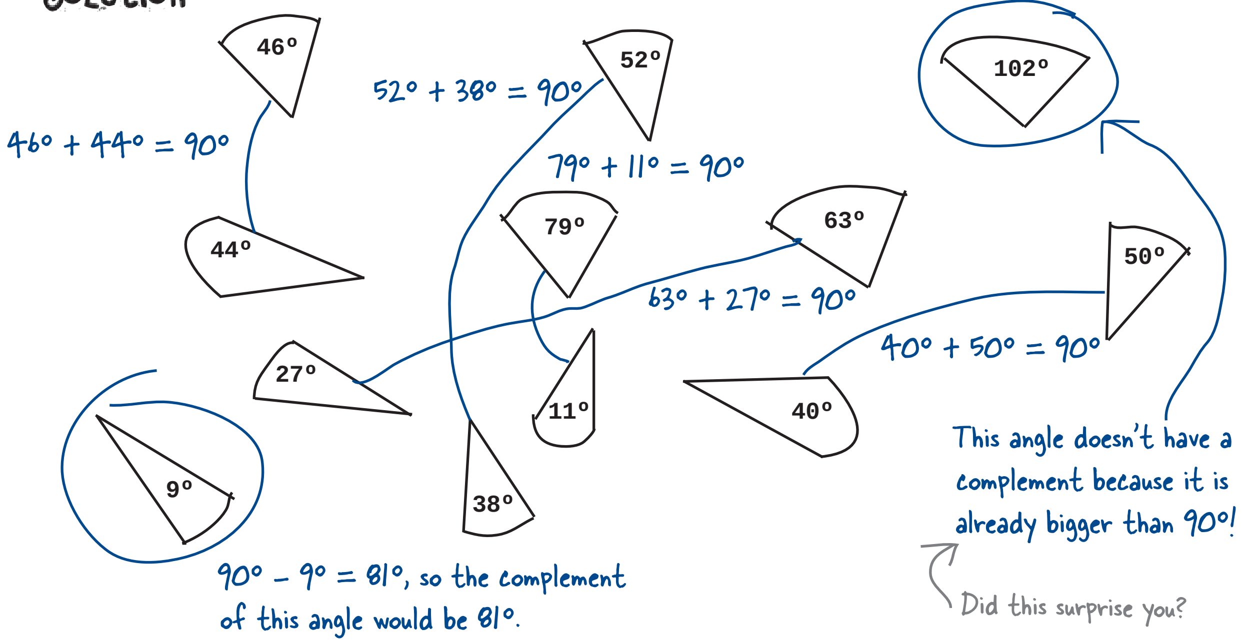
Draw lines to connect up five pairs of complementary angles.
You'll have two lonely single angles left over. Can you work out their complements?



Hint: Your second answer might be surprising.

Exercise Solution

Draw lines to connect up five pairs of complementary angles.
You'll have two lonely single angles left over. Can you work out their complements?



there are no Dumb Questions

Q: How can I tell which angles do and don't have a complement?

A: Only **acute** angles—angles less than 90º—can be complementary. An **obtuse** angle is already greater than 90º on its own.

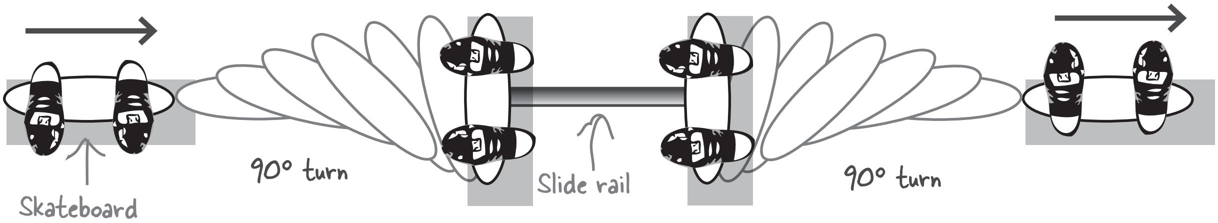
Q: Why doesn't 102º have a negative complement of –12º? Surely that would add up to 90º?

A: Complements can never be negative. There's no real reason for this except that the term "complementary" means two positive angles which add up to 90º.

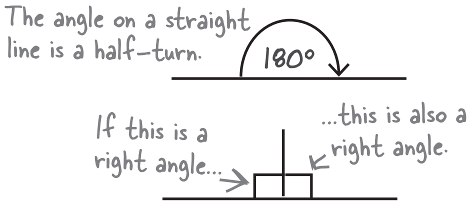
Only acute angles can be complementary.

Right angles often come in pairs

A right angle is a quarter turn, so two right angles add up to one half turn—a 180° on your skateboard, assuming you land fakey (facing the opposite way to when you started).



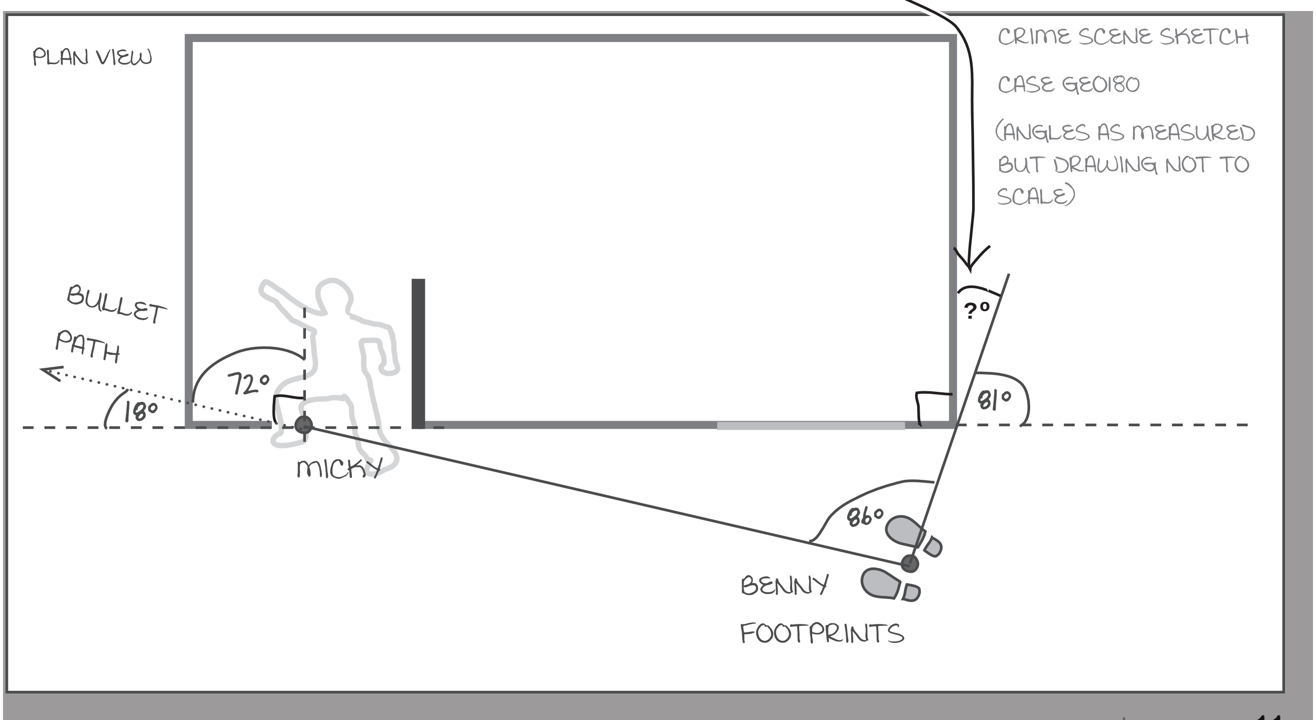
Although it doesn't look like one, a straight line is still an angle—exactly a half turn, or two quarter turns.



Two right angles sit together on a straight line.

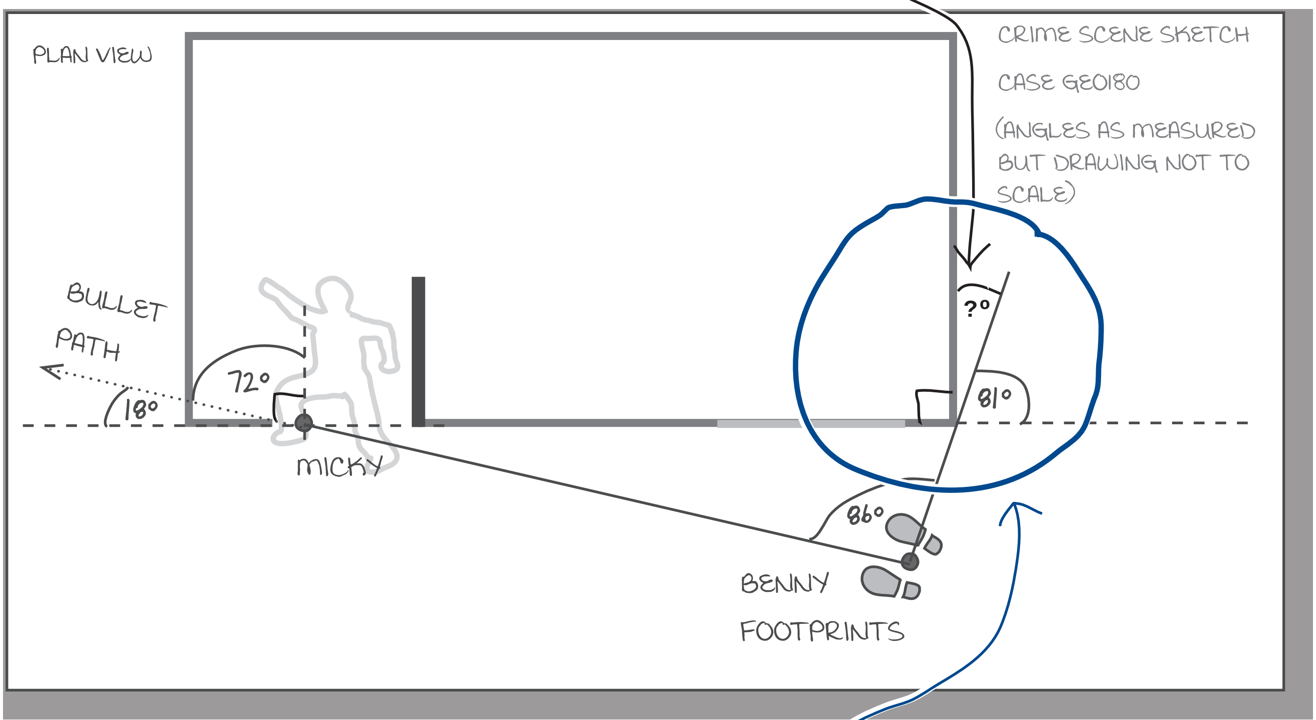
Sharpen your pencil

Find the next mystery angle here...

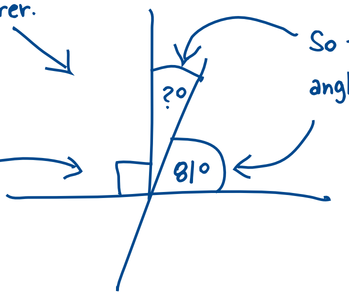


Sharpen your pencil Solution

Find the next mystery angle here…



Often drawing a mini-sketch of just the part you're working out makes things much clearer.

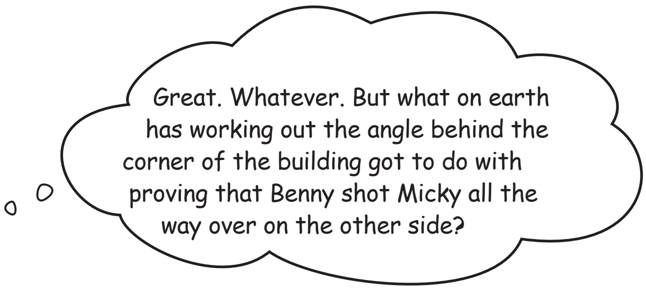


This is a right angle.

So the total angle made by these two angles must also be a right angle.

$? + 81° = 90°$

$? = 9°$





Often you need to solve the puzzle piece by piece.

Each individual angle on the sketch doesn't tell you much, but together they make up an accurate picture that shows you how every part fits in relation to the other parts.

It would be great to jump straight to the most important angle—if you even knew what that was—but usually we need to find a bunch of less exciting stuff to help us get there.

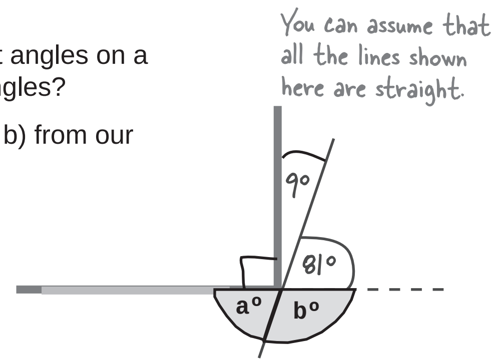
It's like any other puzzle solving really—you're not always fighting the end-of-level boss, sometimes you're just collecting the power-ups you need to beat him/her/it when you get there!



BRAIN BARBELL

We know that you can always fit a pair of right angles on a straight line, but what if we don't have right angles?

What might this pair of mystery angles (a and b) from our crime scene sketch add up to?

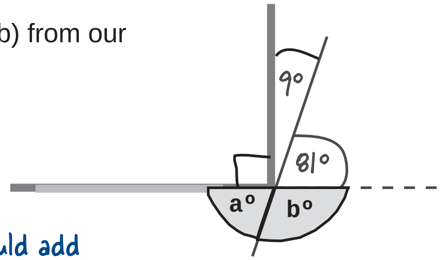


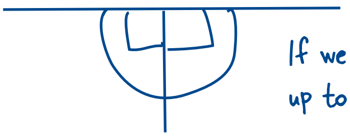


Brain Barbell Solution

We know that you can always fit a pair of right angles on a straight line, but what if we don't have right angles?

What might this pair of mystery angles (a and b) from our crime scene sketch add up to?





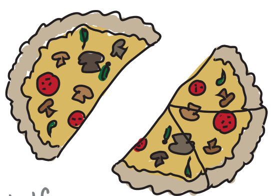
If we had a pair of right angles, they would add up to $90^\circ + 90^\circ = 180^\circ$.

Our pair of mystery angles make up the same total angle as a pair of right angles, so $a^\circ + b^\circ = 180^\circ$.

Angles on a straight line add up to 180°

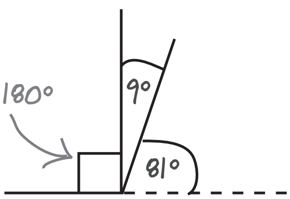
Whether a half turn is made up of two quarter turns or lots of different small turns, the total angle of a half turn is always 180°.

This means that when the angle on a straight line is divided up into smaller angles we can always be sure that they add up to 180°.



Half a pizza is half a pizza, no matter how you slice it.

$90^\circ + 9^\circ + 81^\circ = 180^\circ$

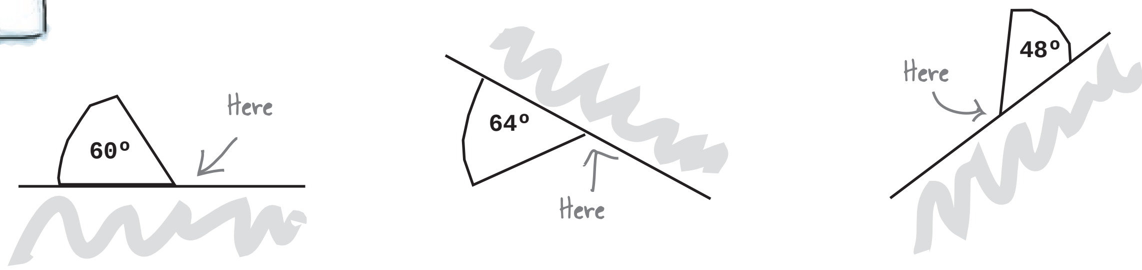


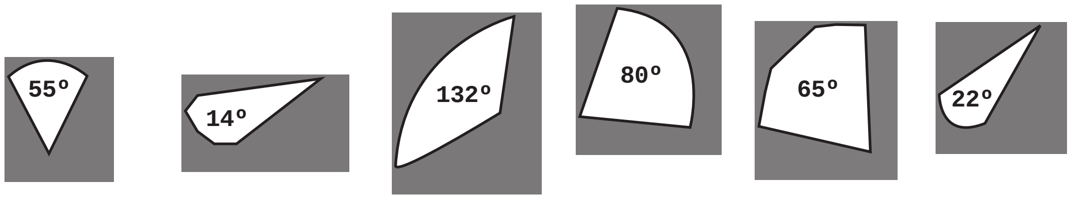


Angle Magnets

Fit the loose angle magnets into the gaps to form three complete straight lines. Some gaps might need more than one magnet to fill them.

You might need to try a few different combinations, so don't be shy!

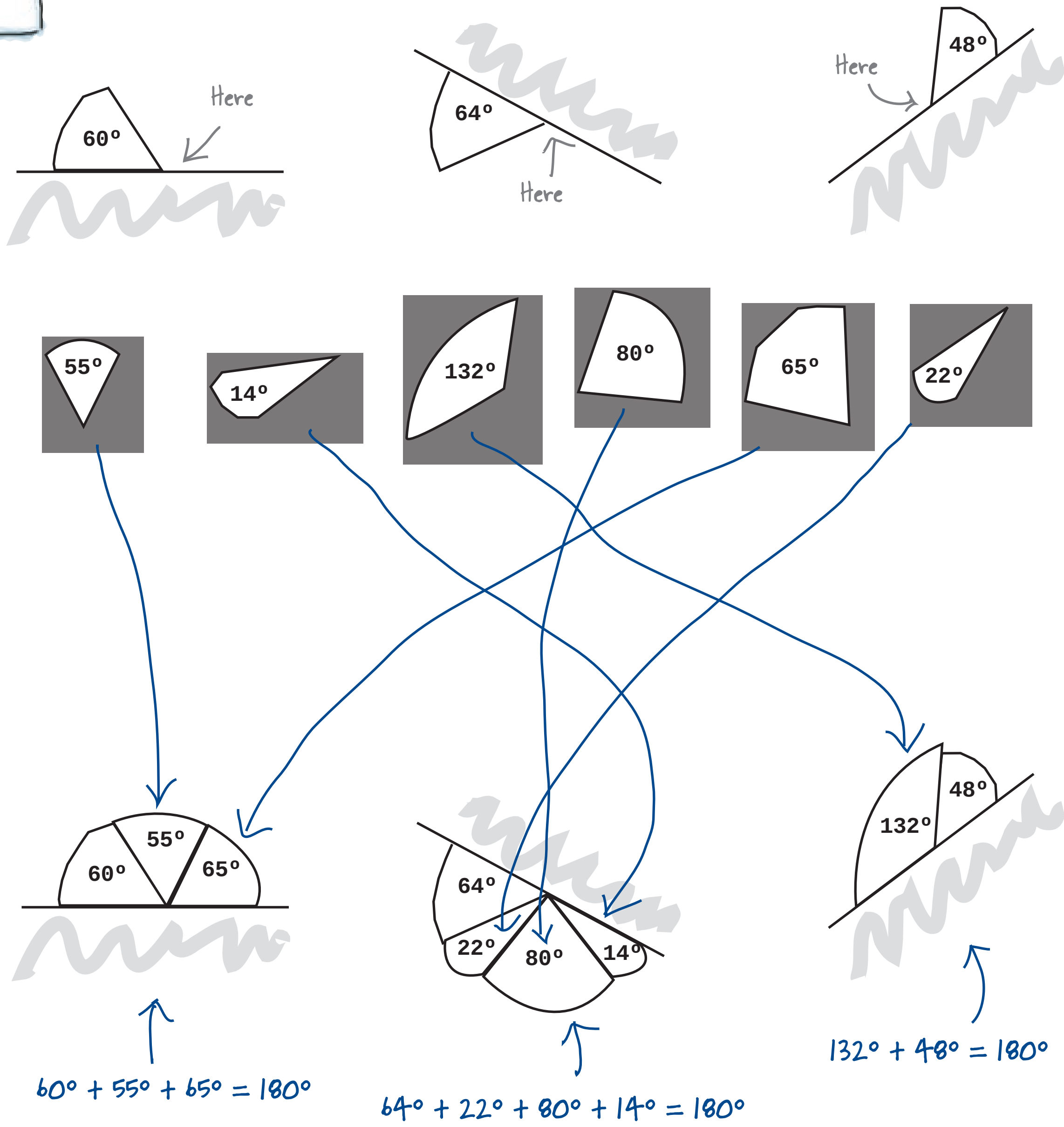






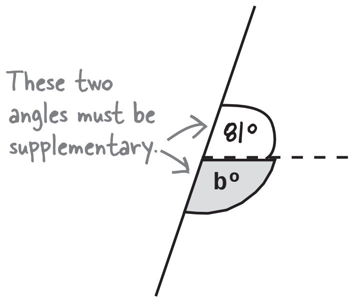
Angle Magnets Solution

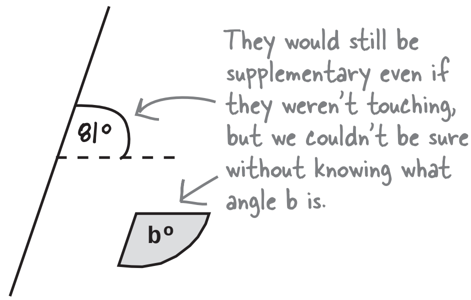
Fit the loose angle magnets into the gaps to form three complete straight lines.



Pairs of angles that add up to 180° are called supplementary angles

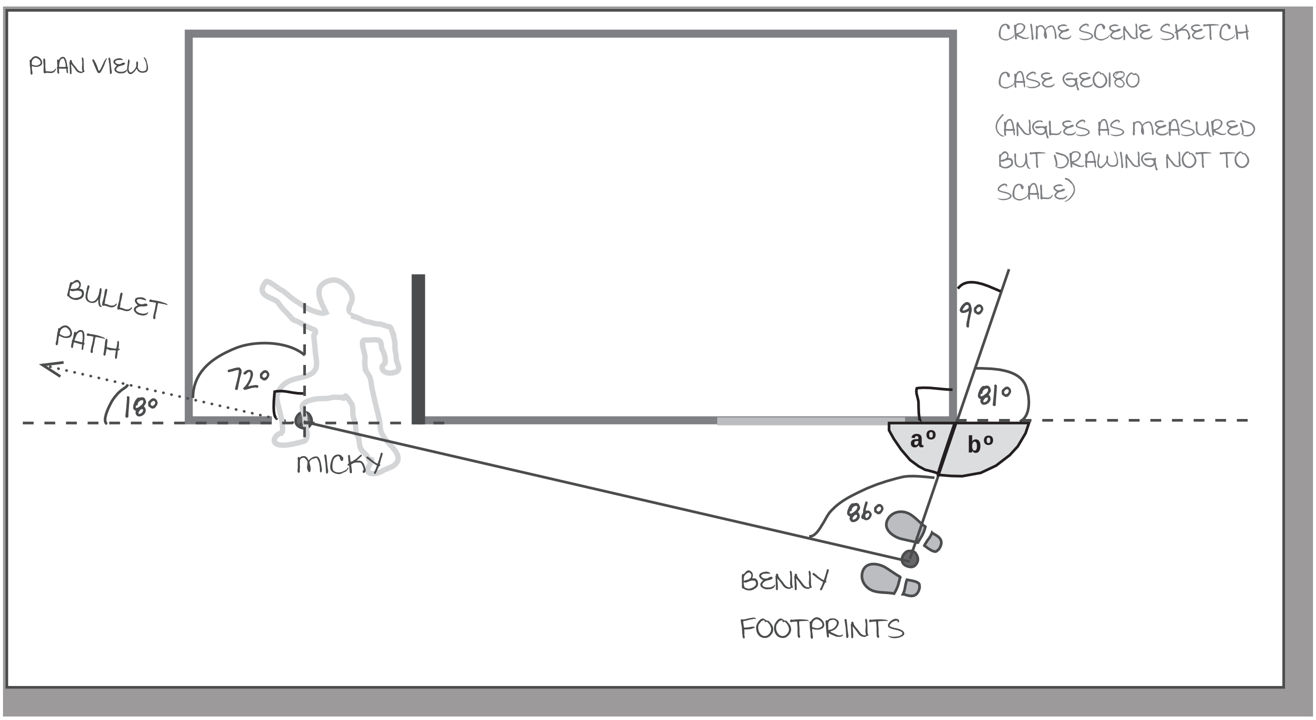
Any **two** angles which add up to 180° are known as **supplementary** angles. They are easiest to spot when they're on a straight line, but they can also be far apart.





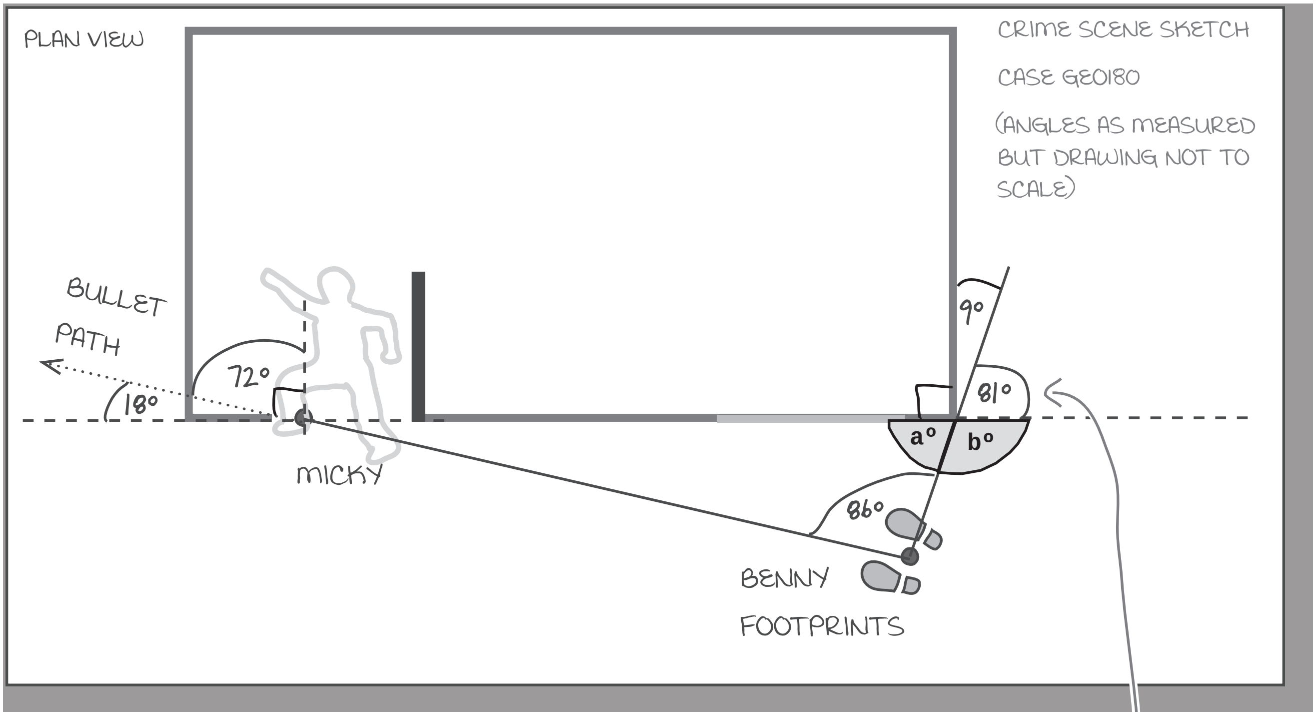
Sharpen your pencil

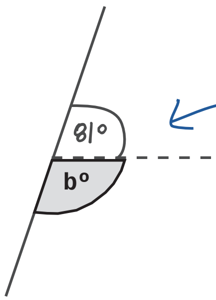
Use supplementary angles to find mystery angles a and b.
Is there anything surprising about mystery angle a?



Sharpen your pencil Solution

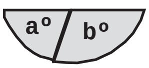
Use supplementary angles to find mystery angles a and b.
Is there anything surprising about mystery angle a?





These two angles are on a straight line and supplementary, so they add up to 180°.

$$81° + b° = 180°$$
$$b° = 99°$$



These two angles are on a straight line and supplementary, so they also add up to 180°.

$$a° + b° = 180°$$
$$a° + 99° = 180°$$
$$a° = 81°$$

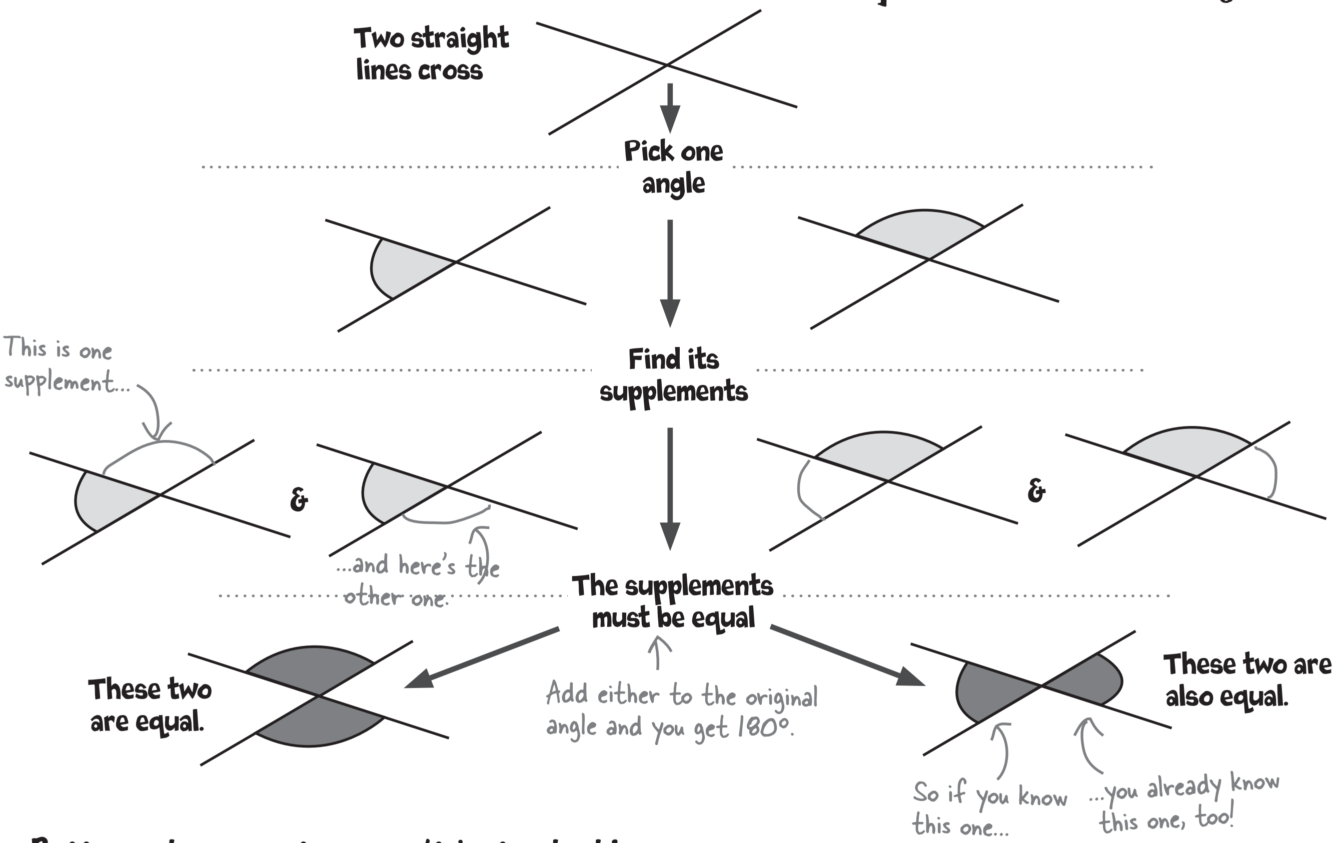
a° is the same as the first angle, which is supplementary with b°!

Vertical angles are always equal

When two straight lines cross they always create two pairs of equal opposite angles, called **vertical angles**.

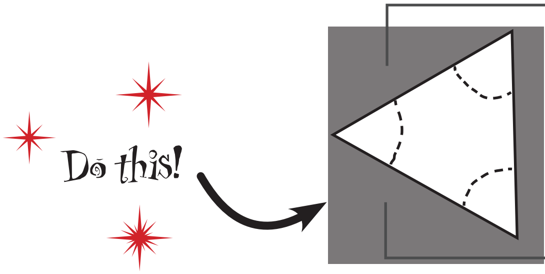
Each angle has to form a supplementary pair with either of the angles on the other side of it, so they must be equal.

Save yourself a ton of math by spotting equal pairs of vertical angles.



Patterns in geometry aren't just coincidence

These **relationships** are how geometry works…and to see a really freaky one, try this:

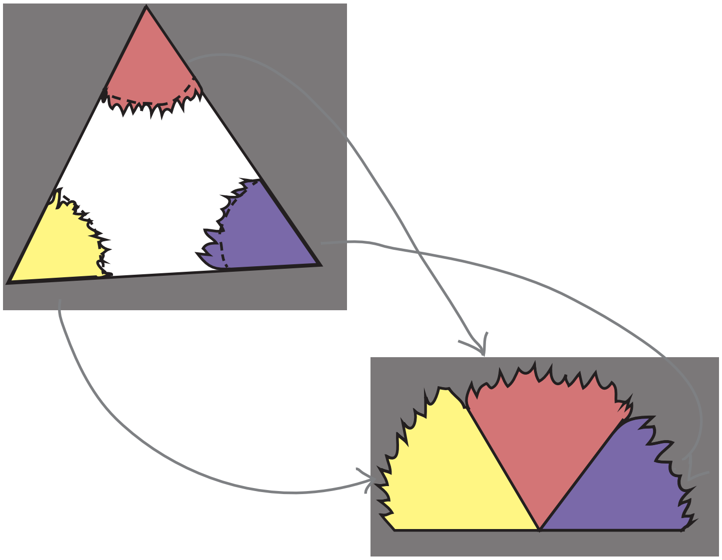


On some scrap paper, draw a triangle using a ruler.
Cut the triangle out.
Tear the corners off.
And then put the three corner pieces together with the points in the middle.

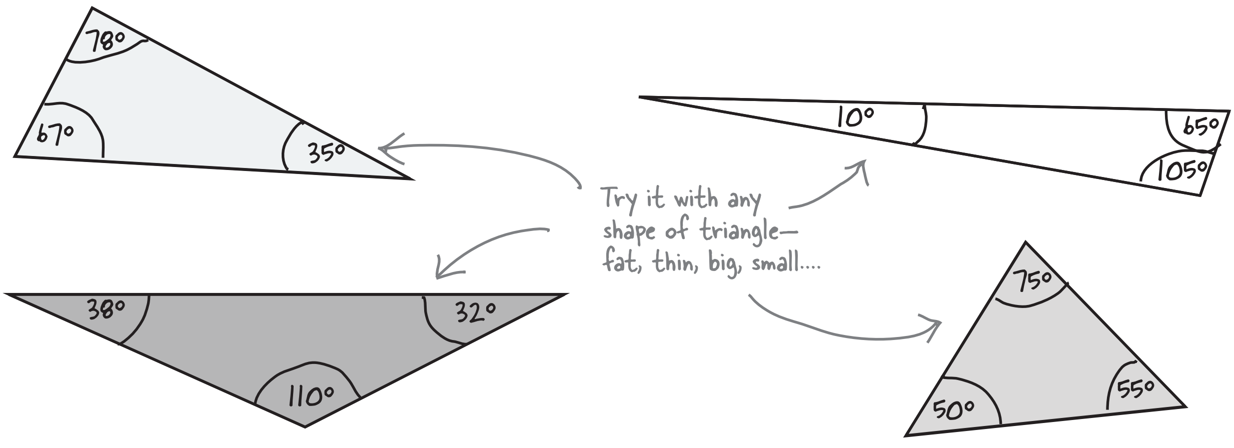
Weird, huh?

The corner angles of a triangle always add up to a straight line

The corner angles of a triangle always make a straight line, no matter what kind of triangle it is. Angles on a straight line always add up to 180°, so the corner angles of a triangle must always add up to 180°, too.



This is true for every triangle you could possibly draw!



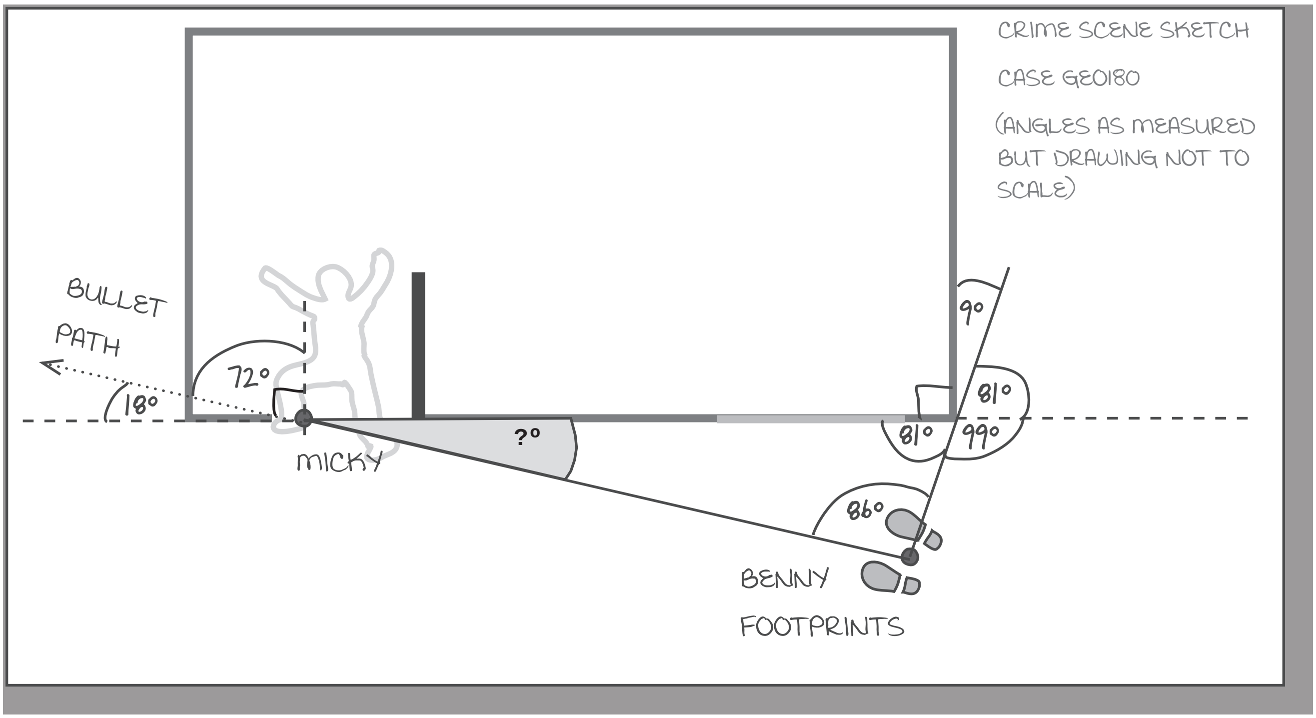
Find one more angle to crack the case

There's only one unknown angle left on the diagram. Once you've figured out this one you can charge Benny with the shooting for sure.



Find this last mystery angle to complete your investigation and prove that Benny was the shooter.

How does it fit in with what you've already found out?

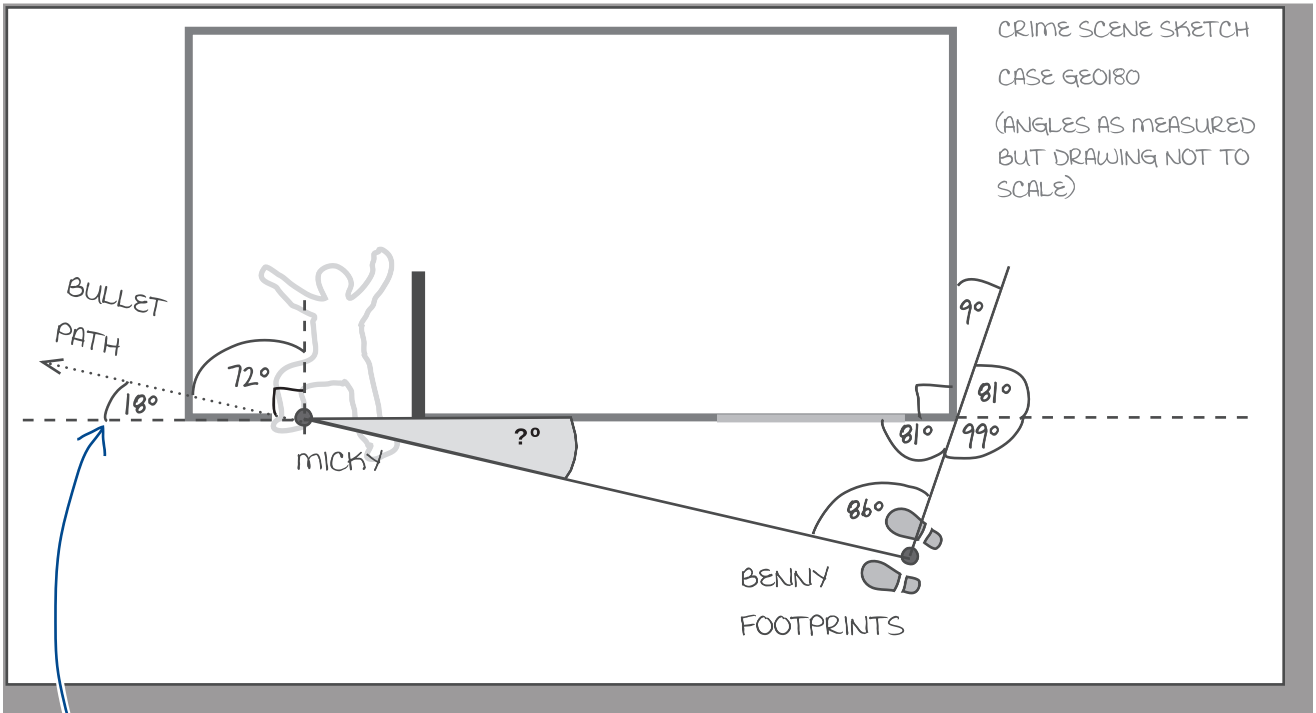




Exercise Solution

Find this last mystery angle to complete your investigation and prove that Benny was the shooter.

How does it fit in with what you've already found out?



There are 2 different ways to find this mystery angle—you might have done either one.

1)

Angle c is opposite this angle here
And vertical angles are equal,
so ? = 18°

But this doesn't fit with the triangle angles:

18° + 81° + 86° = 185°

They should add up to 180°.

Hmm...something funny is going on. It doesn't matter how you worded it, as long as you noticed something didn't add up.

2)

Angles in a triangle add up to 180°
so 81° + 86° + ? = 180°
? = 180° – 81° – 86° = 13°

But this doesn't fit with the 18° angle it's opposite. They should be equal.

Something doesn't add up!

Using the two different methods for finding the mystery angles gives us two different results.

And vertically opposite angles are equal, so ? = 18°

Angles in a triangle add up to 180° so 81° + 86° + ? = 180°
? = 180° − 81° − 86° = 13°

This is not good. The whole investigation could be compromised by bad math!



Joe: This is a disaster. How can there be two different answers? It's totally confusing.

Jill: Benny's lawyer is gonna have a field day. I don't believe it. We've got the right guy in custody—it's obvious he did it—and we're gonna have to let him go because of some messed up math! Can we fix it?

Frank: The weirdest thing is that I'm looking again at the math for either method and it looks good. I mean—corners on a triangle **always** add up to 180 degrees, and vertically opposite angles are **always** equal. Those are the rules.

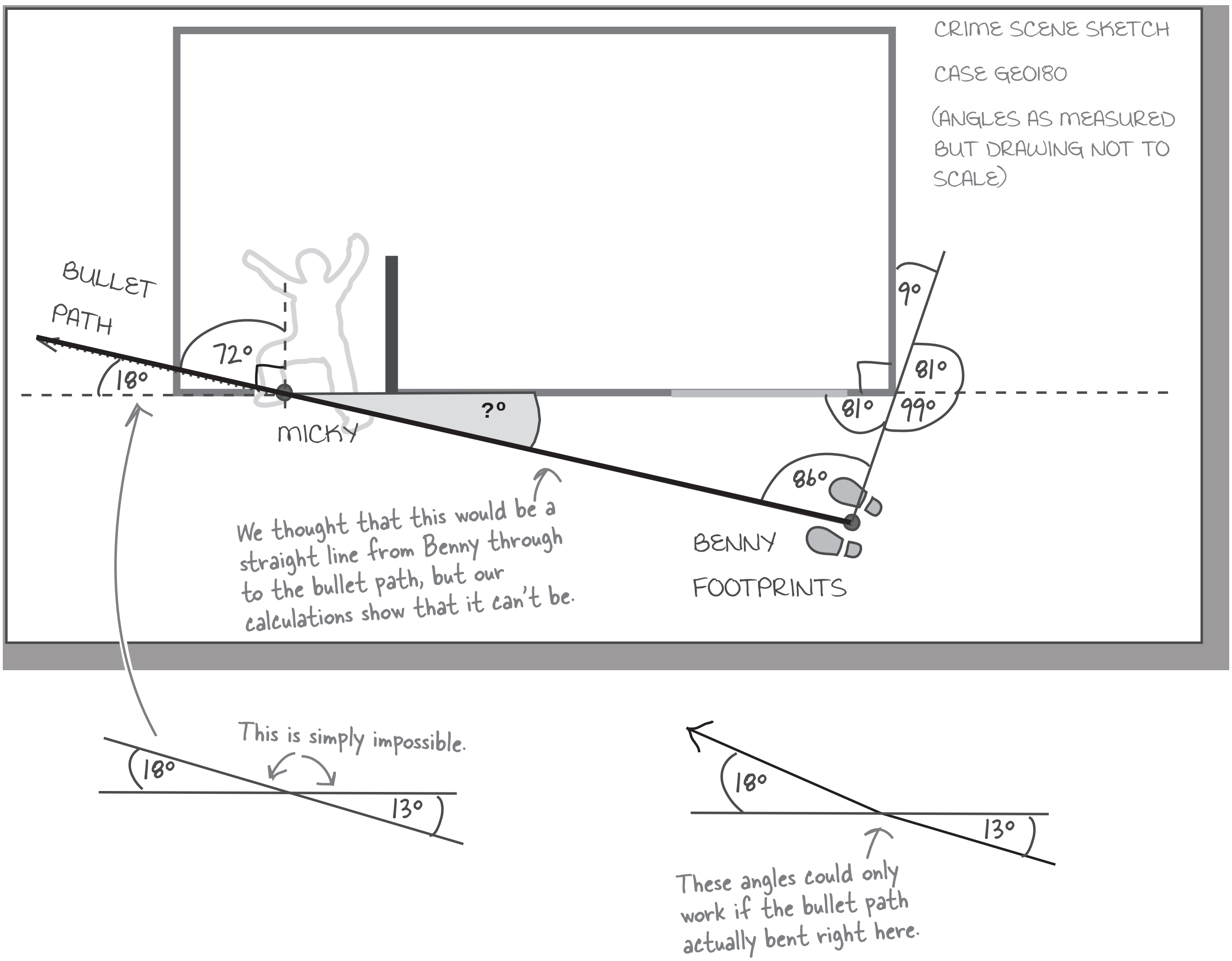
Joe: I knew we were relying too much on these coincidences. My guess is that these "rules" don't always actually work!

Jill: I really don't think that's what it means. If we haven't messed up the calculations then the fact that we got two different answers simply has to mean something else.…

What do you think it means?

If it doesn't all add up, then something isn't as it seems

The mystery angle is 18° **and** the mystery angle is 13°…well, that clearly doesn't add up. If the mystery angle is 13°, then it can't be vertically opposite our 18° bullet path angle as well. This means that the line on the sketch from Benny to Micky doesn't join up perfectly with the bullet path.



You've proved that Benny couldn't have shot Micky!

Unless Benny has a magic gun, he couldn't have shot Micky AND had the bullet enter the wall at the point that the CSIs found at the scene.

Benny is not our shooter, so the charges have been dropped and he's been released from custody, which has produced an interesting development....



What I tried to tell them when I was arrested is that I don't even own a gun! But there was this other guy there, Charlie, inside the building. I think he took a shot at me. I heard a gunshot and breaking glass so I ducked. The bullet hit the car that was parked behind me, and then it sped away. I don't know how Micky ended up shot, but at least you know it wasn't me!

Benny, seriously relieved to be back in his own wardrobe choices

And the chief is on the phone...

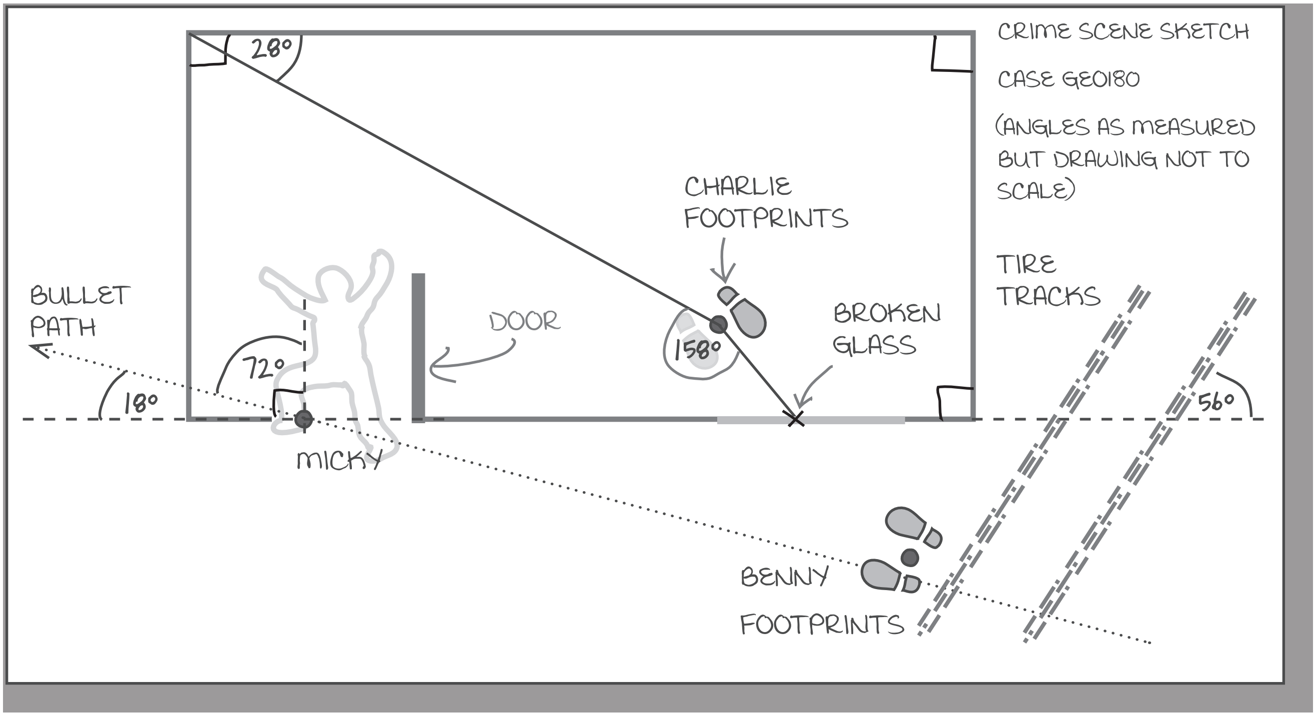
Great work. This case was a mess before you started working up the ballistics. But who the heck is Charlie? We'll get the team back on the scene to search for evidence. Stand by—we're gonna need your help.



We've got a new sketch—now for a new ballistics report

Following up on Benny's story, officers have returned to the scene, and found footprints, a broken window, and a set of tire trackes that might indicate that he's telling the truth. But there's a big problem—from where Charlie appeared to be standing, he can't have shot Micky through the door, plus the bullet that killed him came from the outside the building!

We do know that the bullet must have traveled in a straight line headed in the direction shown on the sketch, but how it got there is the big question.



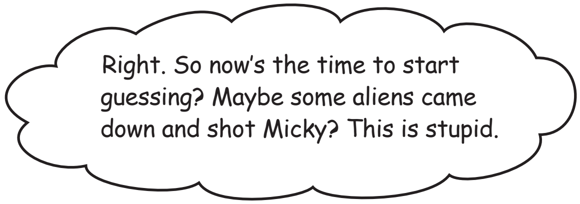
there are no Dumb Questions

Q: The dotted line extending the bullet path—how do we know it's really straight?

A: We're going to start by saying that this line MUST be straight heading backward from where the bullet entered the wall, and then work up the sketch to find out how the bullet came to be traveling in exactly that trajectory.

We need a new theory

Although you don't **know** what happened, if you can come up with some imaginative suggestions about what **might** have happened, you can test them by working out whether the angles add up, just like when you proved that Benny couldn't have shot Micky.





It's not just a matter of guessing.

You can use the statement Benny gave and all the relevant information on the crime scene sketch to guide you. Our points, lines, and angles are still as true as ever.

Statement by Benny on release from custody

What I tried to tell them when I was arrested is that I don't even own a gun! But there was this other guy there, Charlie, inside the building. I think he took a shot at me. I heard a gunshot and breaking glass so I ducked. The bullet hit the car that was parked behind me—it sped away. I don't know how Micky ended up shot, but at least you know it wasn't me!

benny smith 23/8/[illegible]

Sharpen your pencil

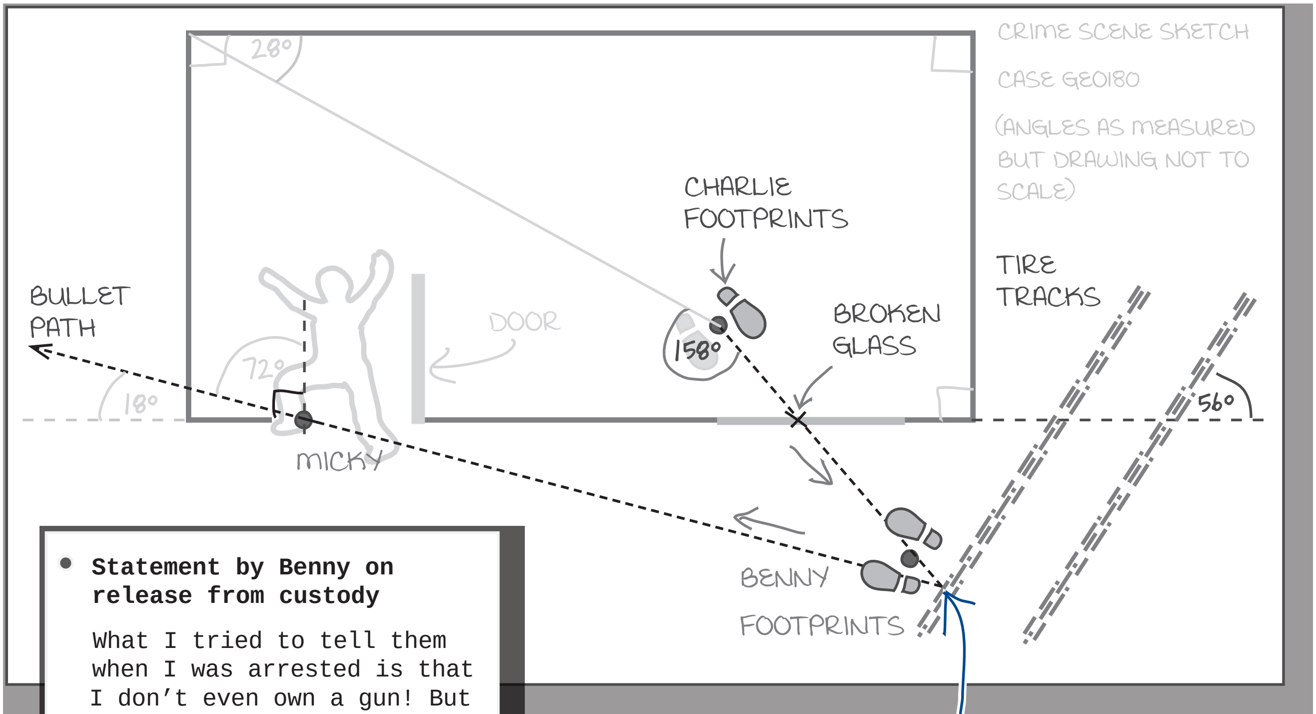
Read over Benny's statement. Using the sketch on the left page, or a blank sheet of paper, can you come up with a new idea about how Micky got shot?

(You can assume it didn't involve aliens.)

Sharpen your pencil Solution

Read over Benny's statement. Using the sketch on the left page, or a blank sheet of paper, can you come up with a new idea about how Micky got shot?

A new theory: Charlie shot at Benny, breaking the glass in the window, but missed, hit a parked car, and the bullet bounced off and hit Micky



Statement by Benny on release from custody

What I tried to tell them when I was arrested is that I don't even own a gun! But there was this other guy there, Charlie, inside the building. I think he took a shot at **me**. I heard a gunshot and **breaking glass** so I ducked. The bullet hit the **car that was parked behind me**—it sped away. I don't know how Micky ended up shot, but at least you know it wasn't me!

Point where the bullet bounced off the car that left the tire tracks

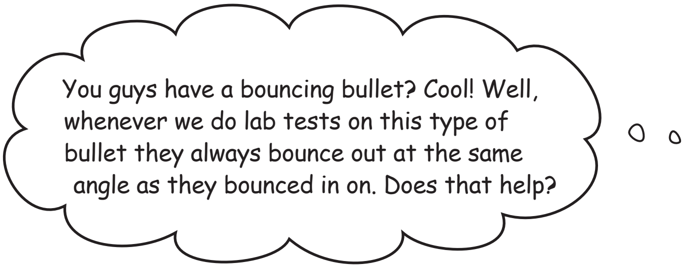
Benny's statement contains some references to points and lines, which are on our sketch.

Work out what you need to know

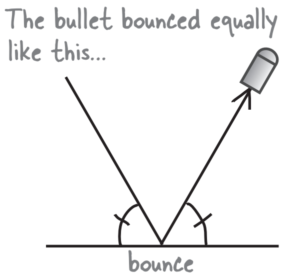
Based on your sketch, you need to work out what your theory actually means in terms of points, lines, and angles.

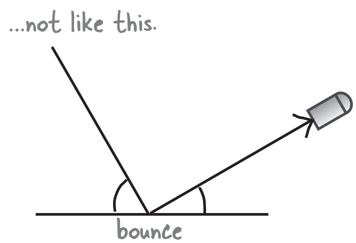
In this case we've got a few important details. We have a line from the point where Charlie was standing, through the hole in the window, until it hits the parked car. We also sketched a line moving backward from the bullet, until that hits the parked car, too.

If our theory is true, and Charlie was the shooter, the way the bullet bounced off the car is important.



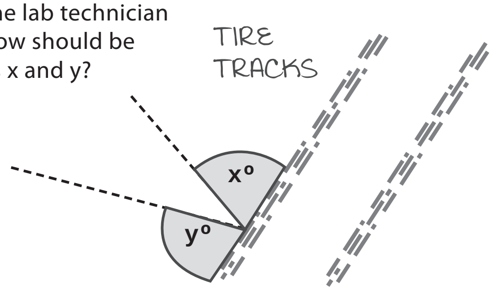






Sharpen your pencil

If our bullet bounced as the lab technician described, what do we know should be true about mystery angles x and y?

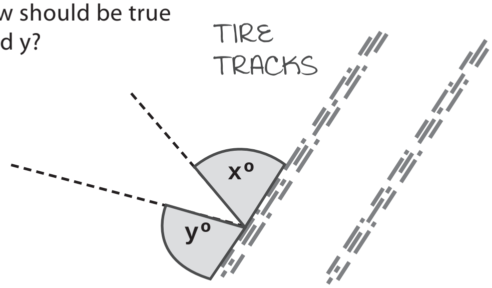


- [ ] x and y are supplementary
- [ ] x is less than y
- [ ] x is equal to y
- [ ] nothing—they could be any angles

Sharpen your pencil Solution

If our bullet bounced as the lab technician desribed, what do we know should be true about mystery angles x and y?

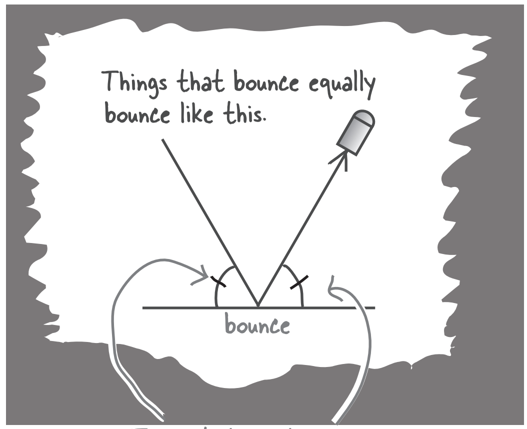
- [ ] x and y are supplementary
- [ ] x is less than y
- [x] x is equal to y
- [ ] nothing—they could be any angles



Tick marks indicate equal angles

Tick marks on angles are used to show that the angles are equal, even if we don't know what size the angles are.

Each set of equal angles has a different number of ticks, so you can mark more than one set on the same sketch if you need to.



These tick marks on the angles show that they are equal.

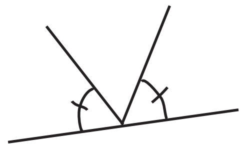
Watch it!

Remember—your sketch isn't usually accurate enough to see by eye whether two angles are equal.

Unless you're told otherwise, a sketch is just a sketch. Even if the angles drawn look pretty close to the real angles, you can't rely on your eye or a protractor—you've gotta work the angles out.

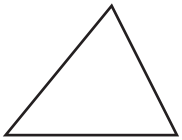
Use what you know to find what you don't know

By spotting patterns around the angles that you do know, you can work out the angles that you don't yet know. Sometimes, like with vertical angles, you don't even have to do any math.



Equal angles

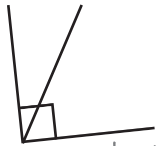
Here are some patterns for you to look out for:



Triangles



Angles on a straight line



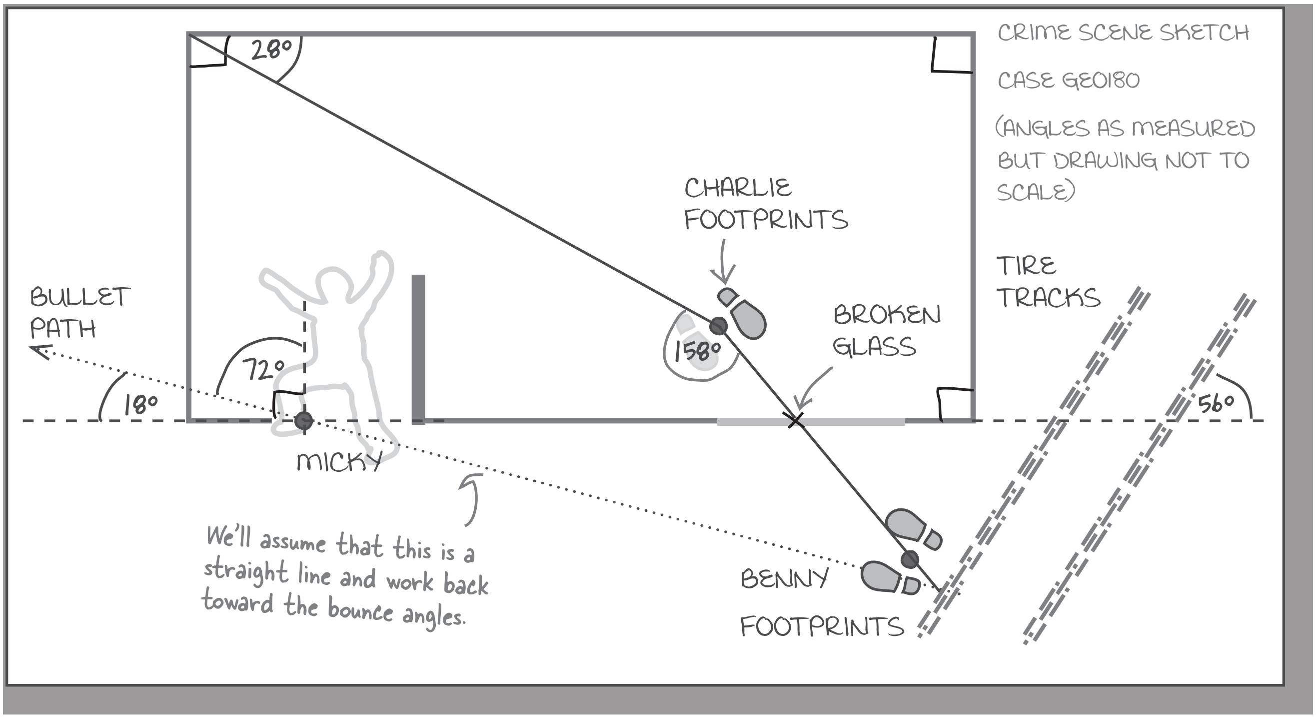
Complementary angles



Vertical angles

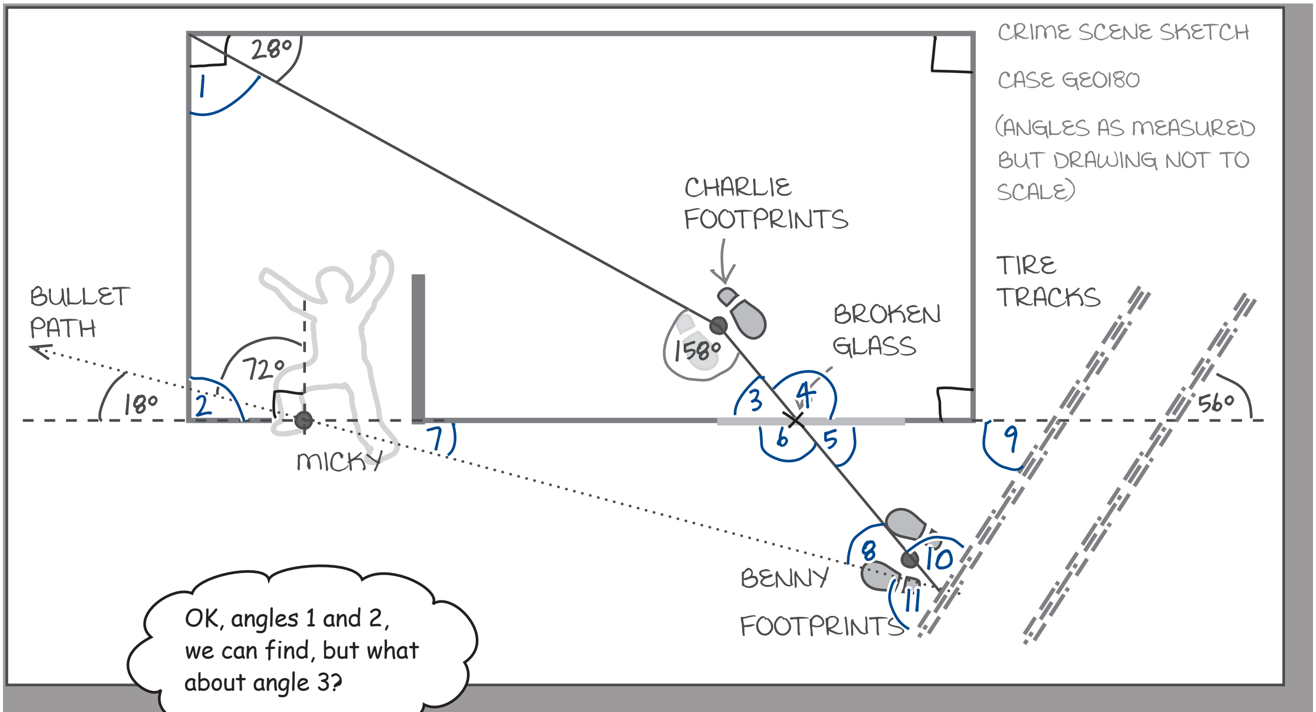
Sharpen your pencil

On the sketch, mark all the new angles you need to find to solve the crime (there are at least 11). ← These aren't the angles you found before.



Sharpen your pencil Solution

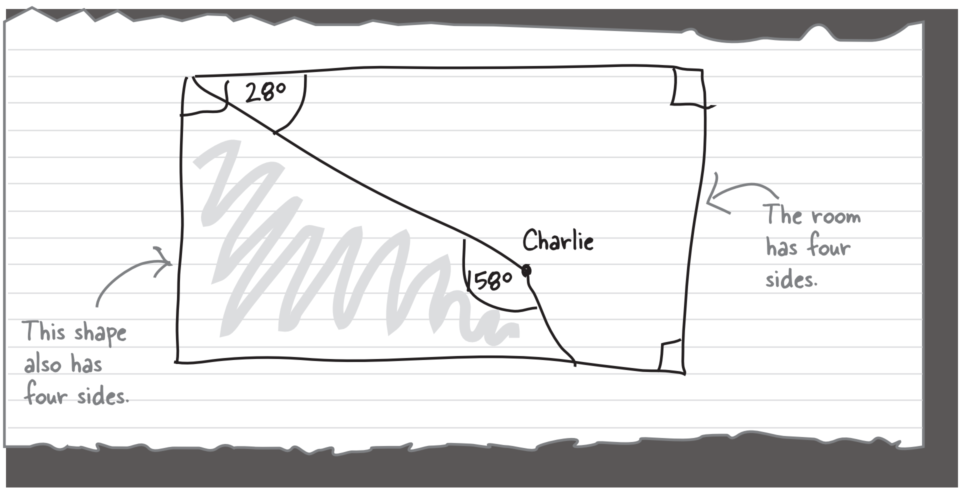
Mark on the sketch all the new angles you need to find to solve the crime (there are at least 11).



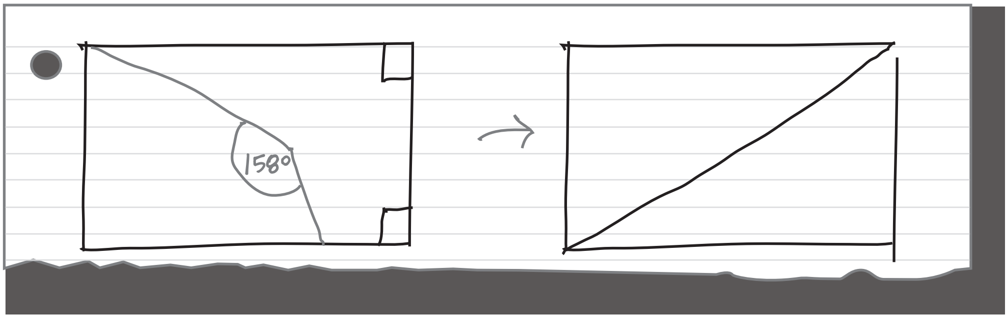


Jill: What's the problem? We just keep going like we have been.

Frank: But the room isn't triangular—it has four sides.

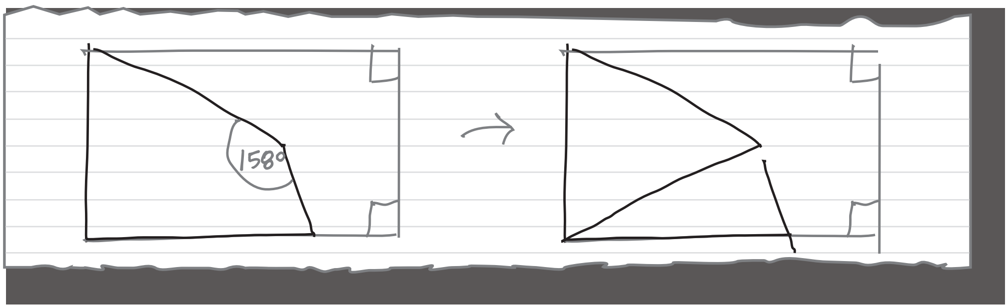


Jill: Oh, good point. Could we add a line of our own and chop the room into two triangles like this?

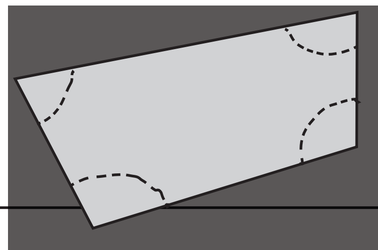


Joe: How on earth does that help?

Jill: Well…at least we know that angles in a triangle add up to 180 degrees. So if we make triangles out of the room, we can keep going like we have been. In fact, there's another shape with four sides there that could help us with angle 3—we could divide that into triangles as well maybe? Like this?



Joe: OK, but that seems like a lot more work. Maybe we could try that thing we did with the paper triangle? With the corners? That might show us something about four-sided shapes in general, and we can use that to find those missing angles?



Sharpen your pencil

What is it that you need to find out about four-sided shapes in order to find the missing angles?

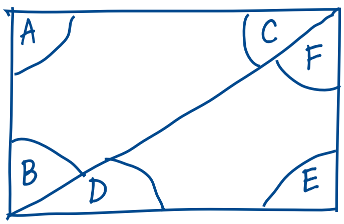
See whether you can figure it out, either by adding up the angles using two triangles, or tearing the corners from a four-sided shape to see what they add up to.

Sharpen your pencil Solution

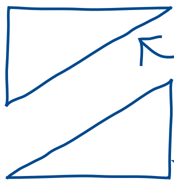
What is it that you need to find out about four-sided shapes in order to find the missing angles?

See whether either method can help you figure it out.

To find the missing angles, we need to know what angles in a four-sided shape add up to.

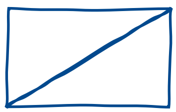


One way of finding out is to split the four-sided shape into two triangles.



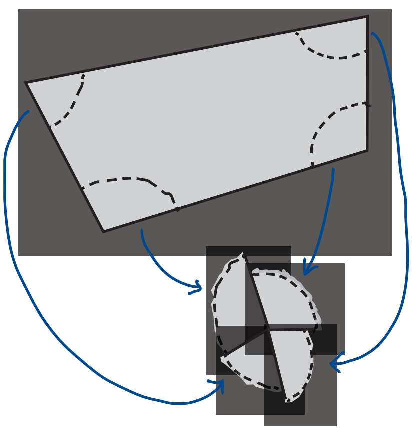
$A + B + C = 180°$

$D + E + F = 180°$

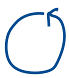


$A + B + C + D + E + F = 180° + 180°$

$= 360°$



Using a different method, if you cut out a four-sided shape and tear the corners off, and put them together with the points in the middle, you'll find they make a whole turn.

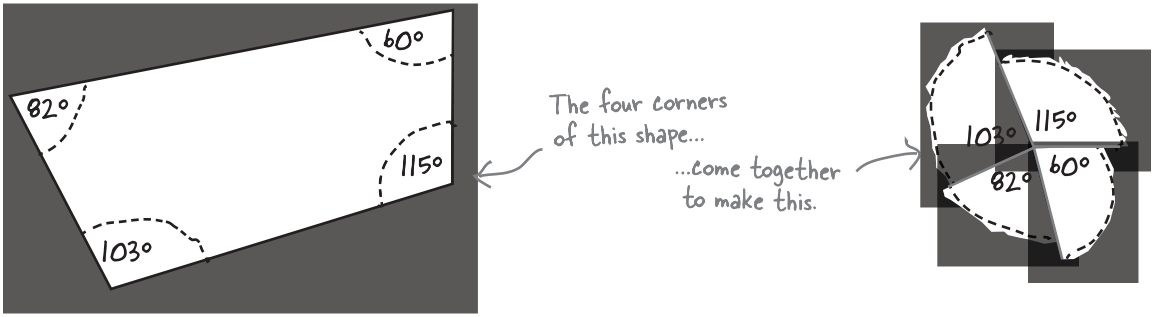


And a whole turn (two half turns) is 360°.

Both methods give us the same answer: 360°.

The angles of a four-sided shape add up to 360°

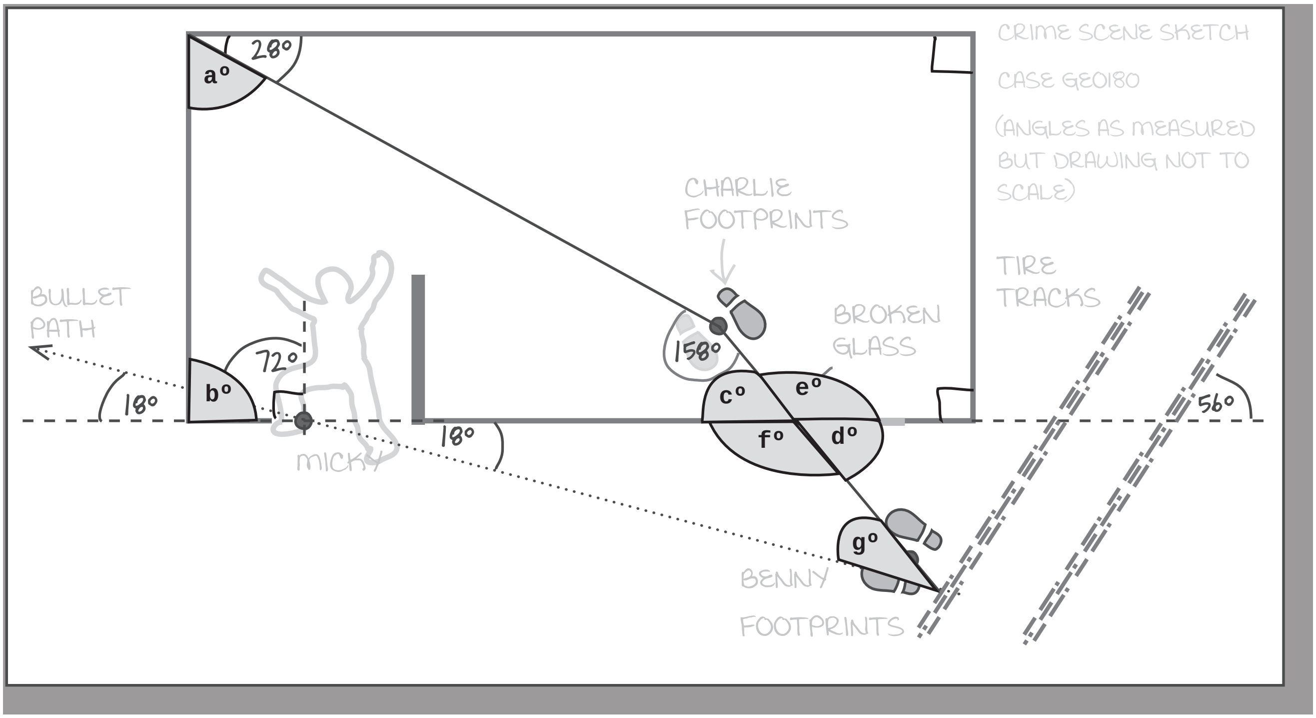
Squares and rectangles have four equal angles, a quarter turn each, and even the most uneven four-sided shape still has four angles adding up to 360°.



Sharpen your pencil

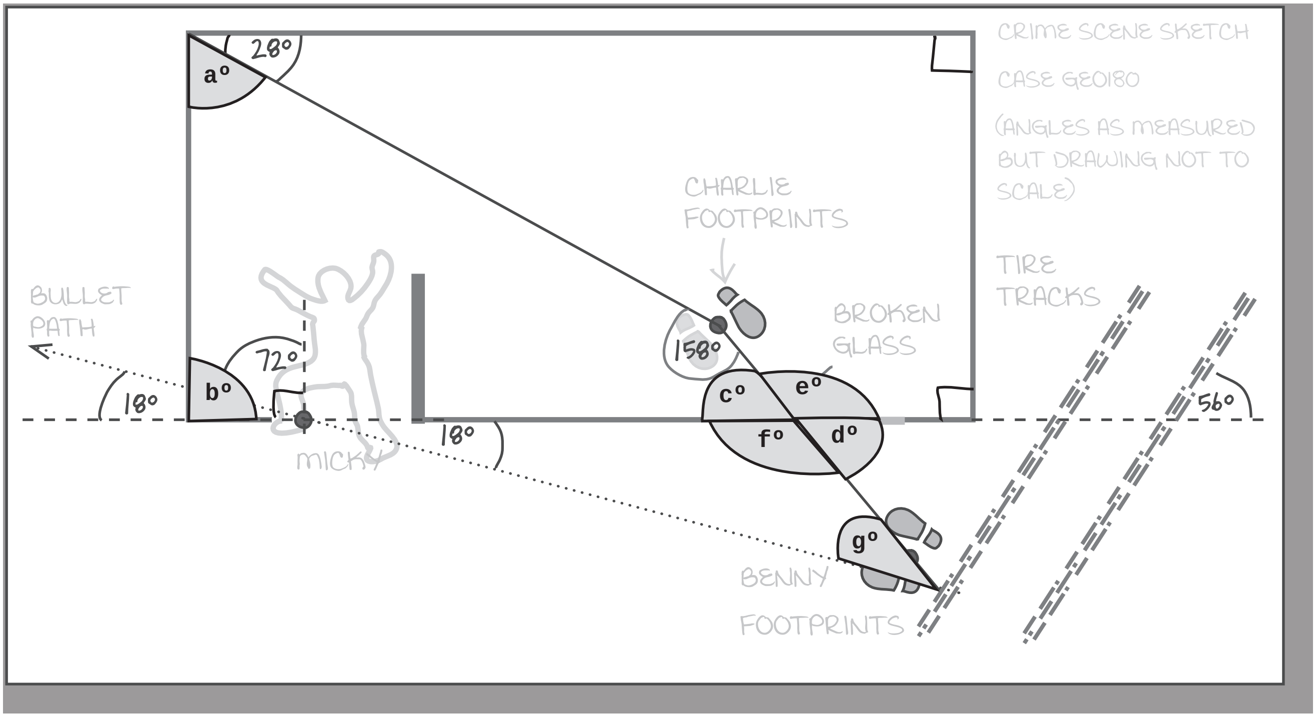
Let's step the investigation up a gear! Find the mystery angles marked on the crime scene diagram, a, b, c, d, e, f, and the all important angle, g, which lets us work out the bounce angles.

Hint: it's worth going in alphabetical order.

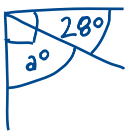


Sharpen your pencil Solution

Let's step the investigation up a gear! Find the mystery angles marked on the crime scene diagram, a, b, c, d, e, f, and the all important angle, g, which lets us work out the bounce angles.



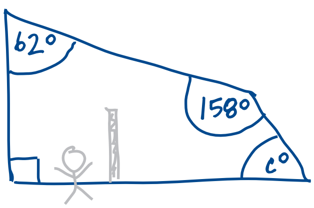
a)



a and 28° are complementary.
$a + 28° = 90°$, so $a = 62°$

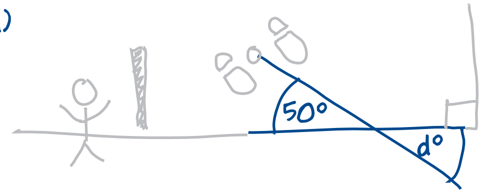
b) The room is a four-sided shape and the other three corners are right angles, so
$b = 360° - (90° + 90° + 90°) = 90°$

c)



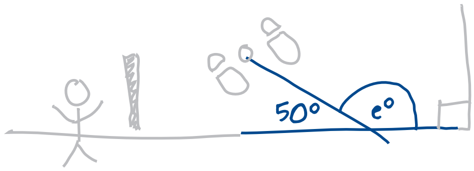
c is part of a four-sided shape, with a° and the corner angle b° we just found, so
$c = 360° - (158° + 62° + 90°) = 50°$

d)



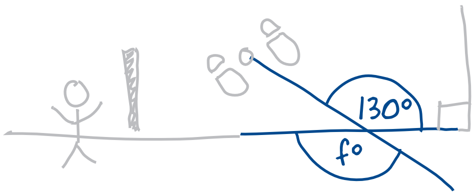
d is vertically opposite c, so d = 50°

e)



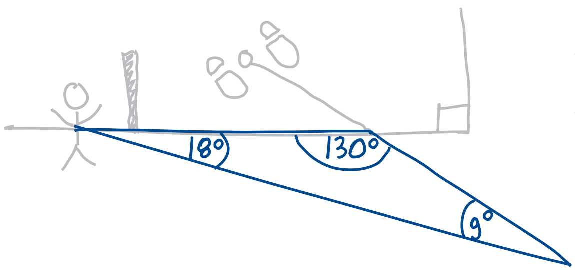
e forms a straight line with c, so
e = 180° – 50° = 130°

f)



f is vertically opposite e, so f = 130°

g)

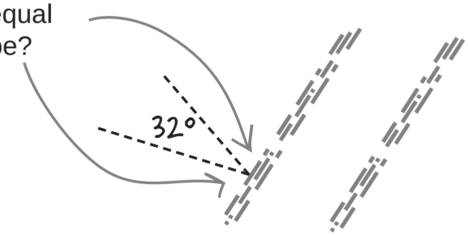


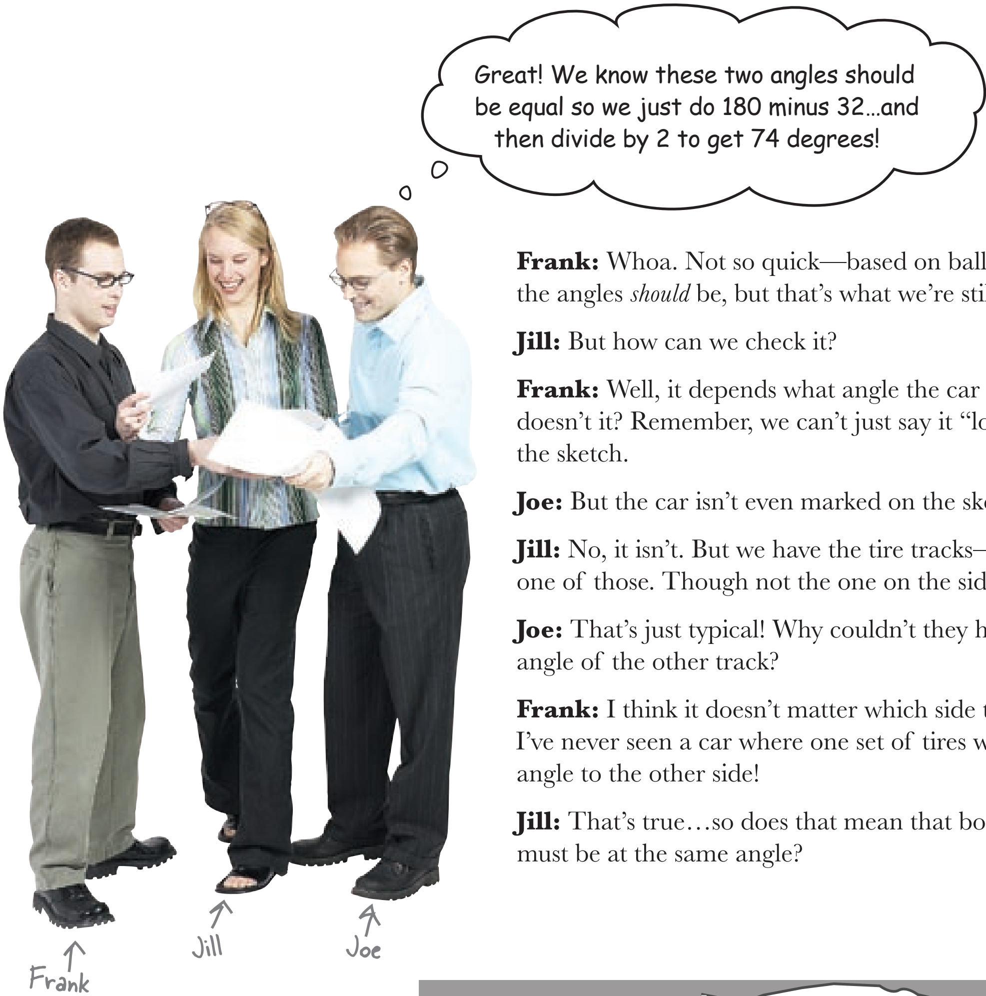
g is part of a triangle with 18° and f, so
g = 180° – (18° + 130°) = 32°

Brain Power

Since we believe that the bullet bounced in and out at equal angles, what do you think these angles will turn out to be?

Is it possible to tell yet whether Charlie shot Micky?





Frank: Whoa. Not so quick—based on ballistics, that's what the angles *should* be, but that's what we're still trying to establish.

Jill: But how can we check it?

Frank: Well, it depends what angle the car was parked at, doesn't it? Remember, we can't just say it "looks about right" on the sketch.

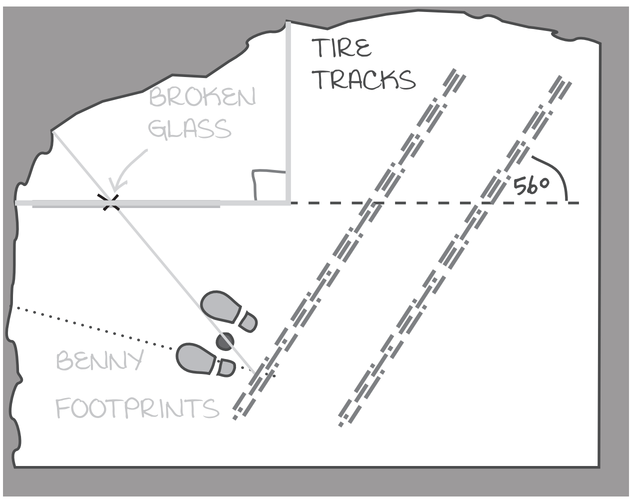
Joe: But the car isn't even marked on the sketch.

Jill: No, it isn't. But we have the tire tracks—and the angle of one of those. Though not the one on the side that the bullet hit.

Joe: That's just typical! Why couldn't they have measured the angle of the other track?

Frank: I think it doesn't matter which side they measured… I've never seen a car where one set of tires was at a different angle to the other side!

Jill: That's true…so does that mean that both sets of tire marks must be at the same angle?

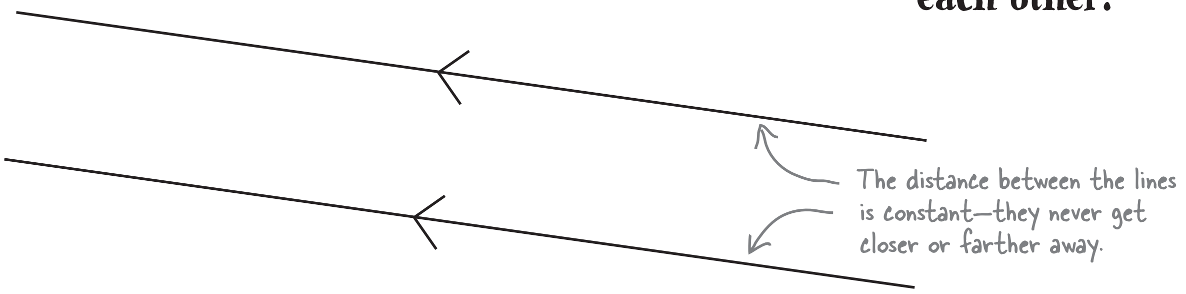


Parallel lines are lines at exactly the same angle

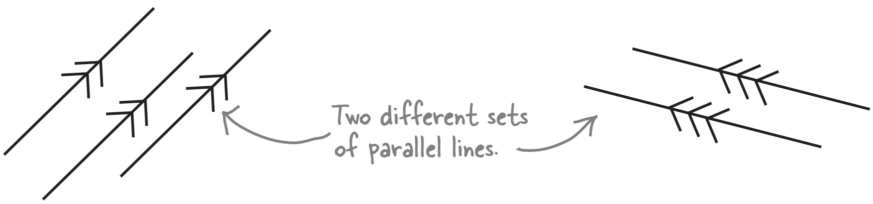
Two, or more, lines which are at exactly the same angle (like the tire tracks from the car must be) are called parallel lines. We use little v-shaped tick marks to indicate sets of parallel lines or line segments on a sketch.

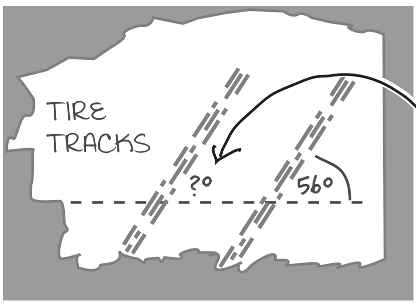
Parallel lines can never meet or cross each other, even if you stretch them for miles and miles and miles.…

Even if you stretch them forever, parallel lines never cross each other.



If you have more than one set of parallel lines on the same sketch then you need to use a different number of tick marks on each set to be able to tell them apart.

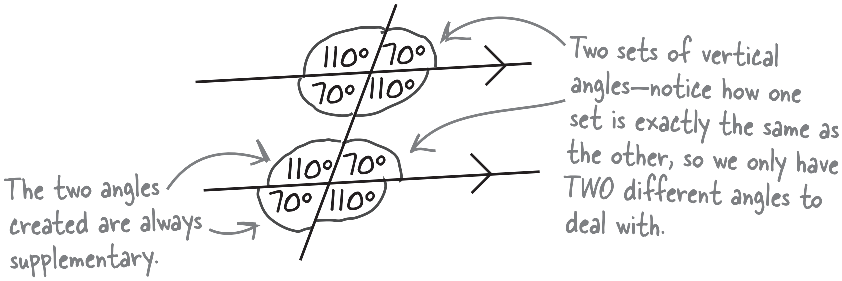


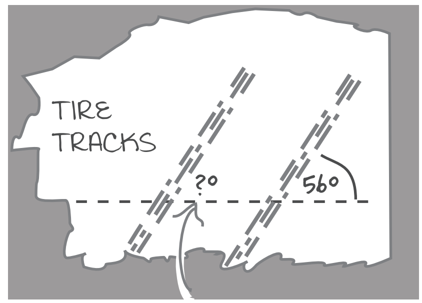


If parallel lines are always at the same angle, and our tire tracks are parallel, what should this angle here be?

Parallel lines often come with helpful angle shortcuts

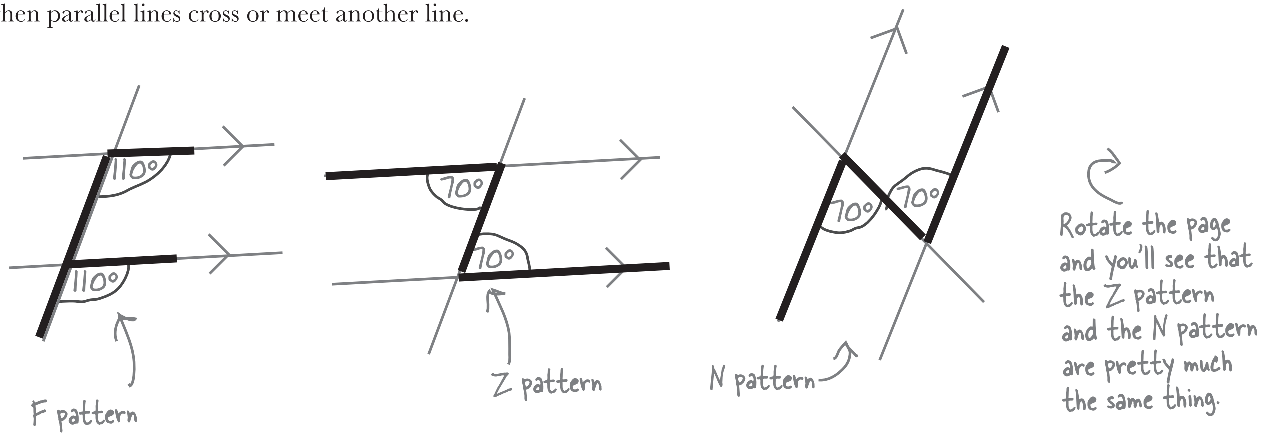
When you have a line which meets or crosses a set of parallel lines, all the sets of (opposite) vertical angles that are created are the same.





So this angle here should be the same—56°.

This means that you can find more missing angles without doing any math at all—just by recognizing a few patterns. The **F Pattern,** the **Z Pattern,** and the **N Pattern** are formed when parallel lines cross or meet another line.



Look out for F, Z, and N patterns around parallel lines for a shortcut to finding missing angles.

Watch it!

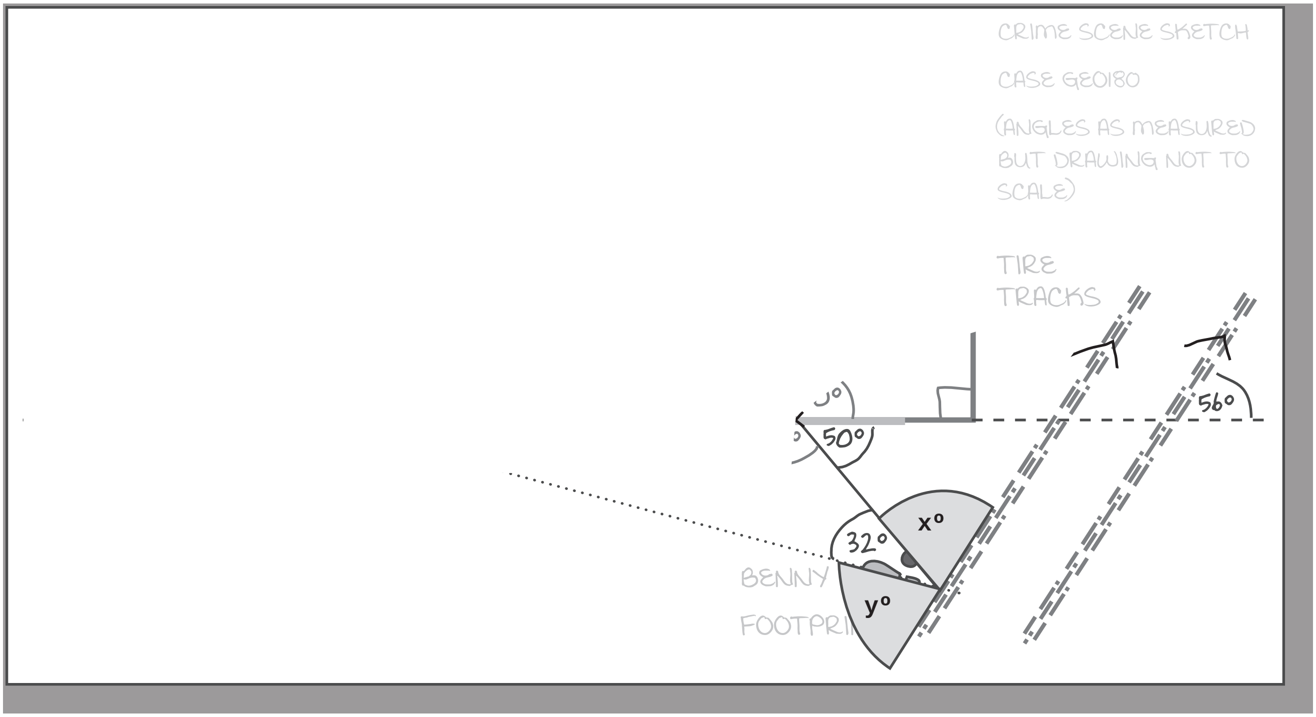
The F, Z, or N can be hidden

Sometimes they're upside down, back-to-front, or both!

Sharpen your pencil

It's the moment of truth.

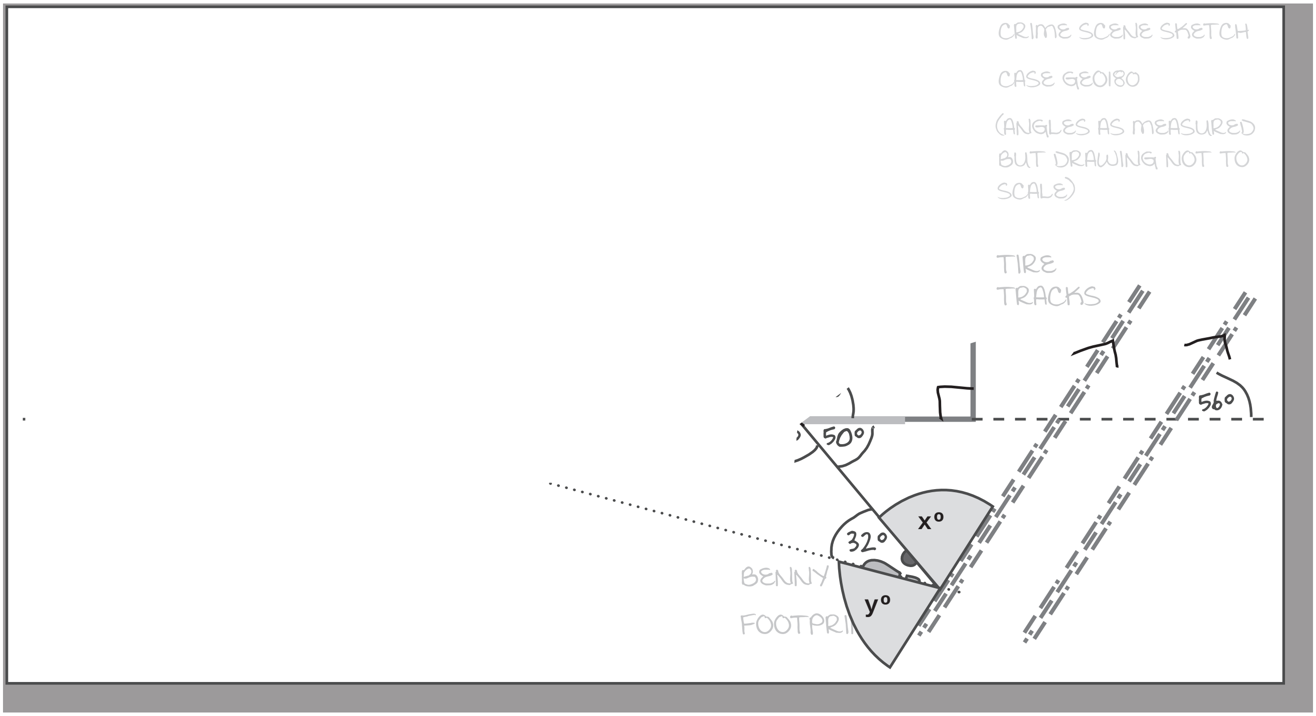
Find the crucial mystery angles, x and y, and prove once and for all whether Charlie is our shooter. If they're equal he's guilty. If they're not then the case is at another dead end.



Sharpen your pencil Solution

It's the moment of truth.

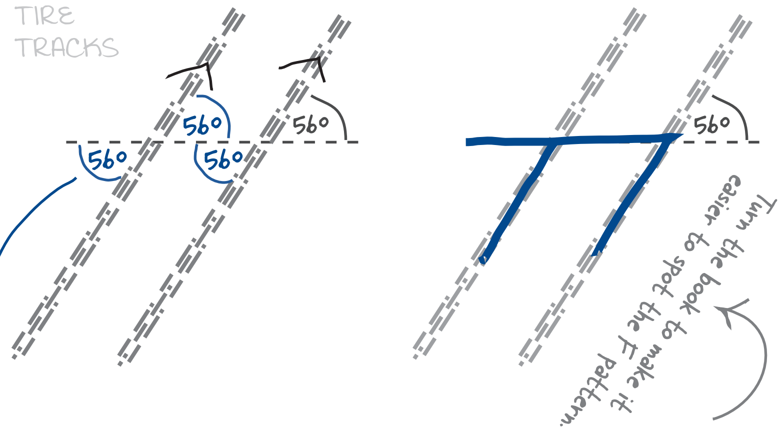
Find the crucial mystery angles, x and y, and prove once and for all whether Charlie is our shooter. If they're equal he's guilty. If they're not then the case is at a dead end.



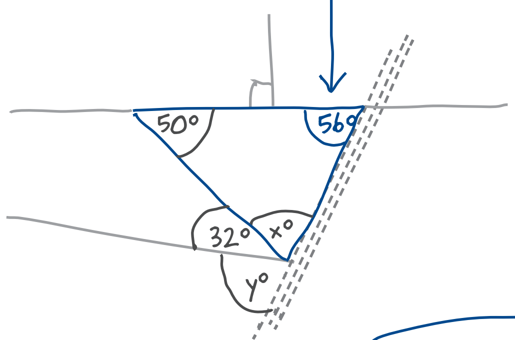
If Charlie shot Micky, then we should find that angles x and y are EQUAL, because that's how the bullet fired from his gun should bounce.

The tire tracks are parallel.

So these sets of vertically opposite angles are the same.



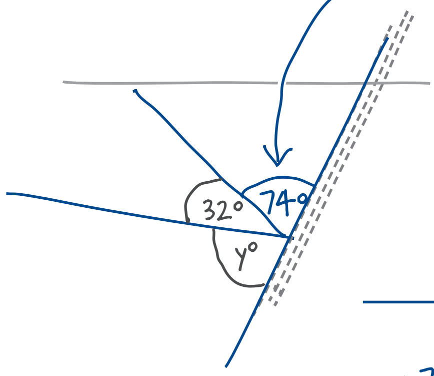
x)



x is in a triangle with the 56° we just found and 50°, so

$x = 180° - (56° + 50°) = 74°$

y)



y forms a straight line with x and 32°, so

$y = 180° - (74° + 32°) = 74°$

$x = 74°$ and $y = 74°$

The angles either side of where the bullet bounced are equal—it all adds up.

Nice one—you solved the crime!

This means that we have proved that Charlie shot Micky, even if it was Benny he intended to hit!

Great work—you cracked the case!

Thanks to your work uncovering the angles, the right guy is behind bars.

You've mastered a whole bunch of techniques for finding missing angles, uncovered some "sketchy" assumptions, and discovered general rules that the CSI team can use again and again to test out ballistics evidence. Your hard work and talent hasn't gone unnoticed—time to hang up your lab coat and take a hot promotion!



Charlie is gonna be making a lot of license plates....

We're really impressed with how you unravelled the evidence—this was a tricky one! You've been great in the ballistics lab and now you've proved you're ready to become our lead CSI, working your own important cases.

The chief couldn't be happier with you.



Before you collect your bonus and your new CSI badge, the chief would appreciate it if you left the ballistics lab a cheat sheet about how you worked out who shot Micky.

WHO DOES WHAT?

Match each technique for finding missing angles to a sketch. You can use each technique just once, but some sketches will need more than one to find the missing angle.

Angles in a triangle add up to 180º

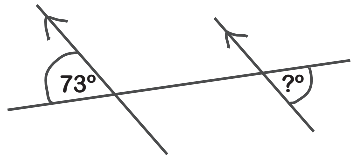
Vertical angles are equal

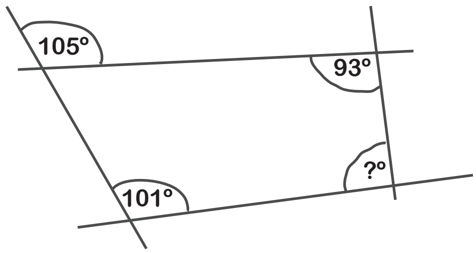
Parallel lines cross other lines at the same angles

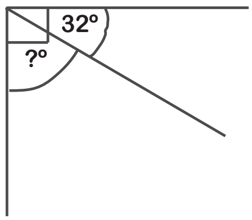
Angles on a straight line add up to 180º

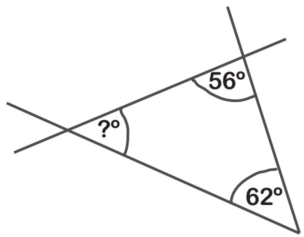
Angles in a four-sided shape add up to 360º

Angles in a right angle add up to 90º









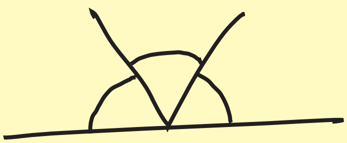
Answers on page 48.



Your Geometry Toolbox

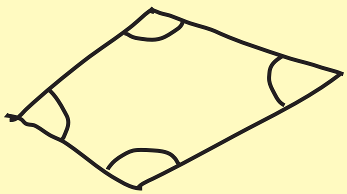
You've got Chapter 1 under your belt, and now you've added techniques for finding missing angles to your toolbox. For a complete list of tool tips in the book, visit www.headfirstlabs.com/geometry.

Angles on a straight line add up to 180°.

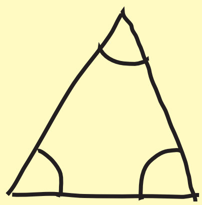


Angles can be made up of other, smaller angles.

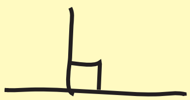
Angles in a four-sided shape add up to 360°.



Angles in a triangle add up to 180°.



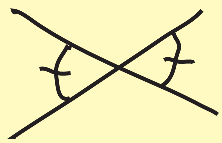
A square mark indicates a RIGHT ANGLE (90°).



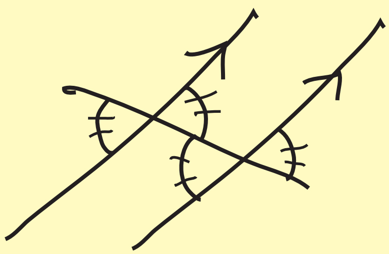
If two angles add up to 180°, they are SUPPLEMENTARY.

If two angles add up to 90°, they are COMPLEMENTARY.

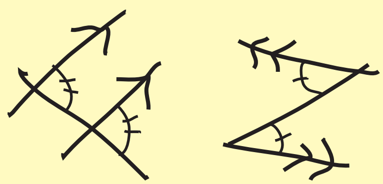
Vertical angles are equal (and opposite each other).



Parallel lines make repeat sets of vertical angles.



F, Z, and N patterns around parallel lines save you a ton of math.



Who Does What? Solution

Match each technique for finding missing angles to a sketch. You can use each technique just once, but some sketches will need more than one to find the missing angle.

Angles in a triangle add up to 180º

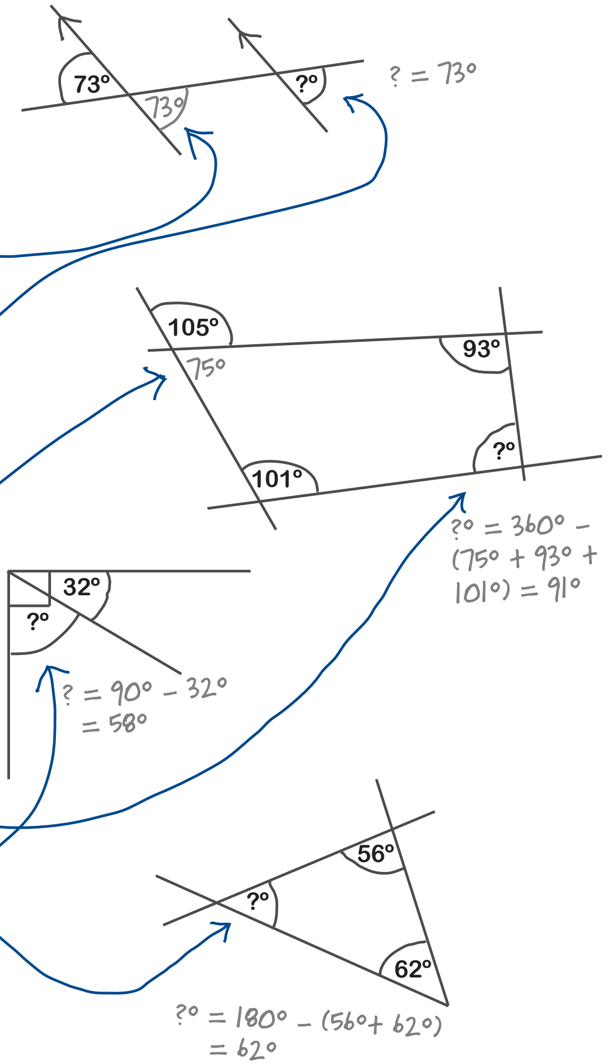
Vertical angles are equal

Parallel lines cross other lines at the same angles

Angles on a straight line add up to 180º

Angles in a four-sided shape add up to 360º

Angles in a right angle add up to 90º



2 similarity and congruence

Shrink to fit



Sometimes, size does matter.

Ever drawn or built something and then found out it's the **wrong size**? Or made something just perfect and wanted to ***recreate it exactly?*** You need *Similarity* and *Congruence*: the **time-saving techniques for duplicating** your designs smaller, bigger, or exactly the same size. Nobody likes doing the same work over—and with similarity and congruence, you'll ***never have to repeat an angle calculation again.***



Welcome to myPod! You're hired

Congratulations! You've landed a dream summer job at myPod, laser etching custom stuff onto iPods, phones, and laptops. If you do well, you'll get your bonus in cool gear.



Bonus gear is up for grabs.



All you have to do is prepare text and designs to be etched onto people's phones, iPods, laptops, and stuff.

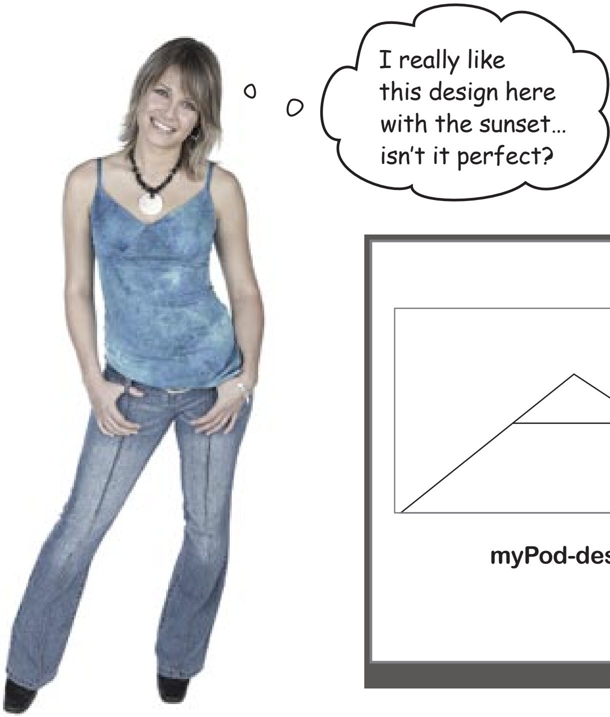
Sounds cool! Think you can etch my new cell phone?



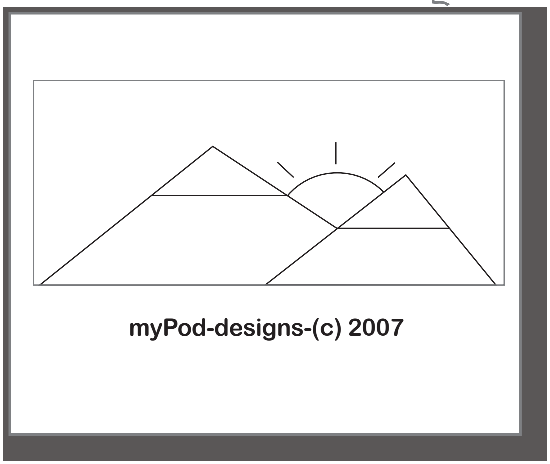
Meet Liz, your first customer.

Liz wants you to etch her phone

Time to get to work. Liz has picked a design she loves for her new cell phone, now all you have to do is engrave it on the back.



This is the design Liz chose.



Sharpen your pencil

Sketch before you etch! Copy the design Liz has chosen (it doesn't matter if it's messy!) and think about what you're paying attention to when you copy.

What three things do you need to know about each line in order to be able to sketch, and then etch, the design accurately?

1) ....

2) ....

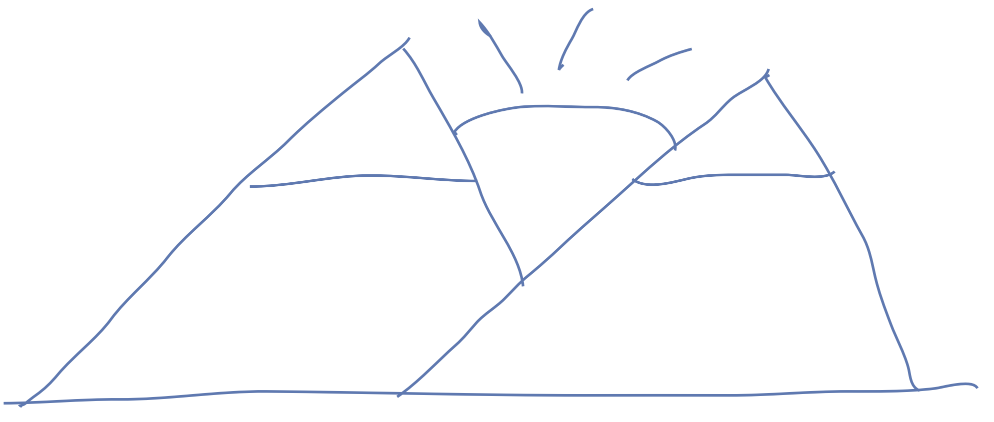
3) ....

Sketch here

Sharpen your pencil Solution

Sketch before you etch! Copy the design Liz has chosen (it doesn't matter if it's messy!) and think about what you're paying attention to when you copy.

What three things do you need to know about each line in order to be able to sketch, and etch, the design accurately?



When you sketch the design, you need to think about these things:

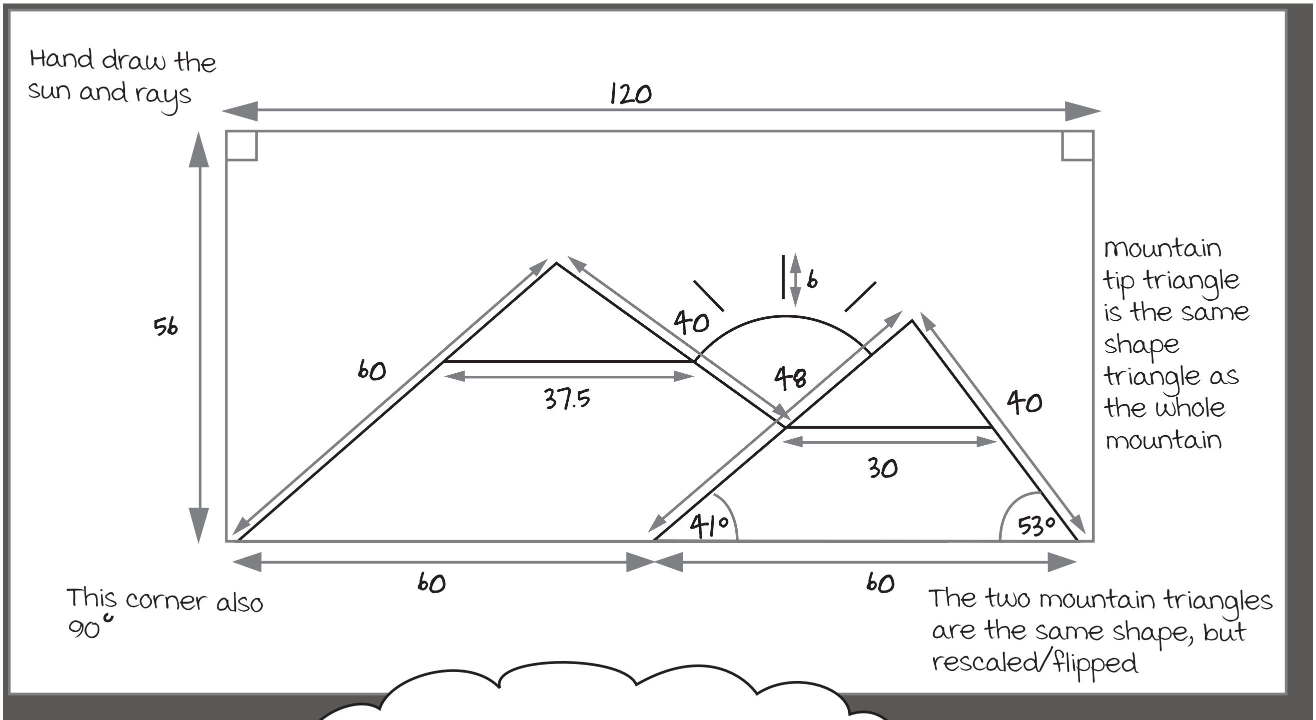
1) How long each line is

2) What angle each line is at

3) Where the line is (what position it starts/ends)

The designer noted ~~all~~ some of the details

The designer wrote a bunch of notes around his drawing. He seems to have filled out all the line lengths, but only included a few of the angles?!?



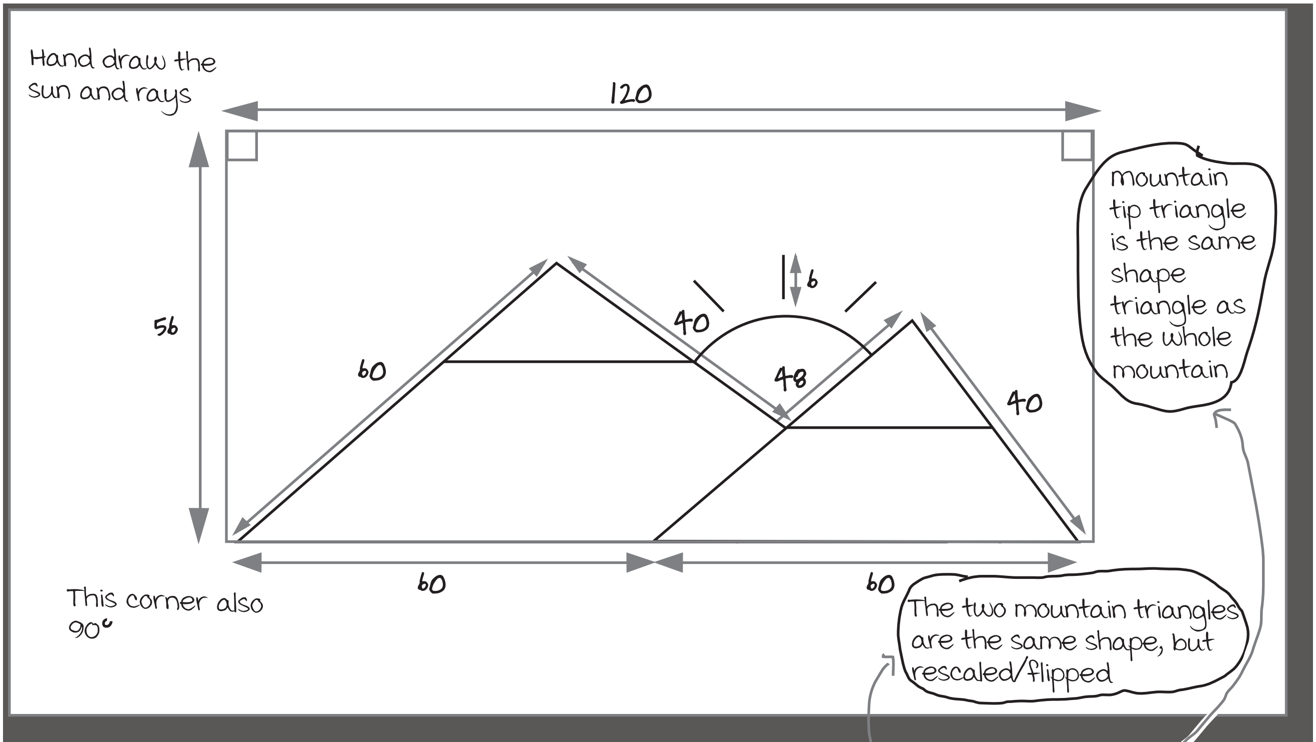


Uh oh! Your first day on the job and you've got a busted design! I can't believe the designer stuck it in the book unfinished like that. I'd get your customer to choose another one....

BRAIN POWER

It certainly looks like there are a lot of angles missing. Has the designer left you any hints about what the angles he hasn't written might be?

The design tells us that some triangles are repeated



These notes are a big hint about the missing information.

The designer made a note saying that the two mountain triangles are the same but rescaled, or different sizes.

There's also a note saying that the mountain tip is the same triangle as the whole mountain.

But what exactly does he mean by "the same triangle"?



Jim: Yeah, I mean, the designer is talking nonsense. One of them is bigger than the other. How on earth can they be the same?

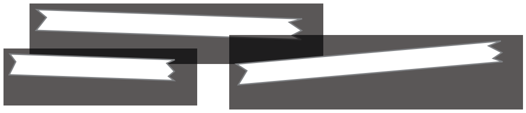
Frank: Well, maybe he was talking about angles. Can you have two triangles with the same angles but with different lengths?



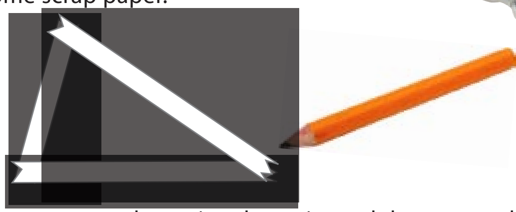
GEOMETRY CONSTRUCTION

Can you have two triangles with the same angles but with different lengths?

1) Cut or tear three narrow strips of paper, making them slightly different lengths.



2) Make them into a triangle and draw around it on some scrap paper.

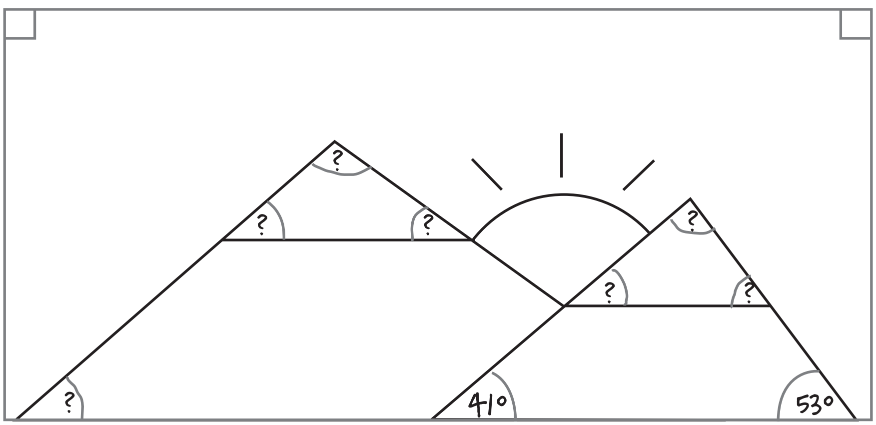


3) Now fold each of your strips of paper in half, join them up to make a triangle again, and draw around it.



Compare your drawings to investigate what happens to the angles of a triangle when you make it bigger or smaller—you just need to make sure that you do the same to each side of your triangle.

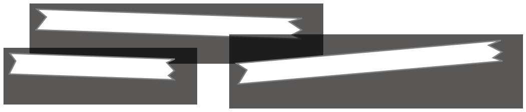
Does your investigation help you to fill in any of the mystery angles on the design?



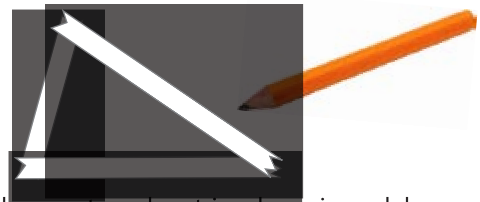
Geometry Construction Solution

Can you have two triangles with the same angles but with different lengths?

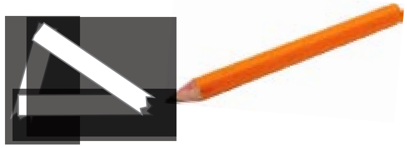
1) Cut or tear three narrow strips of paper, making them slightly different lengths.



2) Make them into a triangle and draw around it on some scrap paper.



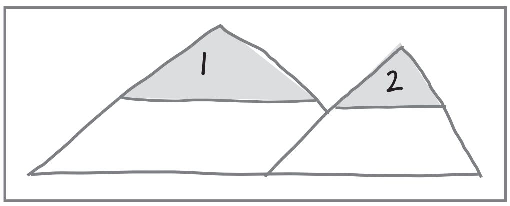
3) Now fold each of your strips of paper in half, join them up to make a triangle again, and draw around it.

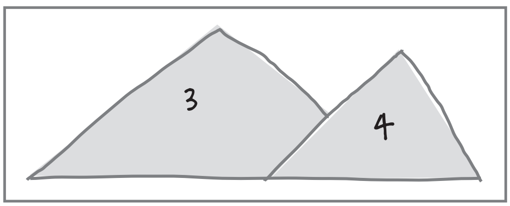


Compare your drawings to investigate what happens to the angles of a triangle when you make it bigger or smaller—you just need to make sure that you do the same to each side of your triangle.

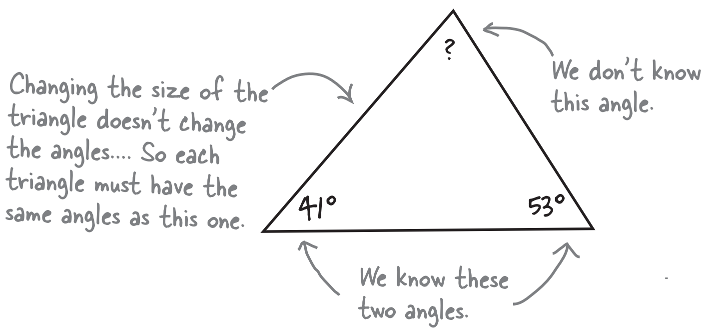
Making a triangle bigger or smaller doesn't change the angles of the corners—providing you change the length of all the sides by the same ratio.

Does your investigation help you to fill in any of the mystery angles on the design?

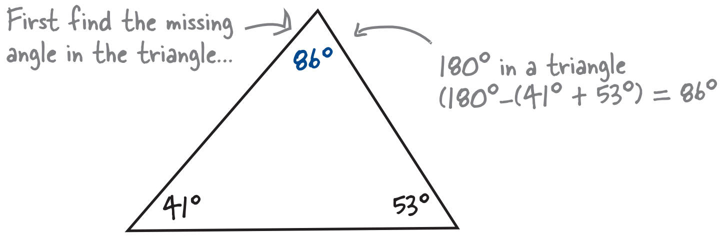


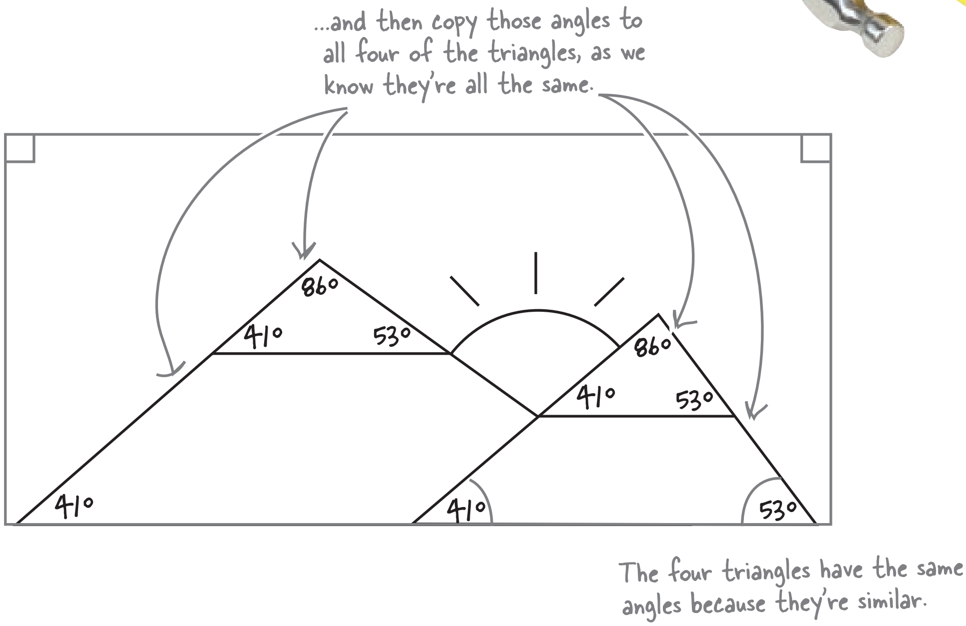


The mountains are basically made out of 4 of the same triangle in different sizes (one tucks in the back but the other corners are the same).







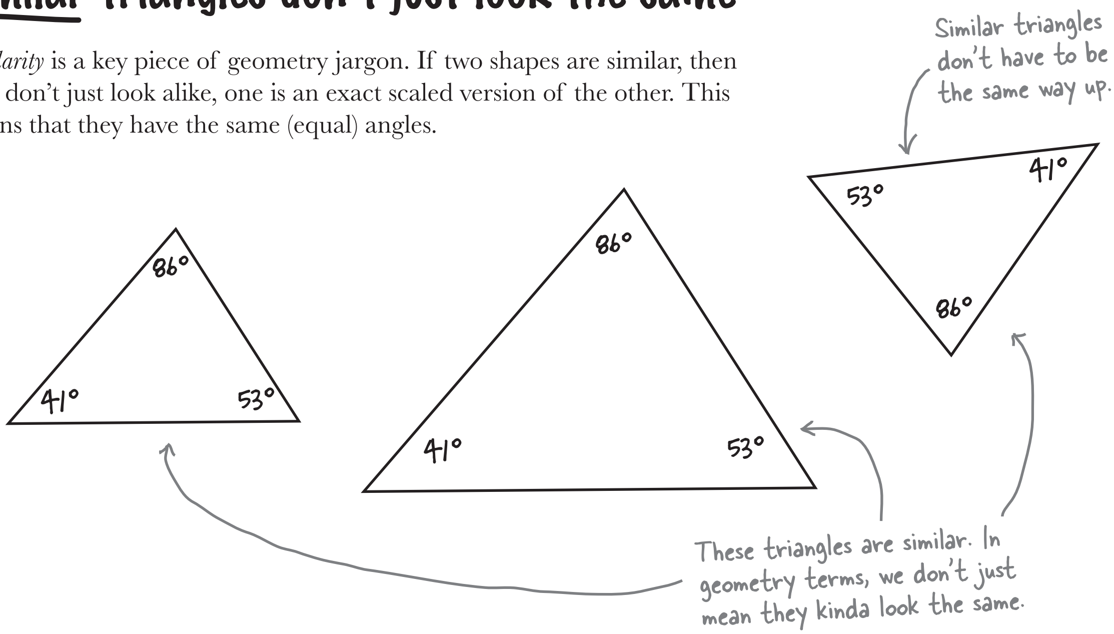


Hold on—didn't you spend Chapter 1 going on about how we couldn't trust our eyes to tell us if things looked right? Now you're saying these triangles are "similar"? Sounds flaky to me—what's the deal?



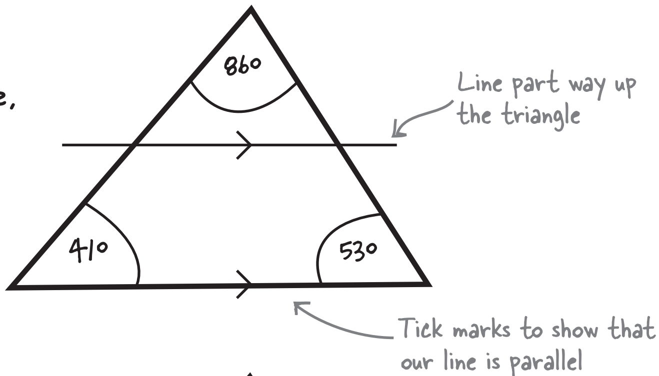
Similar triangles don't just look the same

Similarity is a key piece of geometry jargon. If two shapes are similar, then they don't just look alike, one is an exact scaled version of the other. This means that they have the same (equal) angles.

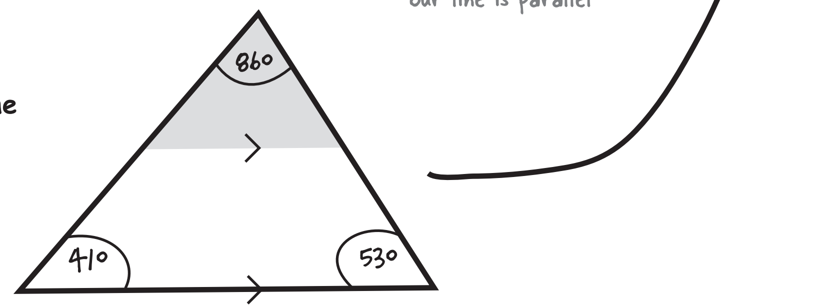


But don't take our word for it...check that it all adds up:

1 Take a big triangle, and draw a line part way up the triangle, parallel with the base.



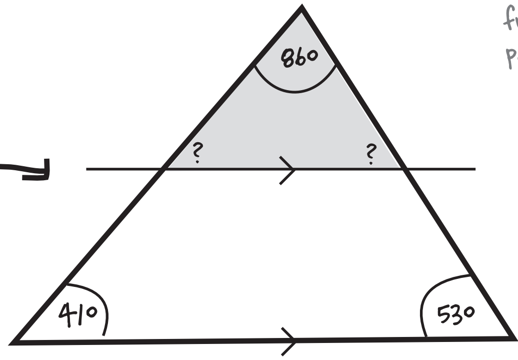
2 This creates a mini triangle which shares the angle at the peak of the big triangle.



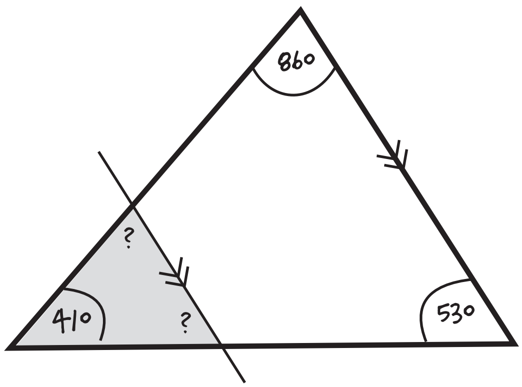
Sharpen your pencil

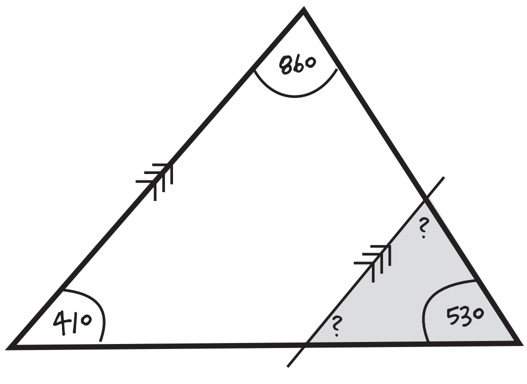
Using what you know about parallel lines and the F pattern, find the two mystery angles to complete the mini triangle you've created.

Remember the F, N, Z patterns from chapter 1? No? Check out page 40.



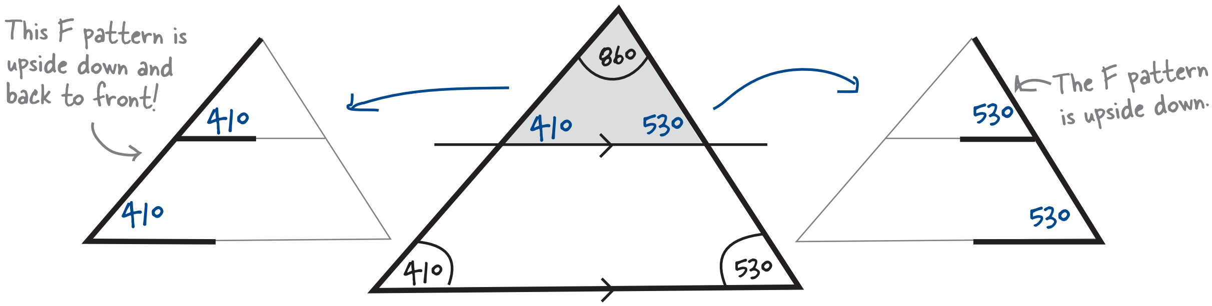
If the line is drawn parallel to one of the other sides, do you still end up with the same angles in your mini triangle?





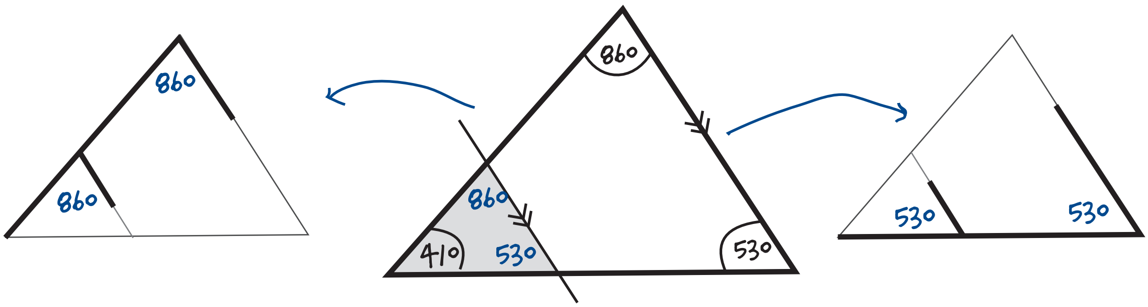
Sharpen your pencil Solution

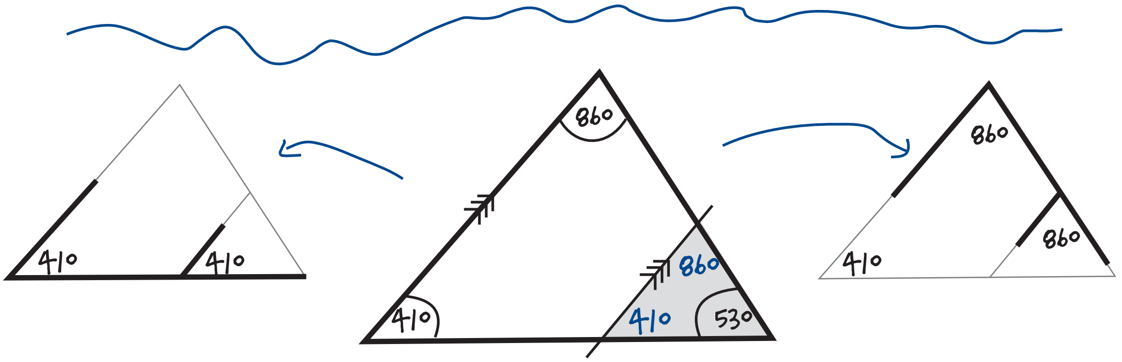
Using what you know about parallel lines and the F pattern, find the two mystery angles to complete the mini triangle you've created.



If the line is drawn parallel to one of the other sides, do you still end up with the same angles in your mini triangle?

The angles always come out the same—it doesn't matter which side the line is parallel to:

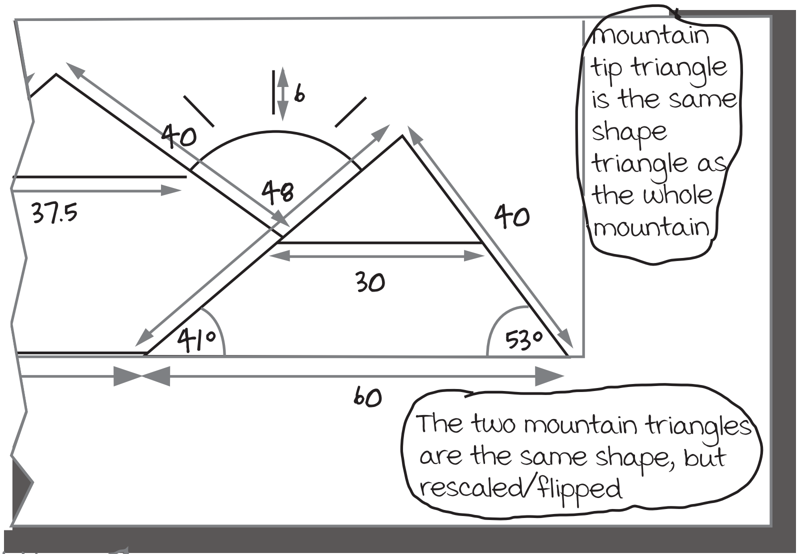




To use similarity, you need to be able to spot it

You won't normally get instructions that actually tell you that shapes are similar. So in order to use similarity, first you need to be able to be certain that shapes are similar.

You can do that by looking for matching sets of angles.

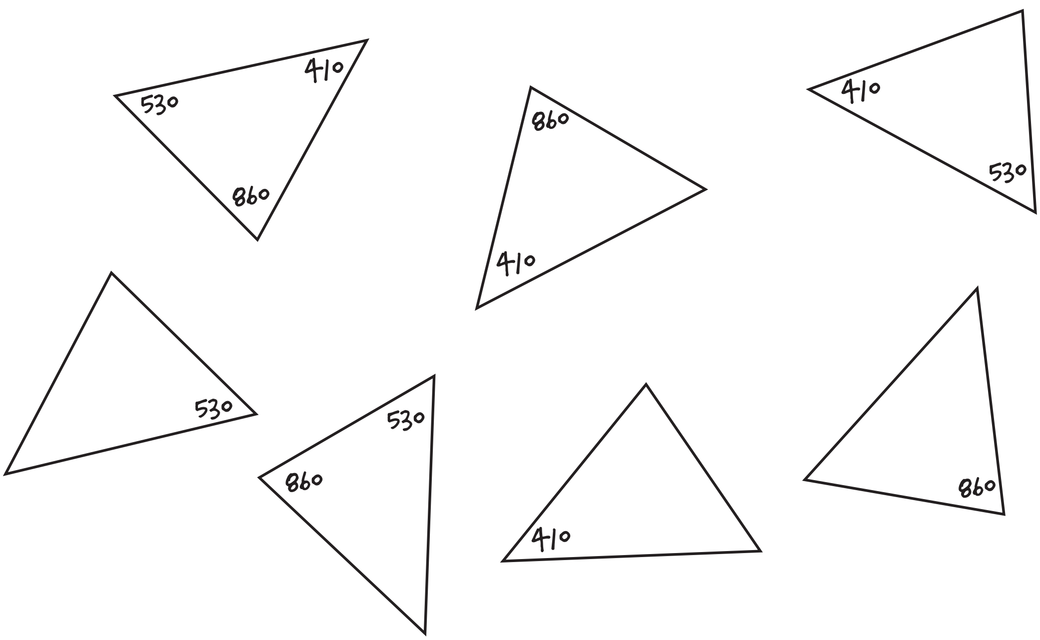


The designer's notes told us the shapes were similar.



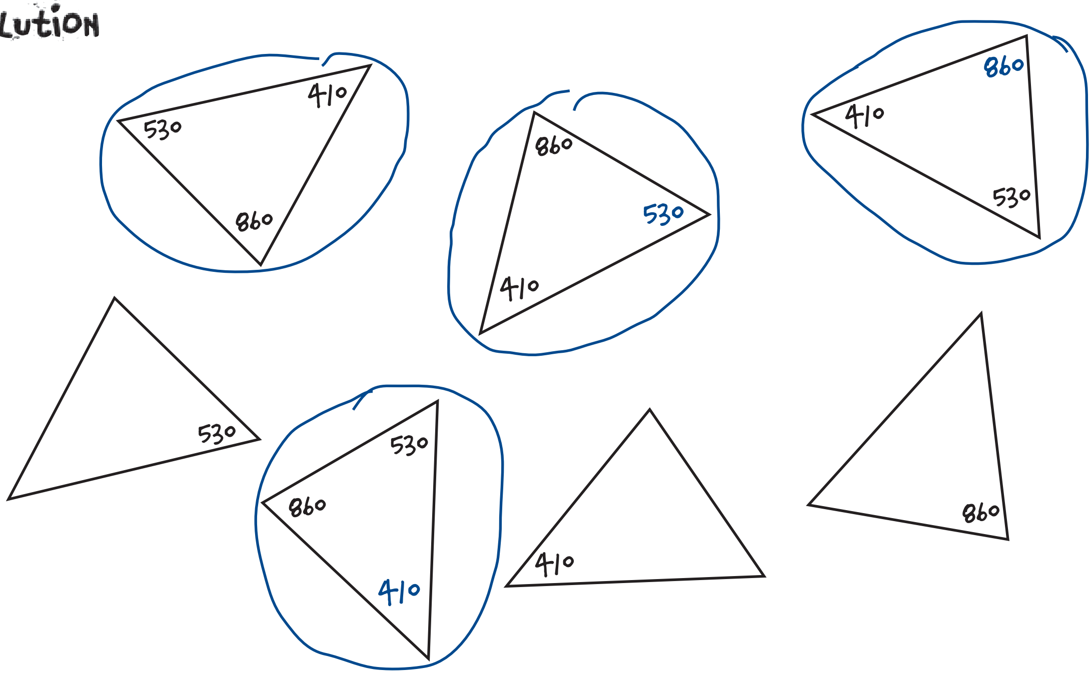
Exercise

Circle the triangles below that you can be SURE are similar to the triangle repeated in the design.



Exercise Solution

Circle the triangles below that you can be SURE are similar to the triangle repeated in the design.

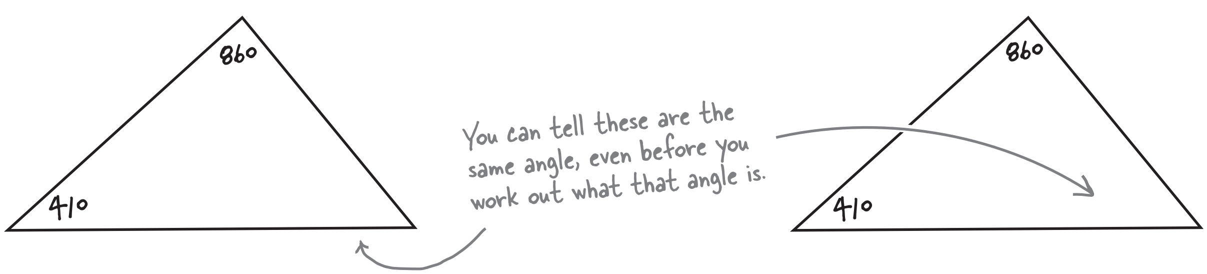


You can spot similar triangles based on just <u>two</u> angles

We know that angles in a triangle add up to 180 degrees, so once you've got two angles in each triangle, you can always work out the third.

And if you've noticed that two angles in one triangle are equal to two angles in another triangle, then you can tell the triangles are similar without even doing any math!

Look for two equal angles to spot similar triangles.



there are no Dumb Questions

Q: What if the triangles are flipped, so one has a 41° on the right and the other has a 41° angle on left? Are they similar?

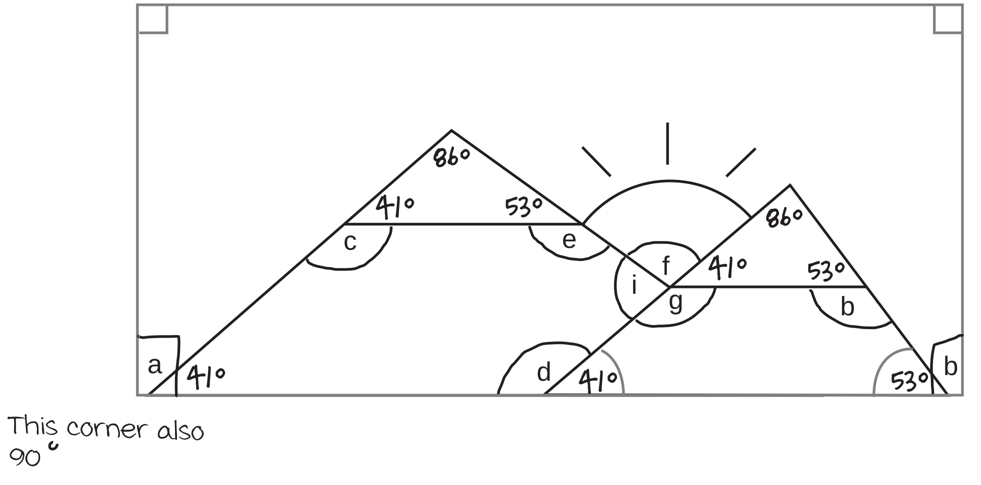
A: As long as you can spot another angle which is in both triangles then yes, they're definitely similar. Similarity is maintained even if your shape is reflected or rotated.

Q: Isn't using similarity kind of like cheating? Shouldn't I be working out all the angles individually?

A: Cheating? We like to think of it as working smarter rather than harder. It does save you plenty of leg work though. Most geometry teachers will be more impressed by use of similarity than repetitive calculations anyway—just make sure to make a note on your work that you used similarity.

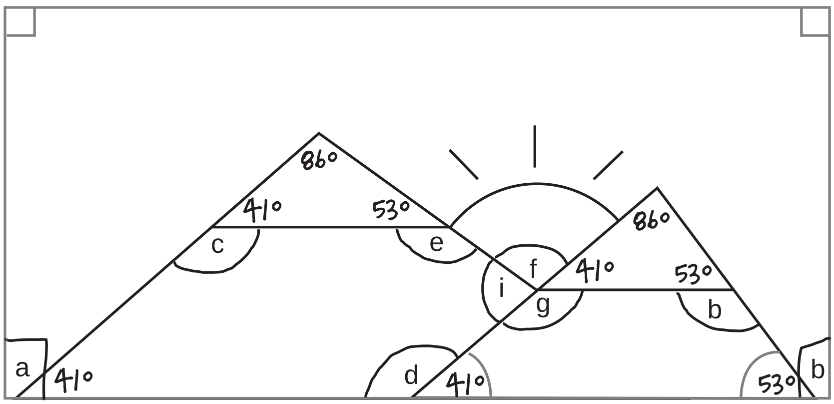
Sharpen your pencil

How many repeated angles are there in total on this diagram, including the ones you've already marked plus the angles **a** through **i**? (Count each value once—if it's repeated don't count it again.)



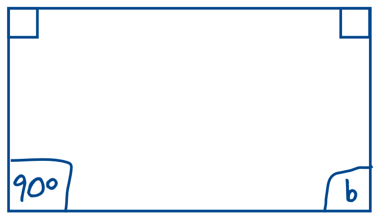
Sharpen your pencil Solution

How many repeated angles are there in total on this diagram, including the ones you've already marked, plus the angles **a** through **i**? (Count each value only once—if it's repeated don't count it again.)



This corner also 90°

a) The diagram tells us angle a is 90°.

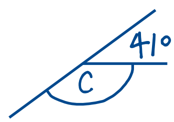


b) angle b completes the four-sided shape, so it must be

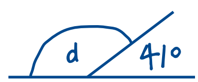
360° – (90° + 90° + 90°) = 90°

Angles in a four-sided shape always add up to 360°.

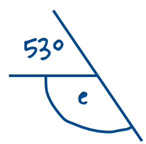
c) angle c is on a straight line with 41°, so C = 180° – 41° = 139°



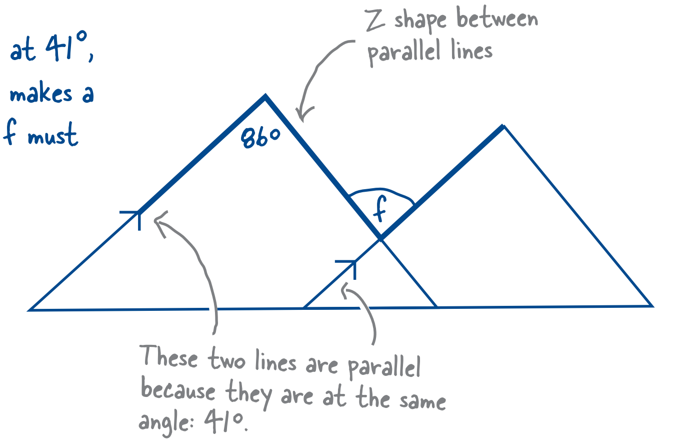
d) angle d is also on a straight line with 41°, so d = 180° – 41° = 139°



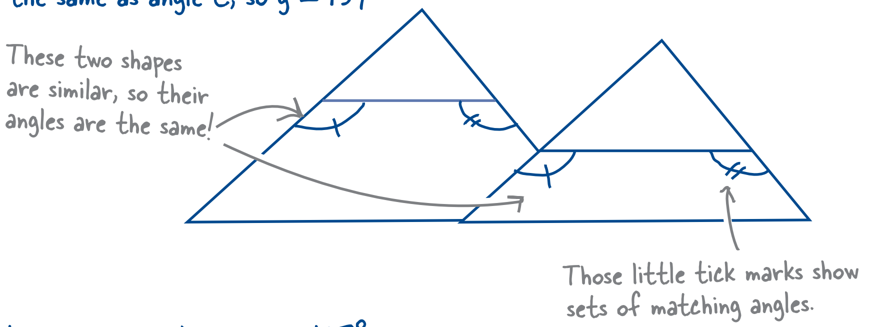
e) angle e is on a straight line with 53°, so e = 180° – 53° = 127°



f) The left sides of the two mountains are both at 41°, so they must be parallel. This means that angle f makes a Z shape (alternate angles) with the 86° peak, so f must also be 86°.



g) by similarity, angle g must be the same as angle C, so g = 139°



h) by similarity, angle i must be the same as angle e, so i = 127°

i) angle i is on a straight line with f, which is 86°, so h = 180° – 86° = 94°

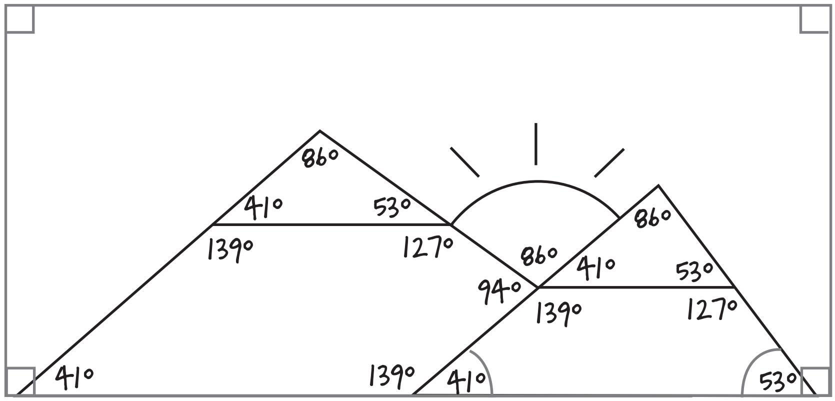
Different angles: 90°, 41°, 53°, 86°, 127°, 139°, 94°... = 7 different angles in total

Repeated angles: 90°, 41°, 53°, 86°, 127°, 139°... = 6 angles are repeated

A bunch are repeated, right? Were you surprised?

Employee of the month already?





Similarity helps you work smarter, not harder, to find missing angles

While it's great to be able to use your Geometry Toolbox for finding missing angles, when you spot similarity, you can zip straight through to filling them in without even breaking out your calculation skills.

BRAIN POWER

How can you tell if two squares or two circles are similar?



Similarity Exposed

This week's interview:
Similarity—faster or smarter—which are you?

Head First: Do you get a lot of pleasure out of saving people so much work? It must be nice to be liked.

Similarity: I do. It's why I do what I do, really. I'm a real stickler for recycling and conserving energy.

Head First: You mean like saving water?

Similarity: I mean like saving brain power! With similarity, you can do a calculation once and then reuse it over and over.

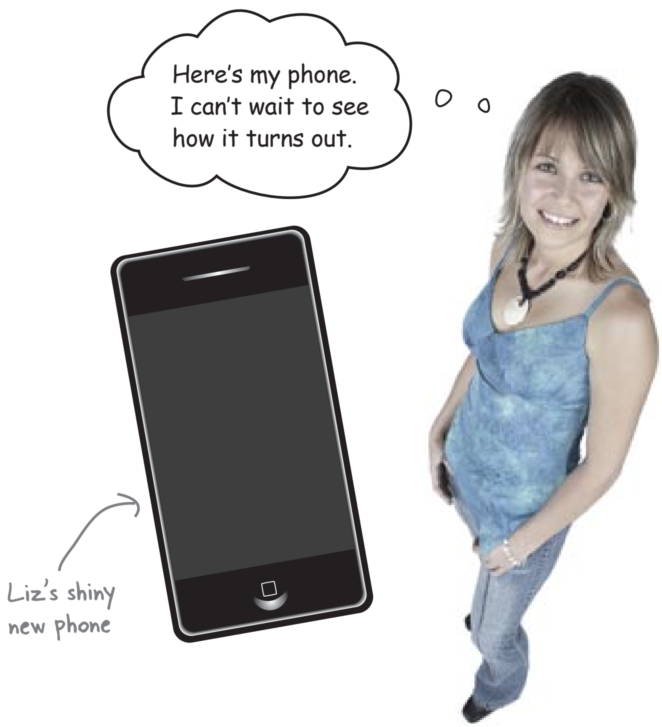
Head First: Oh, I see! Sorry…yes. So, what's the next step for you in your career?

Similarity: Well, really for me the next step is increasing recognition. I need people to know that I'm out here, waiting to get involved in saving them time and energy.

Head First: Well, that's certainly something we'd love to help you with. Thanks for the quick interview.

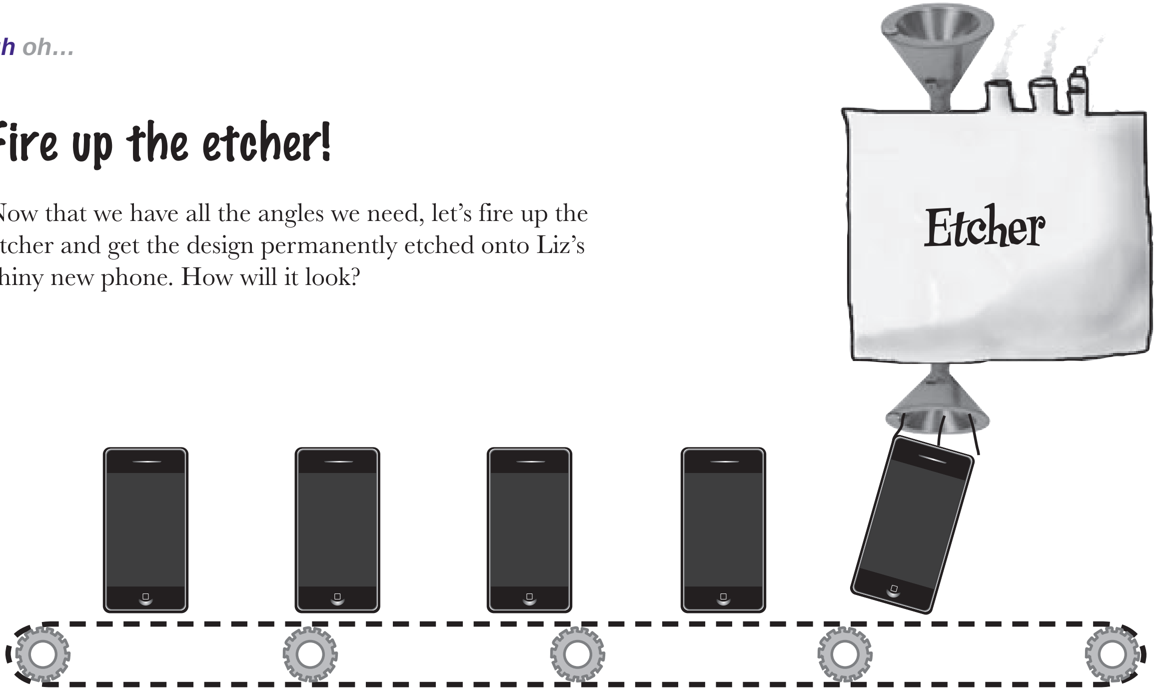
You sketch it—we'll etch it!

Now that you've figured out all of those missing angles, you've calculated everything that's needed to etch the design.



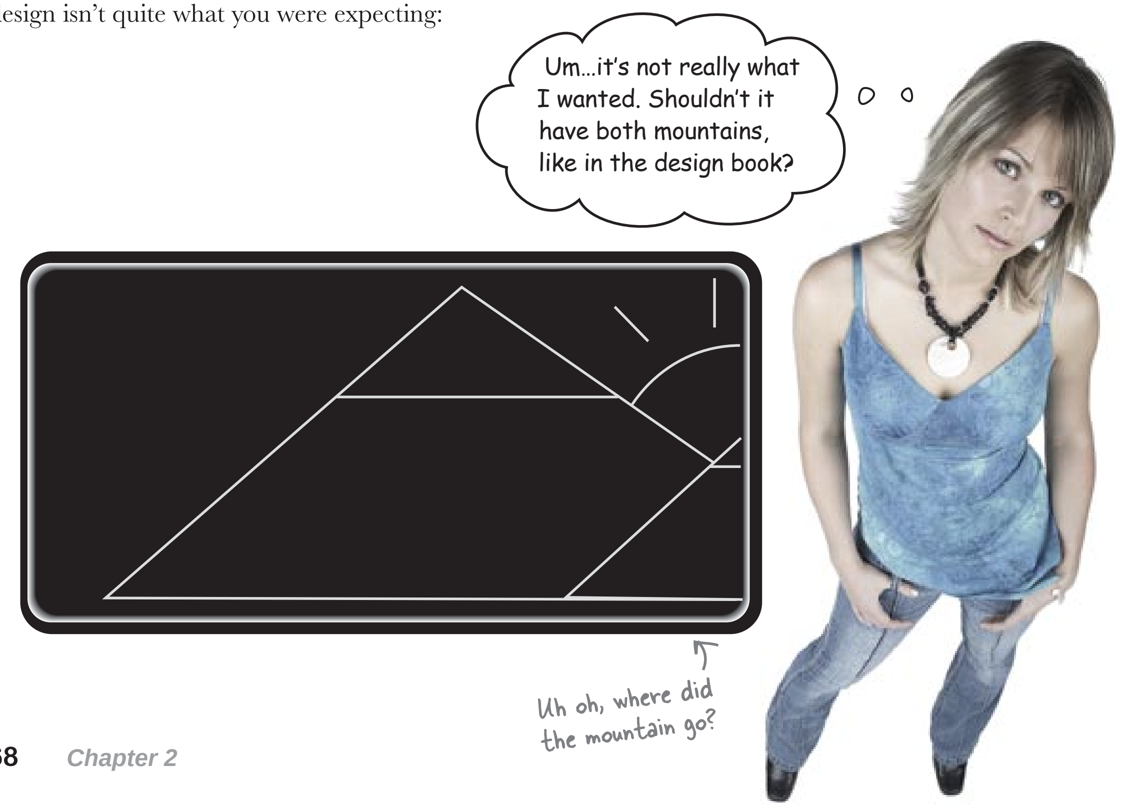
Fire up the etcher!

Now that we have all the angles we need, let's fire up the etcher and get the design permanently etched onto Liz's shiny new phone. How will it look?

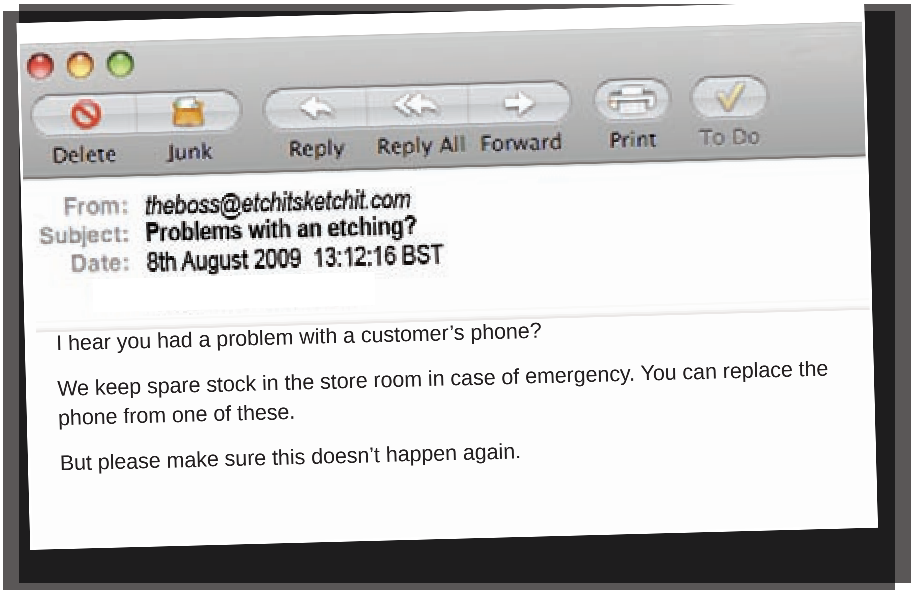


But something's gone horribly wrong...

When you remove the phone from the etcher, the design isn't quite what you were expecting:



The boss isn't happy, but at least you're not fired...


Delete Junk Reply Reply All Forward Print To Do

From: theboss@etchitsketchit.com
Subject: Problems with an etching?
Date: 8th August 2009 13:12:16 BST

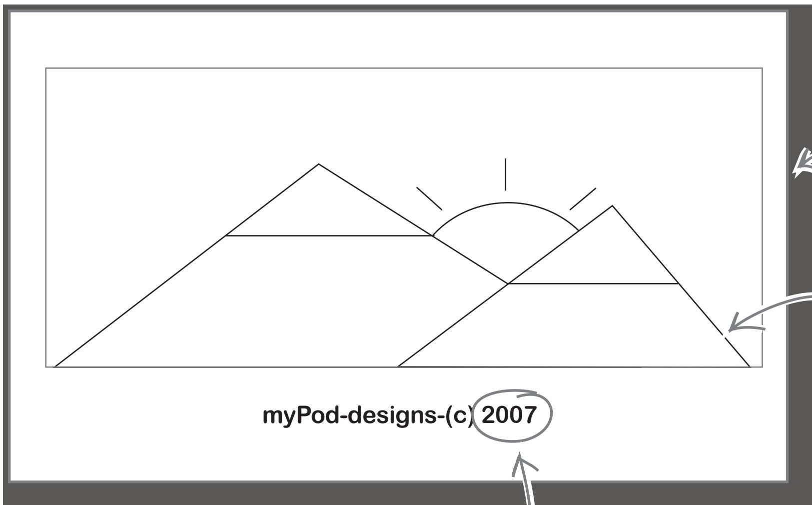
I hear you had a problem with a customer's phone?

We keep spare stock in the store room in case of emergency. You can replace the phone from one of these.

But please make sure this doesn't happen again.

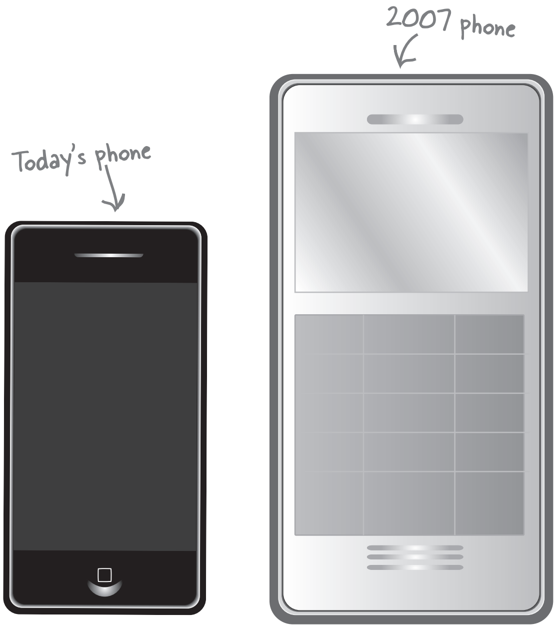
So what happened?

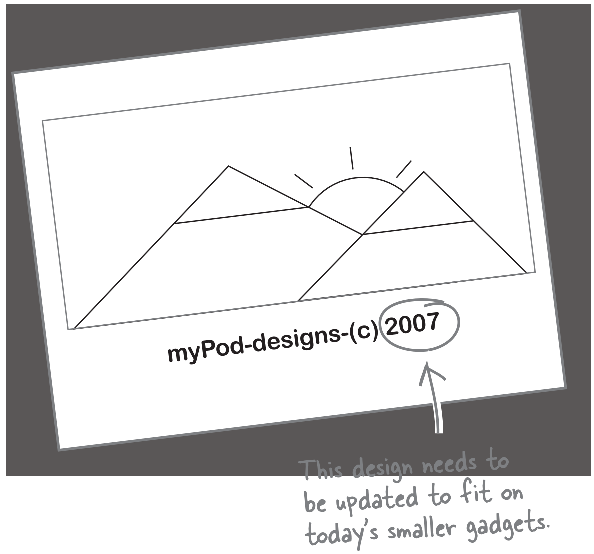
Here's what the design should have looked like. What do you think could have gone wrong?

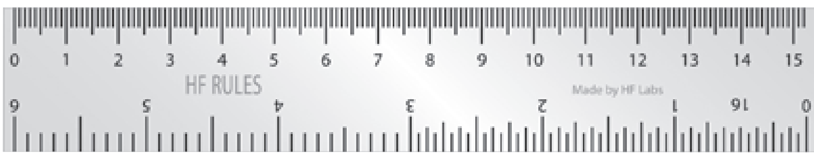


It's a problem of scale...

The design Liz picked was originally created in 2007, when phones were a lot bigger than the one Liz has today. In fact, the design is ***twice*** as big as the space available on the back of Liz's phone for etching!







...but how big a problem is it?

there are no Dumb Questions

Q: How come there weren't any units on the drawing? If they'd put units we could have seen whether it fit before we started etching.

A: You're right, we could have. In this case, though, the lengths on the diagram wouldn't be normal measures like millimeters or inches, but a special measure used by the etching machine.

Q: But shouldn't the drawing still have a scale? Wouldn't we be best to add one now?

A: For now we can just adjust the lengths we've got to work with—we can safely assume that whatever units the etcher uses, they aren't going to change before we etch again. But in the next chapter we're going to look at using scales in a lot more detail.

Pool Puzzle



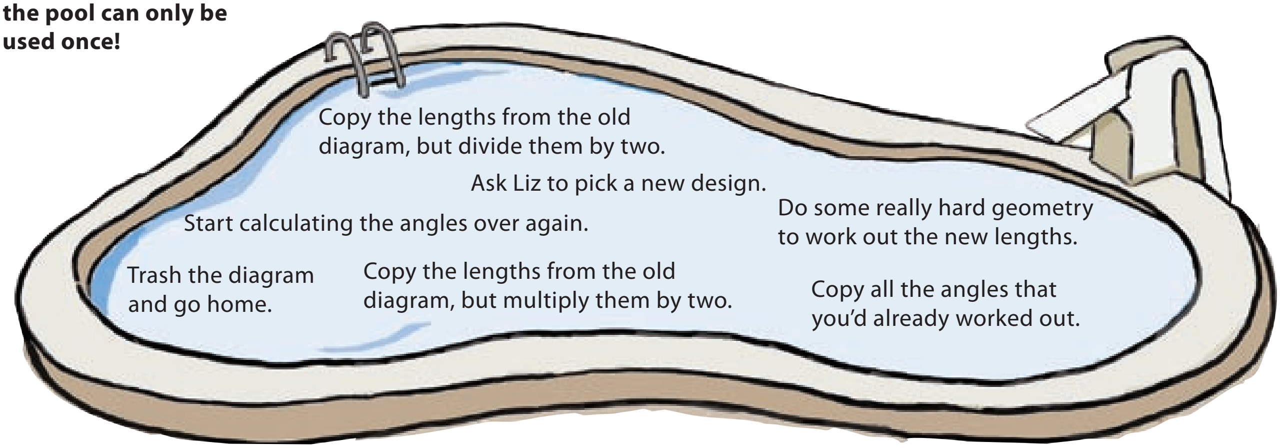
Your job is to take steps from the pool that you think will help you make the design fit on Liz's new phone, and use them to complete your to-do list. (You don't need to use them all.)

1) Create a clean copy of the diagram with no numbers on it.

2) ....

3) ....

Note: each thing from the pool can only be used once!



Pool Puzzle Solution



Your job is to take from the pool the steps you think will help you make the design fit on Katie's new phone, and use them to complete your to-do list.

1) Create a clean copy of the diagram with no numbers on it.

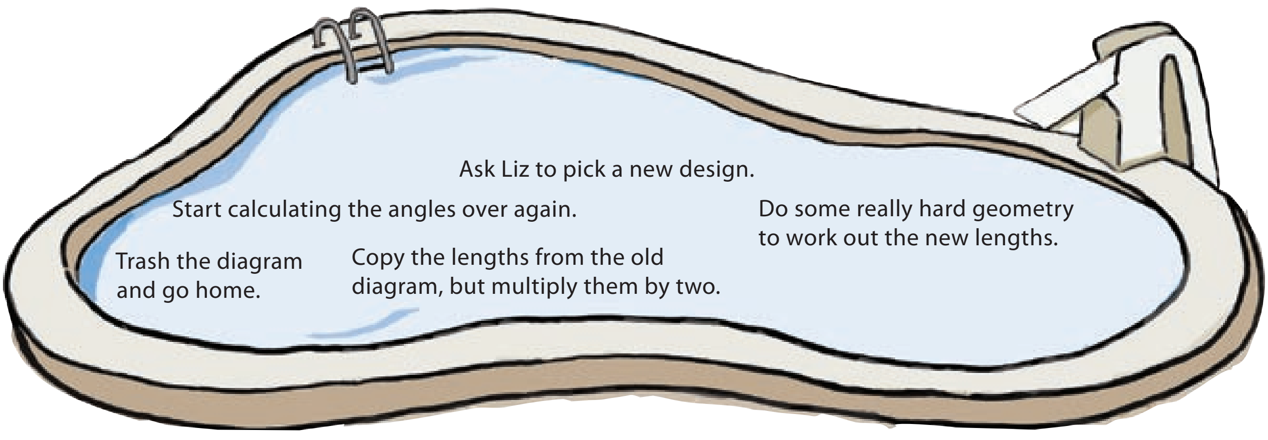
2) Copy all the angles that you'd already worked out.

3) Copy the lengths from the old diagram, but divide them by two.

When you shrink something evenly, the angles don't change.

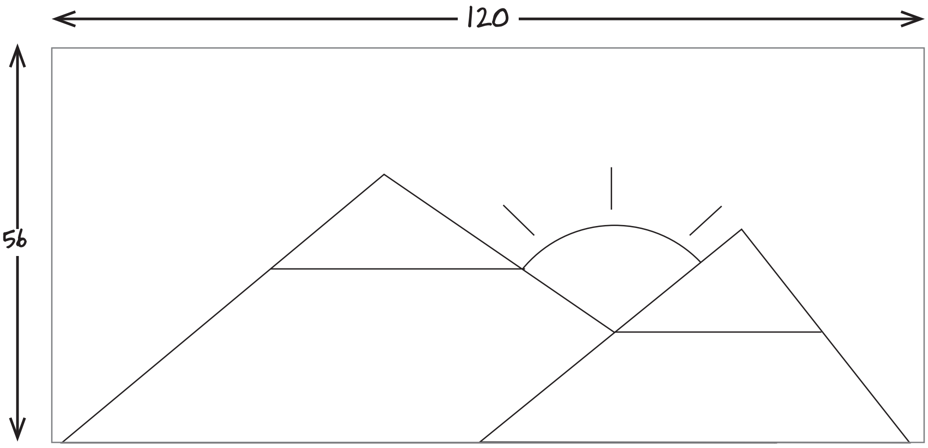
And all the lengths change by the same FACTOR.

A factor is a common multiplier—like if we doubled your pay and also doubled your hours, we would have increased both by a factor of 2.



Complex shapes can be similar, too

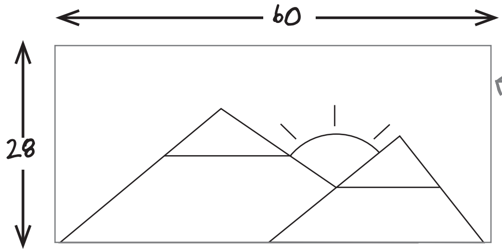
Similarity isn't just for triangles! Provided you shrink or grow your shapes **proportionally**, they can also be similar. When shapes are **proportional**, the **ratios** between the lengths of their different lines are the same.



The ratio of length to width in this design is 120/56.

60/28 and 120/56 are the same ratio.

Similar:



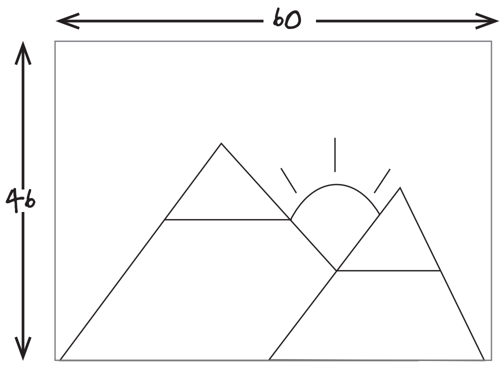
The ratio of length to width in this design is 60/28.

If you're not a wiz at fractions, you could check this on a calculator.

60/46 and 120/56 aren't the same ratio.

The ratio of length to width in this design is 60/46.

Not Similar:



BRAIN POWER

Can you use proportionality to tell if shapes are similar even if you don't know ANY of their angles?



Similarity Exposed II

This week's interview:
Ratios or angles, which is the real similarity?

Head First: You're really becoming popular—a lot of people are saying you're the time-saving technique they wish they'd always known.

Similarity: Yes—it's nice of you to say so! I do like to think I'm rather, um, efficient is the best word, I guess.

Head First: That's certainly true! But there's one thing I'm wondering.…

Similarity: Go on.…

Head First: Well, people recognize you by matching angles—and others use the proportional thing—and I'm just wondering, which is the *real* you?

Similarity: I don't understand. You mean you think I can only be one or the other?

Head First: Well, surely one is what you're really about, and the other is just a convenient alternative way of presenting yourself. I want to get to the heart of the real similarity—who are you when you're just relaxing at home?

Similarity: Well, to be honest, I really am always both! I know it sounds silly, but I've never thought of my different aspects as being separate. With triangles, and a lot of other shapes, too, if the angles are matching, then the sides are also proportional. I can't really pick and choose one or the other!

Head First: And what about if you've got proportionality; if ratios between the lengths of a triangle are the same, but you don't have matching angles? Do you feel something is missing?

Similarity: But that could never happen with a triangle! That's just how it is. Anytime triangles have the same ratios, they have the same angles. You've made me anxious now…but honestly, it's just not possible. Proportionality and angles—with triangles it's always about both, equally together!

Head First: Together? I didn't know you were mixing it up like that. Interesting.… Now, you said, "a lot of other shapes, too"—that suggests that it's not always the case that angles and proportionality go together?

Similarity: Ah, well, there are some shapes that are different. Take rectangles for example. All rectangles have the same angles—90, 90, 90, and 90 degrees. But they aren't all proportional—you can have long skinny ones and short fat ones.

Head First: So you don't work with rectangles at all?

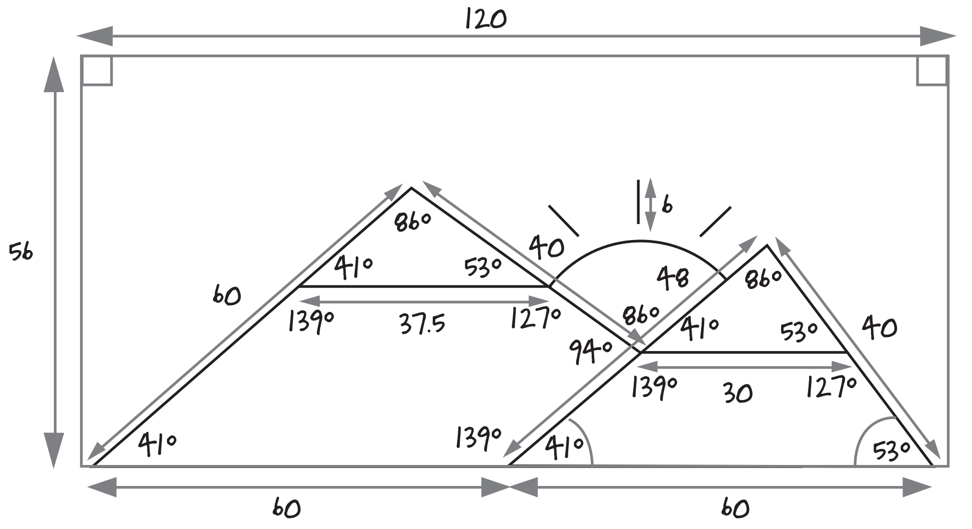
Similarity: Oh, I do. But only proportional ones. Like if you had a rectangle with sides 3 and 6, and one with 4 and 8—you'd know they were similar. And squares! I love squares. All of them are similar. Every single one. Beautiful. Just beautiful.

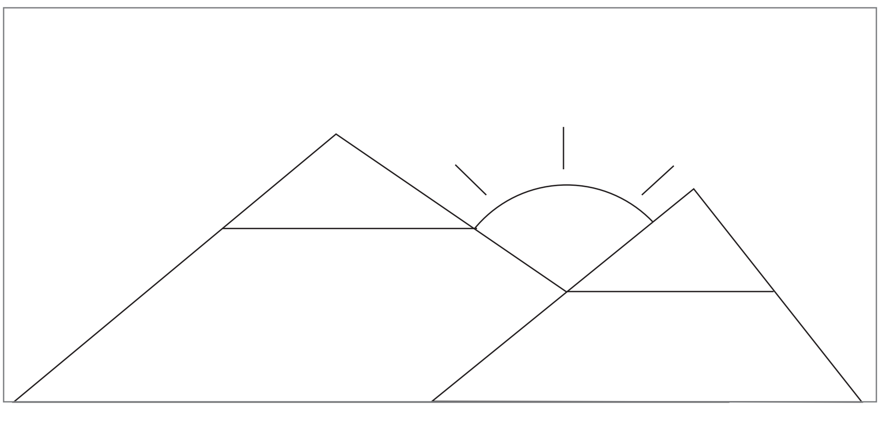
Head First: Right. Beautiful squares, eh? Thanks for the interview.

You can spot similarity using angles or the ratios between lengths or sides, or both.

Sharpen your pencil

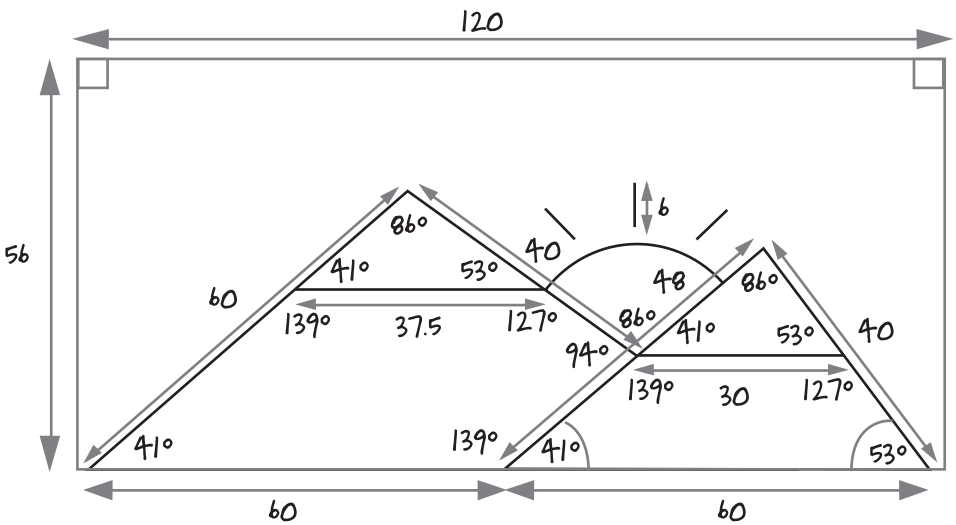
Based on the old diagram and the angles you'd figured out earlier, mark up a fresh design to fit Liz's phone. It needs to be half the size of the original.

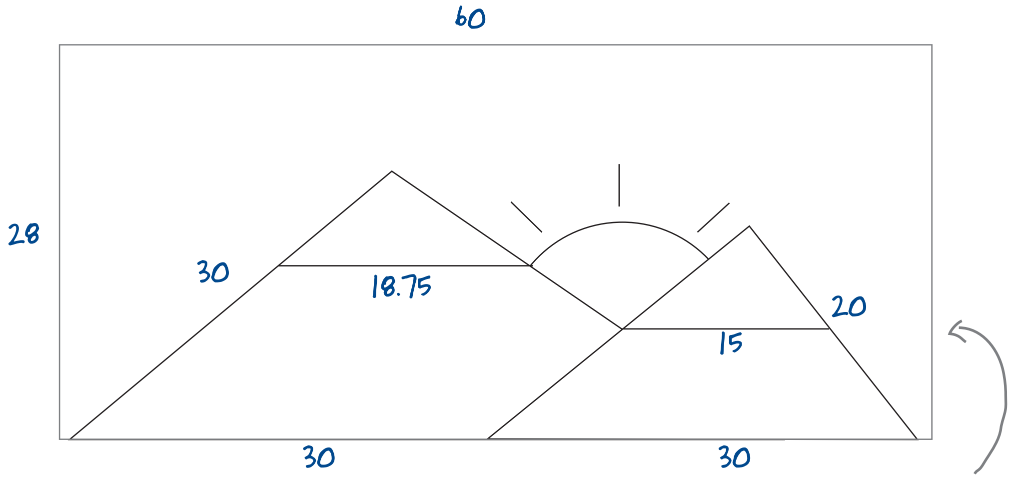




Sharpen your pencil Solution

Based on the old diagram and your angle workings, mark up a fresh design to fit Liz's phone. It needs to be half the size of the original.



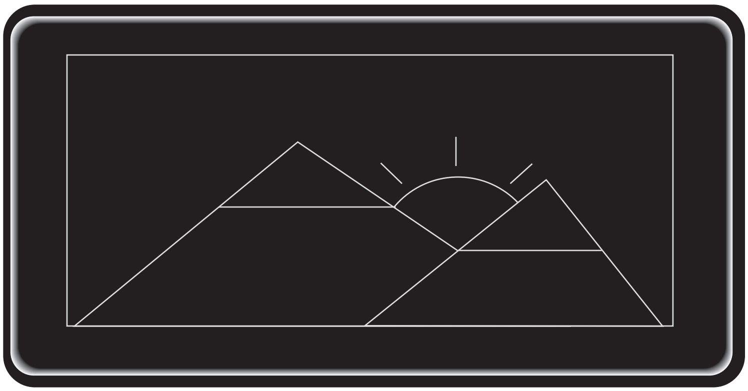


All the angles are the same, but the lengths need to be divided by two.

You sketch it—we'll etch it (to fit)

Liz is a very patient customer, she's hung around while you used similarity and proportionality to resize the design to fit perfectly on her new replacement phone. It does look good though!





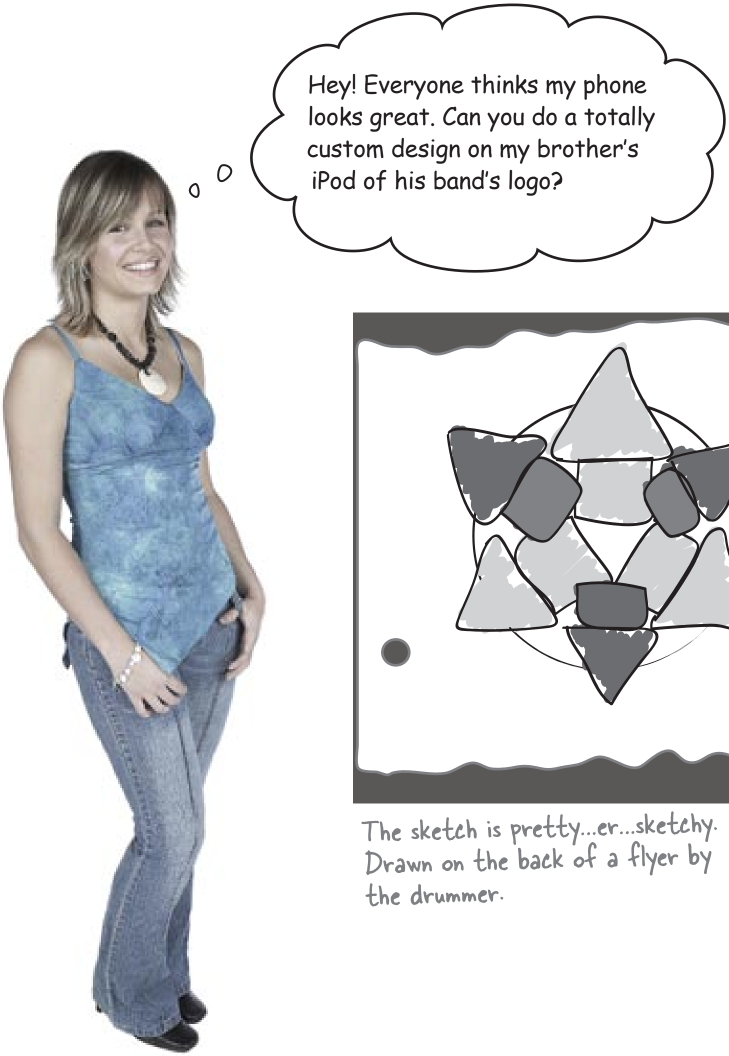
BULLET POINTS

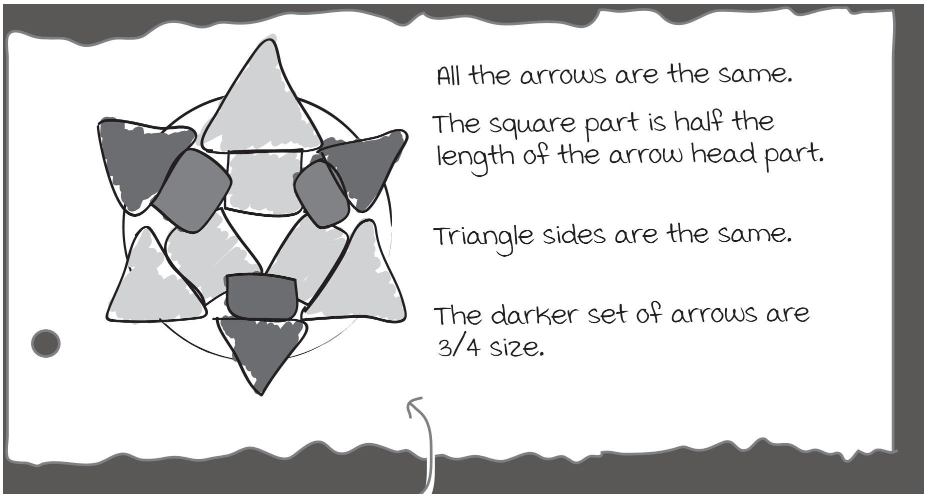
- **Similar** shapes have the **same angles.**
- Similar shapes have the **same ratios** between **lengths of sides.**
- Similar triangles have the same three angles (and you can tell from **just two**).
- Some shapes are always similar.
- All **circles** are similar.
- All **squares** are similar.

Just try drawing a square which doesn't have four angles all 90° and all sides the same ratio: 1/1.

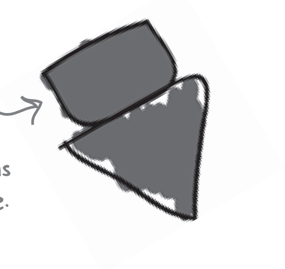
Liz is back with a special request

The great thing about happy customers is that they just keep coming back. Impressed by the effort you put in to getting her phone just right, Liz is trusting you with another great gig.





The sketch is pretty...er...sketchy. Drawn on the back of a flyer by the drummer.



An "arrow" is one of these.

Sharpen your pencil

Before you start sketching the design, what lengths and angles do you need to find? Could you use similarity to save yourself some time and effort?

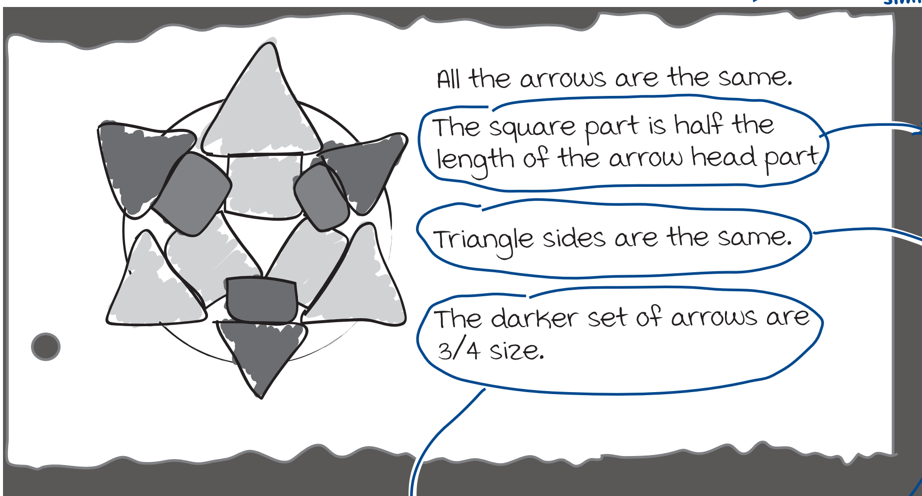
Sharpen your pencil Solution

Before you start sketching the design, which lengths and angles do you need to find? Could you use similarity to save yourself some time and effort?

We need to find the lengths and angles of the sides of one of the small triangles and one of the big triangles, plus the lengths of the sides of one of the small squares and one of the big squares.

Although there aren't any length or angle markings on the diagram, the instructions give us plenty of clues—and there's a ton of similarity going on here.

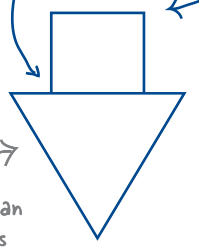
The diagram is made up of six similar arrow shapes like this.



A square with sides half the length of the triangle sides.

A triangle with three sides all the same length.

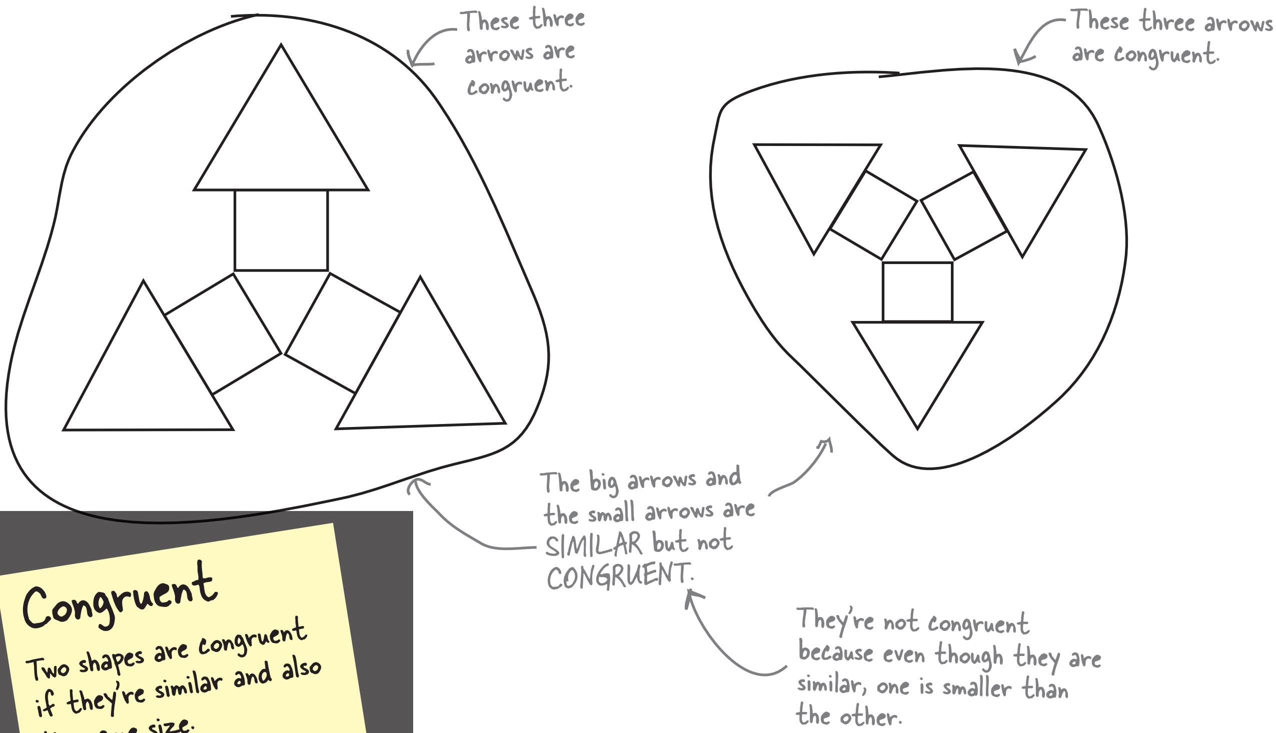
There are three large arrows and three smaller ones, 3/4 of the size of the large ones.



This kind of triangle is known as an equilateral triangle (the note says the three sides are the same), and all three angles are the same as well. $180°/3 = 60°$

Similar shapes that are the same size are congruent

Shapes that are similar have equal angles and are proportional, but if they're actually the exact same size, then we say that they're **congruent**.



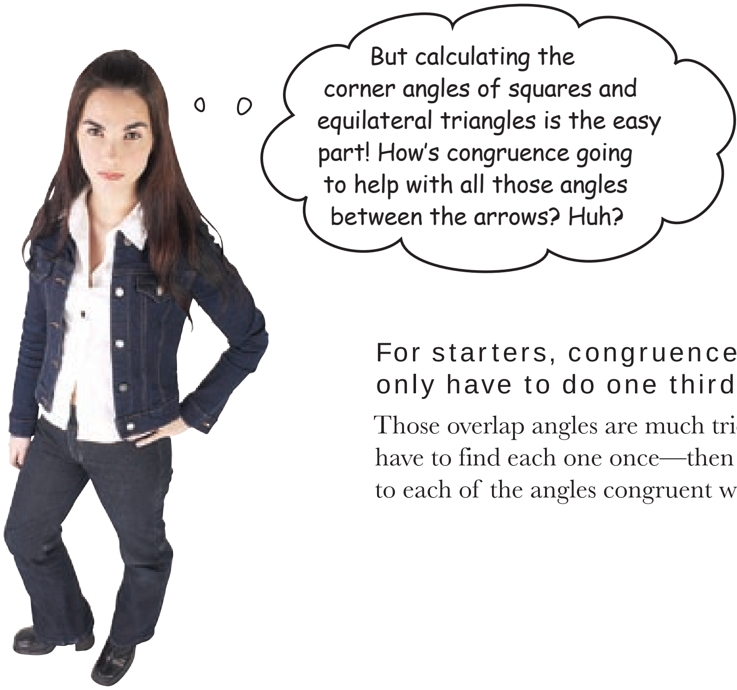
Congruent

Two shapes are congruent if they're similar and also the same size.

Two shapes are incongruent if they are not the same size.

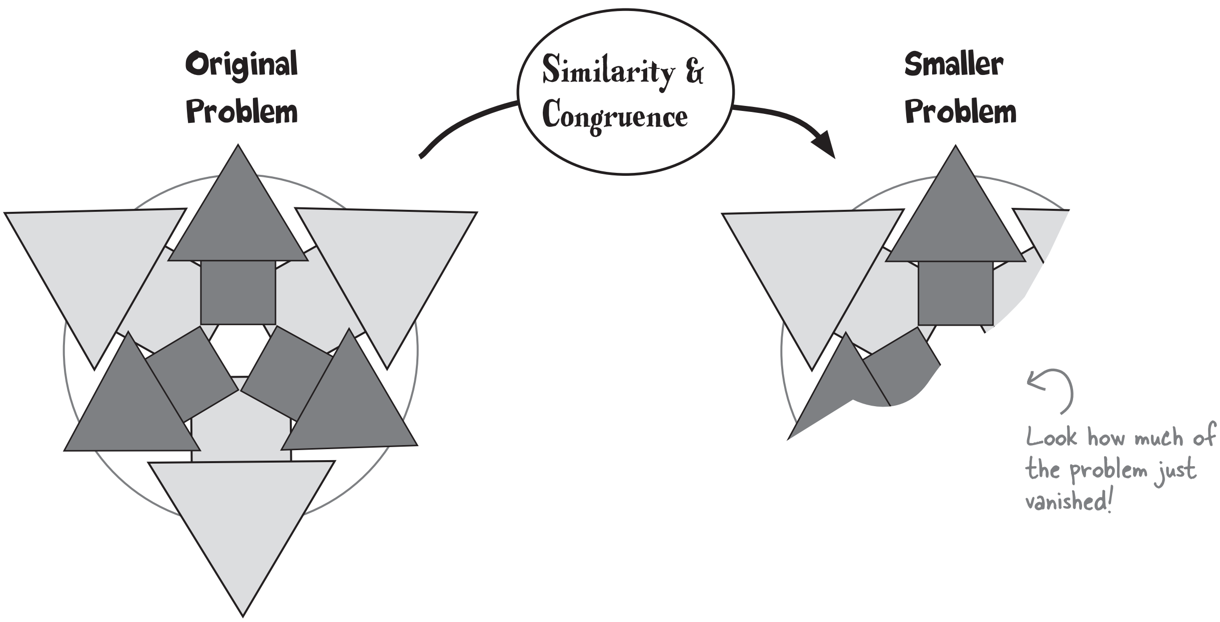
BRAIN POWER

How can spotting congruence save you even more time and work than similarity?



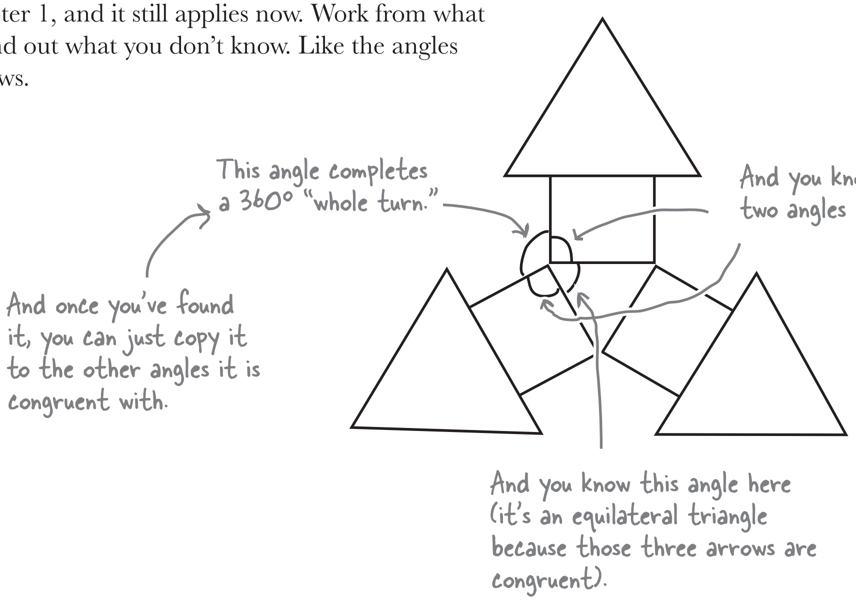
For starters, congruence means you only have to do one third of the work.

Those overlap angles are much trickier, but you'll only have to find each one once—then you can just copy it to each of the angles congruent with it.



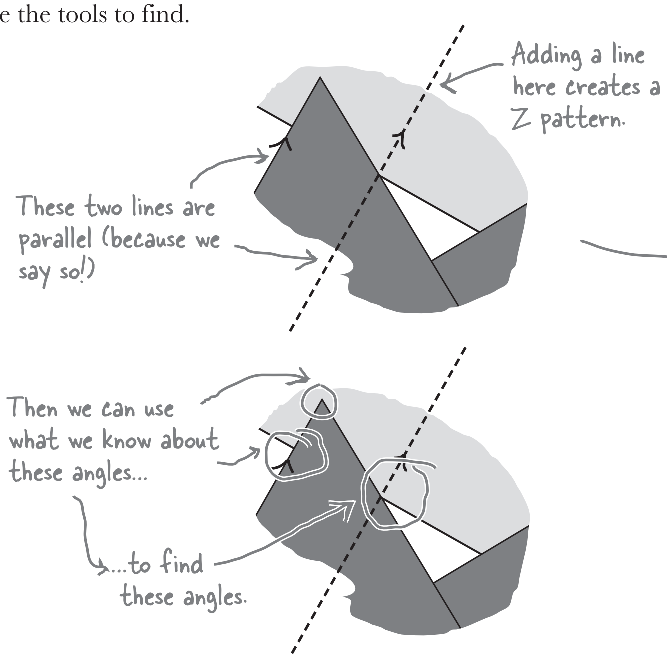
Use what you know to find what you don't know

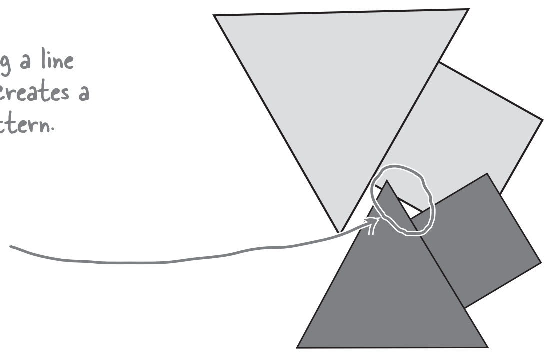
We said it in Chapter 1, and it still applies now. Work from what you do know to find out what you don't know. Like the angles ***between*** the arrows.



And if you don't have what you need, add it!

You can add parallel or perpendicular lines to your sketch to break down the missing angles into parts you have the tools to find.



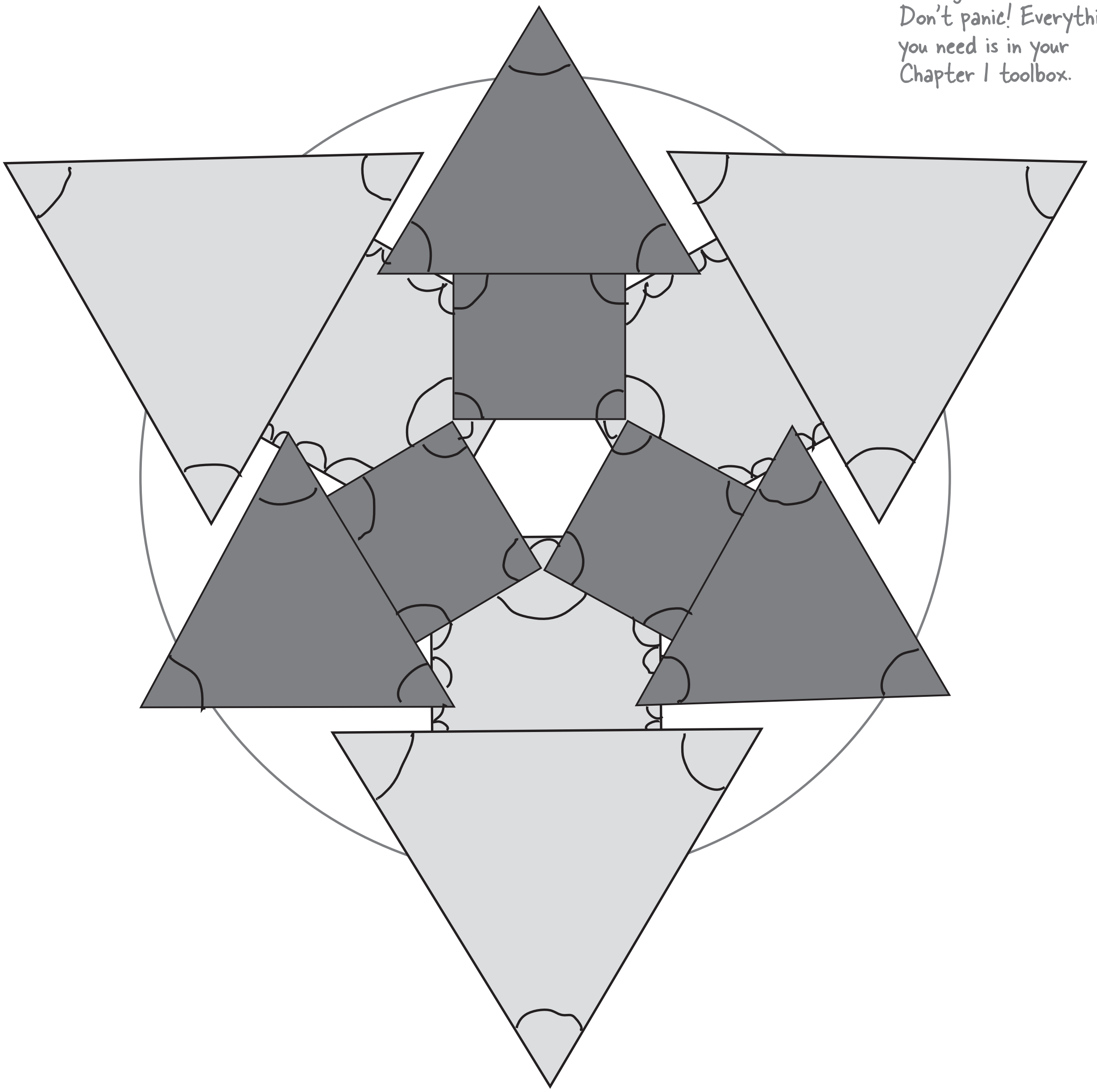


Ready to kick some serious design butt?

Exercise

There are 60 angles on the band's logo design. Use the space on the right to start working on the sketch and calculate them all. How many of each different angle are there?

Feeling overwhelmed? Don't panic! Everything you need is in your Chapter 1 toolbox.



Exercise Solution

There are 60 angles on the band's logo design. Calculate them all. How many of each different angle are there?

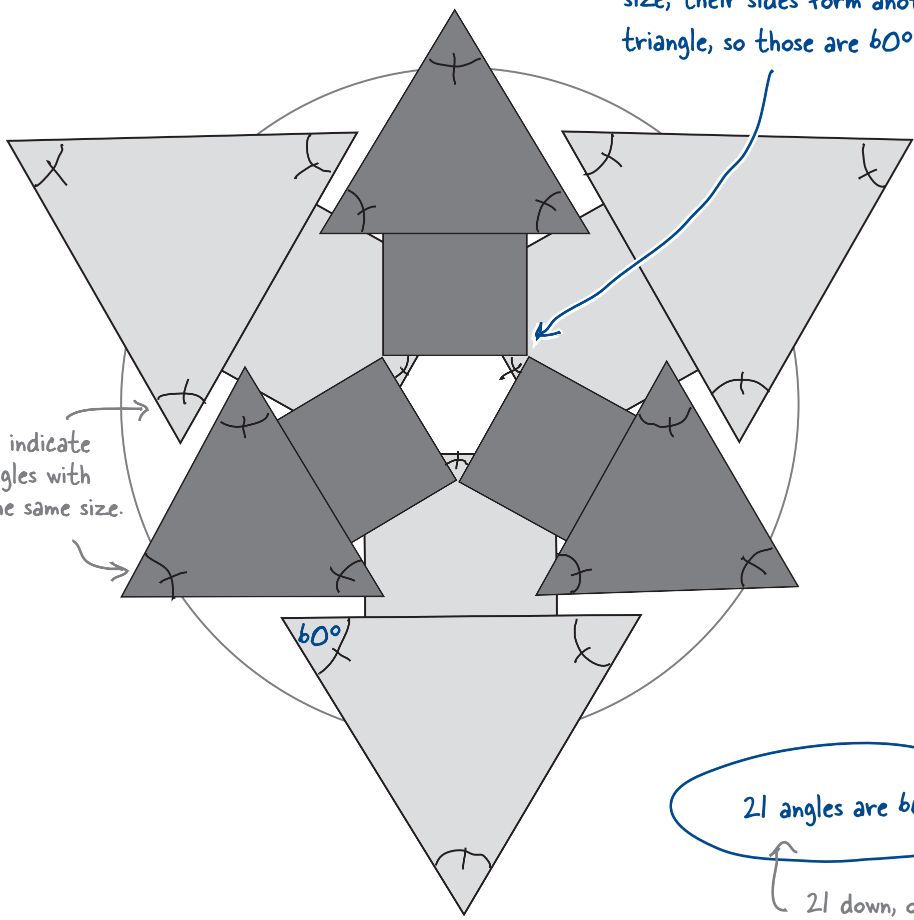
There are 21 angles that are 60°, 24 angles that are 90°, 9 angles that are 120°, and 6 that are 150°.

Here's how you can find them all:



Each arrow head is an equilateral triangle, with 3 equal angles: 180° / 3 = 60°.

At the center of the design the darker arrows meet. Since they are all the same size, their sides form another equilateral triangle, so those are 60° as well.



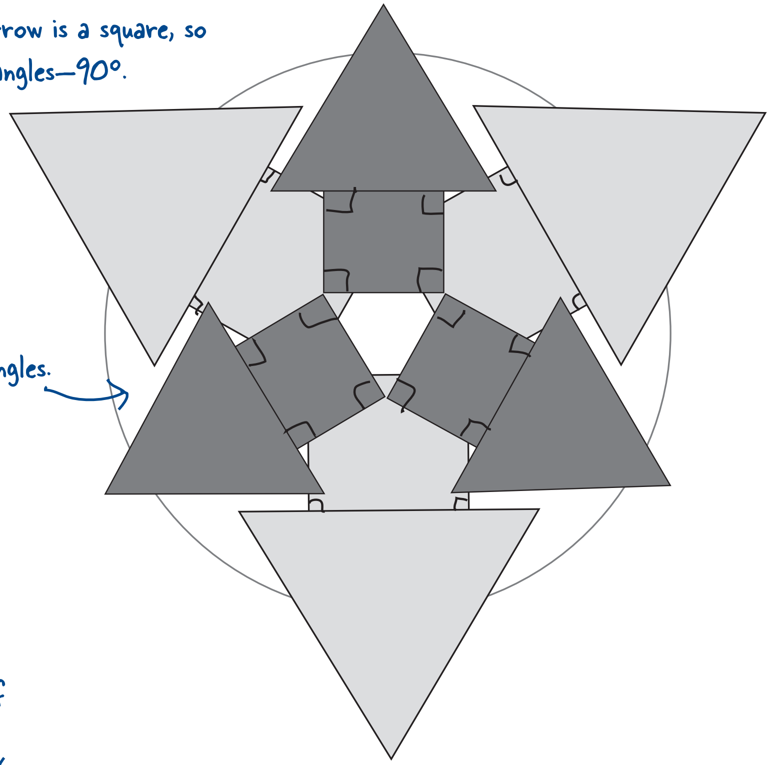
The tick marks indicate that all the angles with one tick are the same size.

21 angles are 60°.

21 down, only 39 to go!



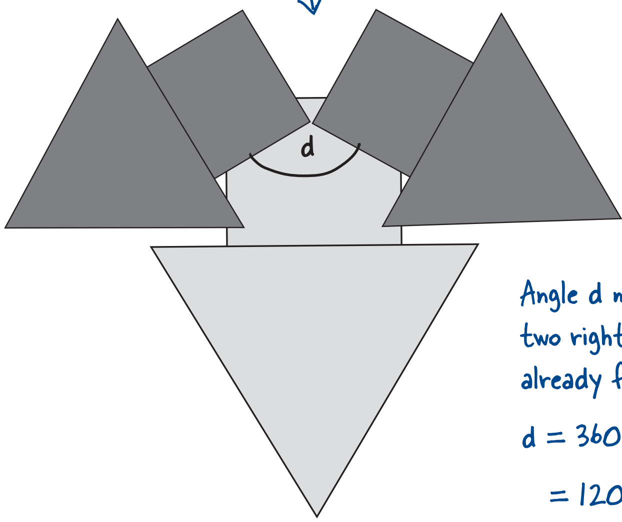
The bottom of each arrow is a square, so those have four right angles—90°.

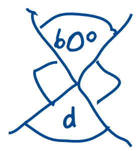


That's 18 right angles.

39 down, only 21 to go!

To find the remaining angles, let's use similarity and just work on a chunk of the design that is repeated.





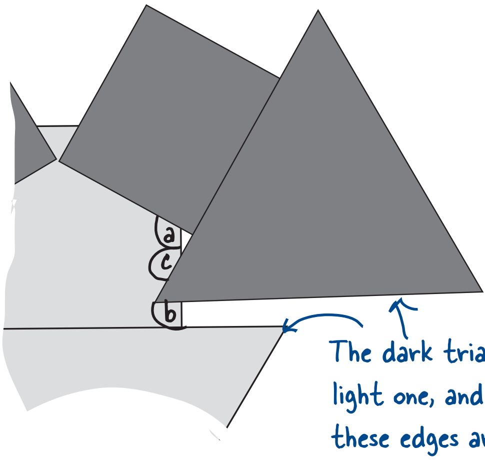
Angle d makes a whole turn (360°) with two right angles and the 60° angle we already found, so:

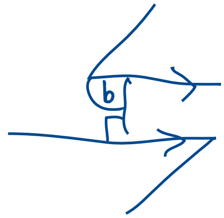
$d = 360° - (90° + 90° + 60°)$

$= 120°$ (there are 3 of these)

42 down, only 18 to go...

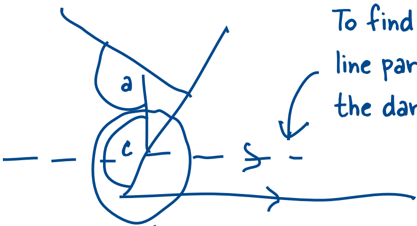
Exercise Solution ...continued



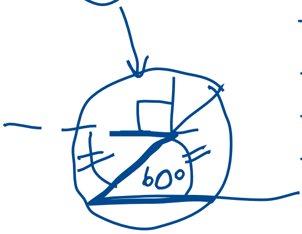


The dark triangle is turned 60° compared to the light one, and each angle of the triangle is 60° so these edges are parallel. A line which is perpendicular to one line is also perpendicular to lines parallel to that line, so b is also 90° (another 6).

48 down, 12 to go....



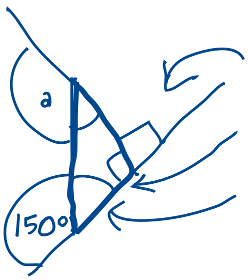
To find angle c, draw a line parallel to the base of the dark triangle.



This creates some equal angles—the Z pattern. Angle c is this Z angle (60°) plus the perpendicular (it's one side of the square and we drew our line parallel to the bottom of the square), so

$c = 60° + 90° = 150°$ (6 of these, too!)

54 down...just six left...

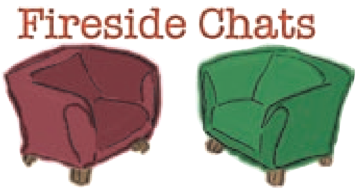


To find angle a, first find the angles of this triangle.

This angle is the supplement of a right angle we already found, so it's 90°.

This angle is the supplement of the 150°we just found, so it's 30°.

The angle next to a is the remaining angle in the triangle = 60°, so a is the supplement of this angle = 180°– 60° = 120° (6 of these as well).



Tonight's talk: **Similarity and Congruence**

Similarity:

I'm worried about you. You seem neurotic, always concerned about being a size zero, or seven, or whatever. You need to relax more.

Congruence:

But size DOES matter. It's all very well having your angles right on paper, but if something is too big or too small, in the real world size actually matters.

Similarity:

I just think if you stopped worrying so much about size and focused on proportionality you'd have more opportunities. I get used all the time—my flexibility is a real asset.

Congruence:

And potentially a real headache! Sometimes it doesn't cut it just being similar—and that's where I come in, when you need to rely on size as well as shape. Plus, I'm faster to use.

Similarity:

Faster? How? I'm pretty fast, you know!

Congruence:

Yeah, you're quick, but even if something's similar, if it's not congruent, then you've gotta do some math for the lengths. OK, it's only multiplication and division, but it all takes time.

Similarity:

True. And I'm a big stickler for efficiency. People would do well to use both of us more!

Congruence:

Yes. Are you getting much non-triangle work these days? I'm mostly getting triangle stuff still, and it's not that I don't like it…but you know, I have so much more potential.

Similarity:

Oh, tell me about it! I think they forget that we're not all about the triangles. And that we travel so well.…

Congruence:

Completely—I love to travel! You can flip me upside down, back to front, spin me around and move me from one place to another, and I still work just as well.

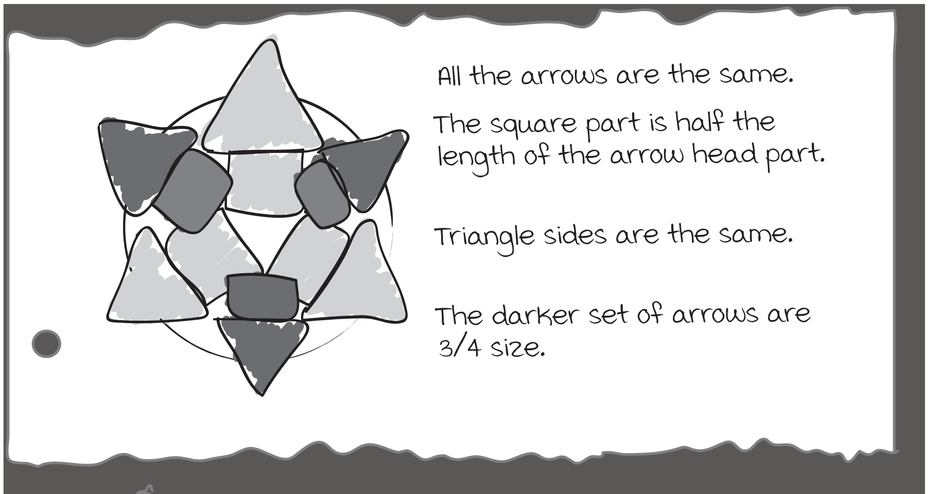
Similarity:

Great to catch up! I'll let you know if I get any work you could help out with.



Hey—I had a cool idea. Is there any way you could work up the design so that maybe we could get some t-shirts printed, too? The band would be psyched!

It's an interesting suggestion—could you represent the design in a way that means that you could use it to etch gadgets of any size, and maybe even print it on T-shirts?

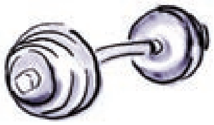


Can you really get this diagram to fit perfectly onto all of these without doing a ton of work for each different size?









BRAIN BARBELL

It would be cool to finish marking up your design of the band's logo so that it would fit on any size gadget—even a T-shirt.

But what does that actually mean you need to do?

a) Draw the design at different sizes and mark up different lengths on each of them, that way you'll always have one to fit.

Write what you'd need to do and why you do or don't think its the best approach.

....

....

....

b) Forget the final size, and just make sure you've captured the relationships between the lengths of the lines, so it stays proportional.

....

....

....

Brain Barbell Solution

It would be cool to finish marking up your design of the band's logo so that it would fit on any size gadget—even a T-shirt.

But what does that actually mean you need to do?

a) Draw the design at different sizes and mark up different lengths on each of them, that way you'll always have on to fit.

Apart from taking forrrevvver, this still wouldn't guarantee that you always had a size that fit...what if they started making an XXL t-shirt? You'd have to fiddle with the numbers again. ✗

b) Forget the final size, and jut make sure you've captured the ratios between the lengths of the lines, so it stays proportional.

By using ratios rather than actual sizes, you can make your design more flexible. You'll still need to do a little bit of math, but nothing major. ✓

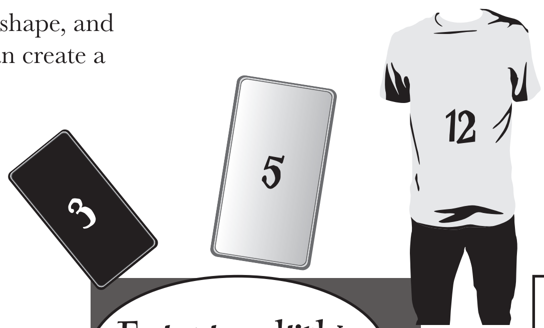
there are no Dumb Questions

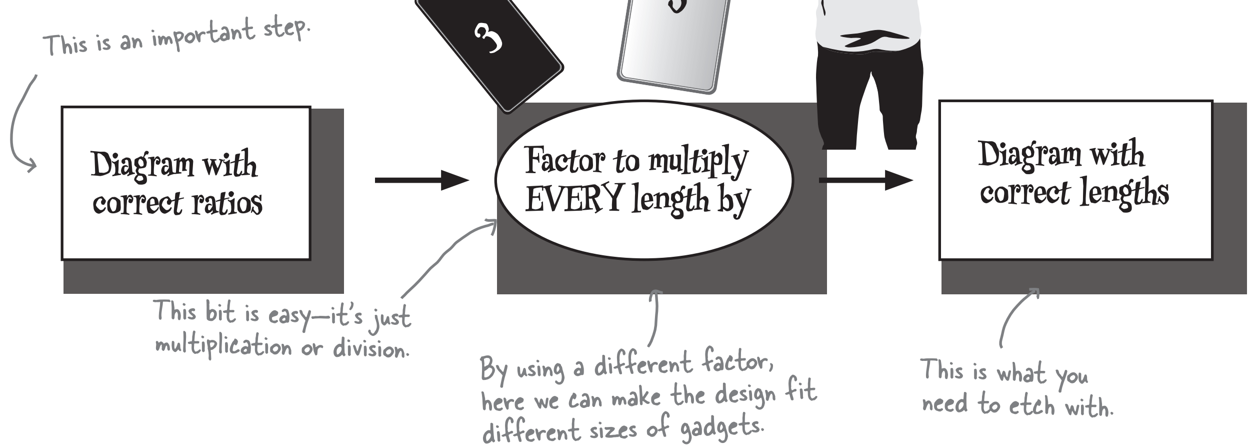
Q: Surely even if I use the proper sizes, as long as it's proportional, I can just do some more math to work out a different size by multiplying by a new factor?

A: You can—it's just that the math will be a lot harder and the ratios much less clear. Technically it's not wrong, it's just not the simplest way to go about creating a scalable design.

Ratios can be more useful than sizes

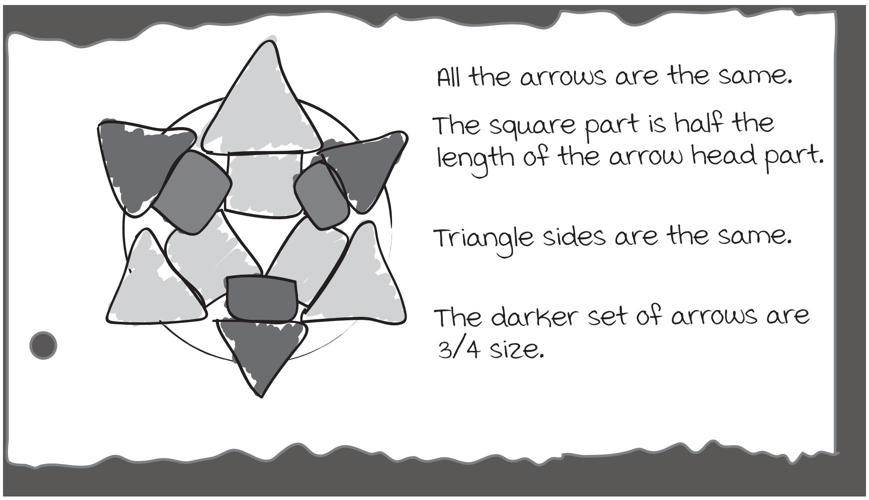
A ratio captures the proportions of a shape, and then by using a different factor, we can create a similar shape of any size.





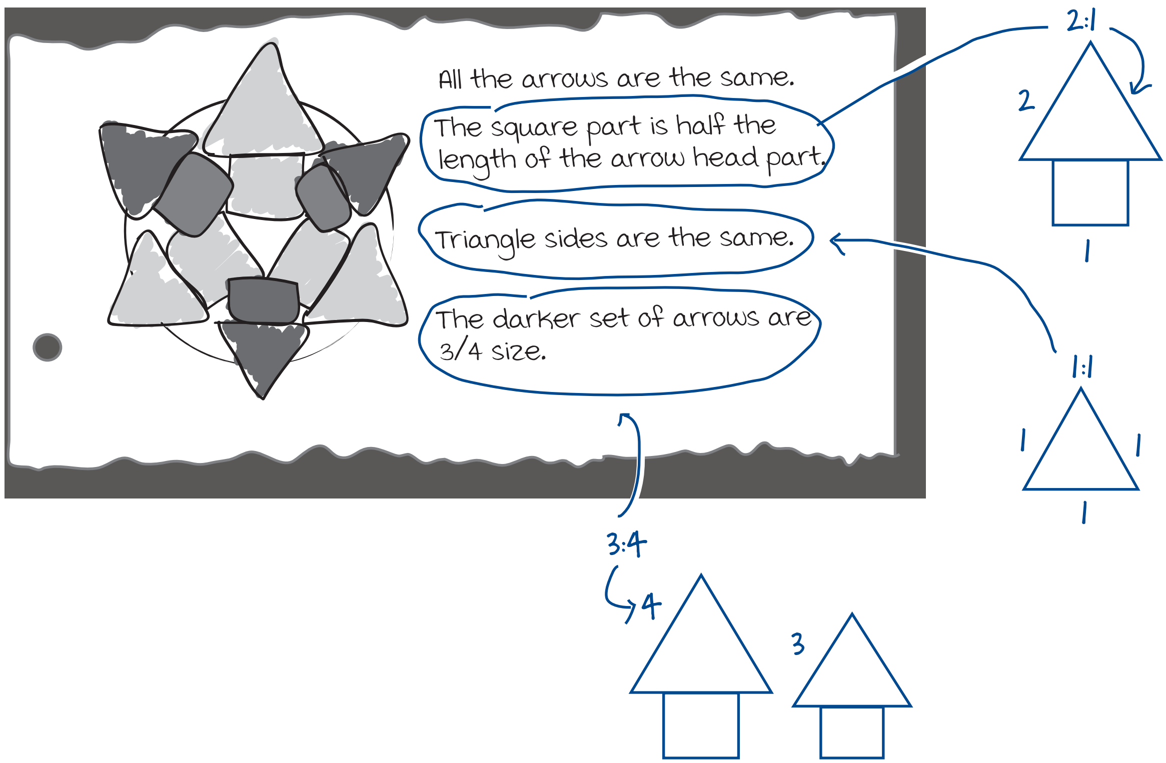
Sharpen your pencil

The notes the drummer made on the diagram tell you three ratios—what are they? Draw sketches if it helps you figure it out.



Sharpen your pencil Solution

The notes the drummer made on the diagram tell you three ratios—what are they? Draw sketches if it helps you figure it out.



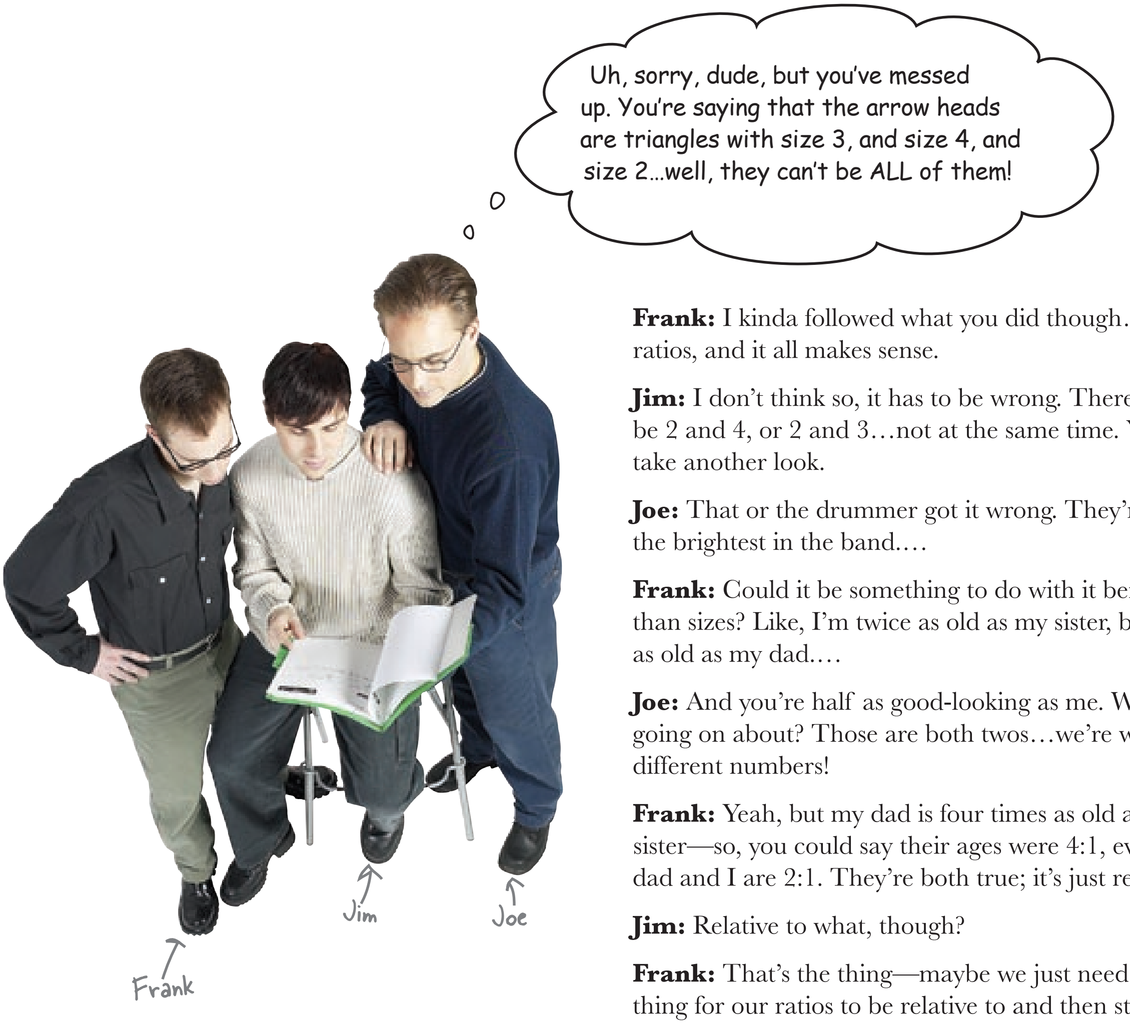
there are no Dumb Questions

Q: How come 1:1 is a ratio? Isn't that a bit pointless?

A: It does sound a bit weird, doesn't it? 1:1 is a way of indicating that it's exactly the same size as the original. If you didn't put the 1:1 in there, then someone reading your work or diagram might wonder whether you'd forgotten to put a ratio in for that item. 1:1 indicates clearly that it's the same size.

Q: The drummer wrote that the dark arrows are 3/4 size, so why have you written it as 3:4?

A: 3/4 and 3:4 are just different ways of indicating the same relative proportion. 3/4 is fraction notation and 3:4 is ratio notation. Unless you've been specifically told to use one or the other then they're mostly swappable.



Frank: I kinda followed what you did though…with the ratios, and it all makes sense.

Jim: I don't think so, it has to be wrong. There's no way it can be 2 and 4, or 2 and 3…not at the same time. You need to take another look.

Joe: That or the drummer got it wrong. They're not always the brightest in the band.…

Frank: Could it be something to do with it being ratios rather than sizes? Like, I'm twice as old as my sister, but I'm also half as old as my dad.…

Joe: And you're half as good-looking as me. What are you going on about? Those are both twos…we're worried about different numbers!

Frank: Yeah, but my dad is four times as old as my kid sister—so, you could say their ages were 4:1, even though my dad and I are 2:1. They're both true; it's just relative.

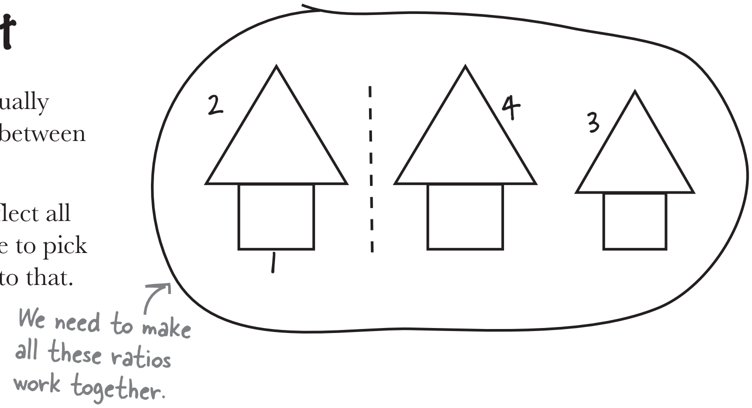
Jim: Relative to what, though?

Frank: That's the thing—maybe we just need to choose one thing for our ratios to be relative to and then stick with it.

Ratios need to be consistent

The ratios we took from the diagram are individually correct, but they describe different relationships between the lengths of our lines.

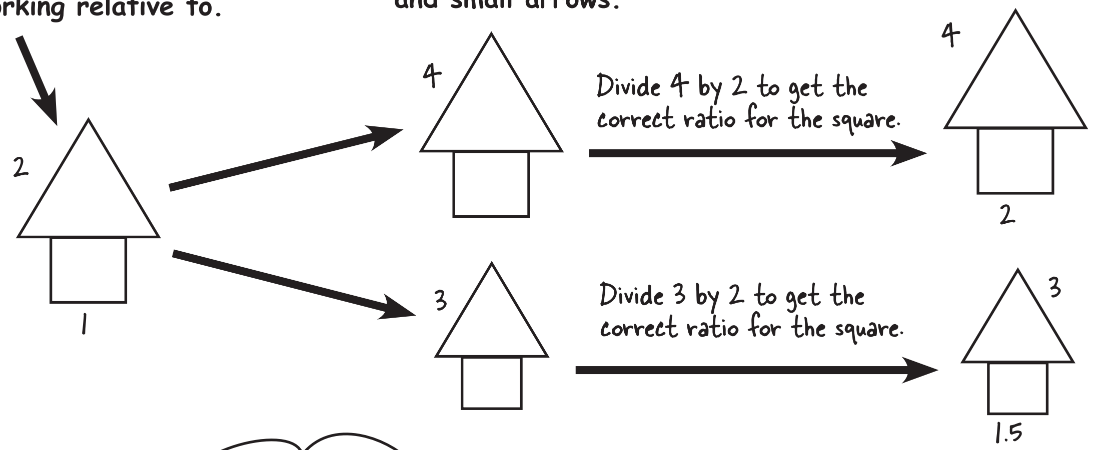
On our design, we need to make sure that we reflect all the ratios at the same time, which means we have to pick one thing and then work everything out relative to that.

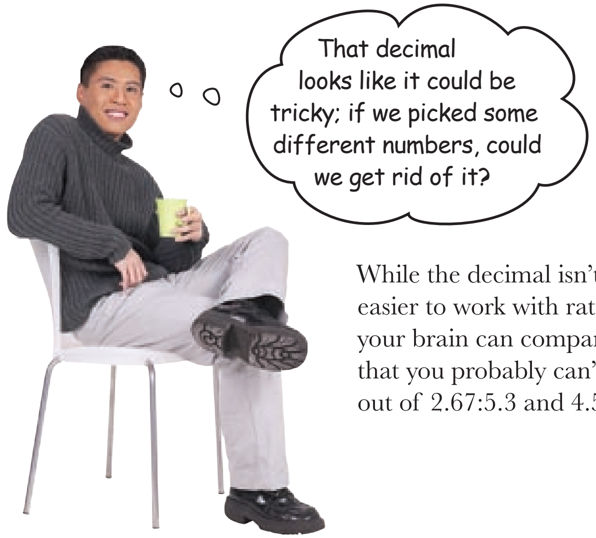


1 Let's choose this biggest arrow to be the thing we're working relative to.

2 Now we need to include the 4:3 ratio between the large and small arrows.

3 We're ready to use ratios!





While the decimal isn't really a problem, it's certainly easier to work with ratios that are only whole numbers: your brain can compare ratios like 3:4 and 7:8 in a way that you probably can't just figure out which is bigger out of 2.67:5.3 and 4.56:6.2.

Sharpen your pencil

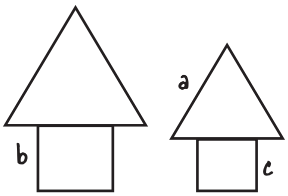
The ratios have turned out to be 4, 3, 2, and 1.5.

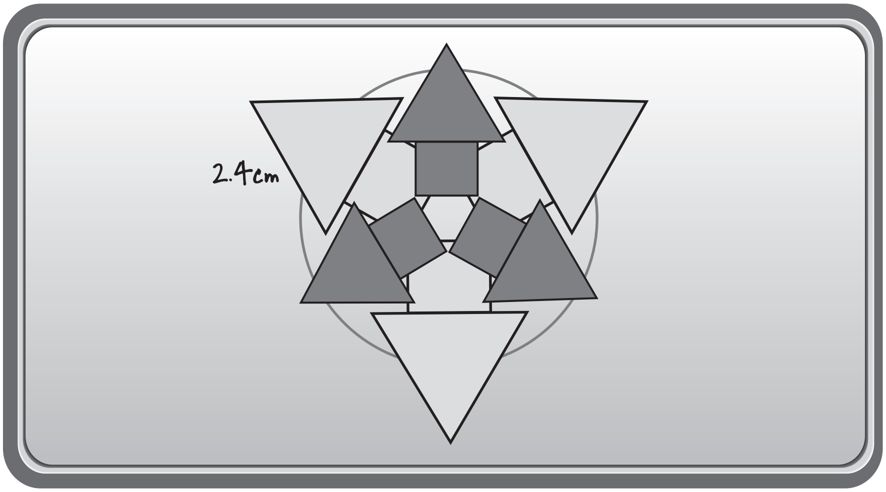
What would be the smallest set of whole numbers you could substitute and still keep the ratios the same?

Use your answers from here in this answer.

Exercise

Time to get etching! If the design will fit on Liz's brother's iPod with the biggest arrow head edges at 2.4cm long, what lengths do the other lines—a, b, c—need to be?



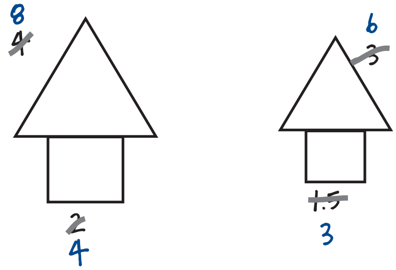


Sharpen your pencil Solution

The ratios have turned out to be 4, 3, 2, and 1.5.

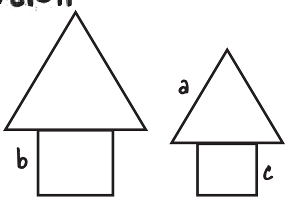
What would be the smallest set of whole numbers you could substitute and still keep the ratios the same?

If we multiply all the ratios by 2, then we get a set of whole numbers with the same ratios: 8, 6, 4, and 3.



Exercise Solution

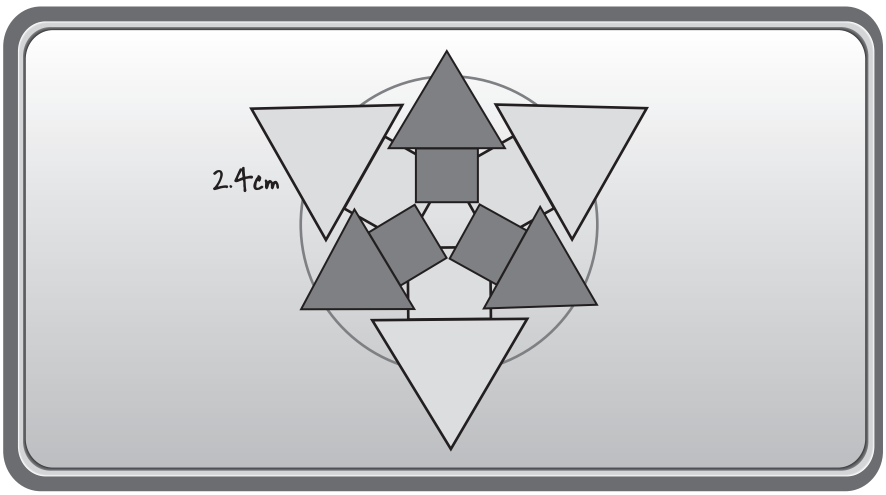
Time to get etching! If you make the big arrowhead edges 2.4cm long, what lengths do the other lines—a, b, c—need to be?



First we need to find the factor for scaling a, b, and c:

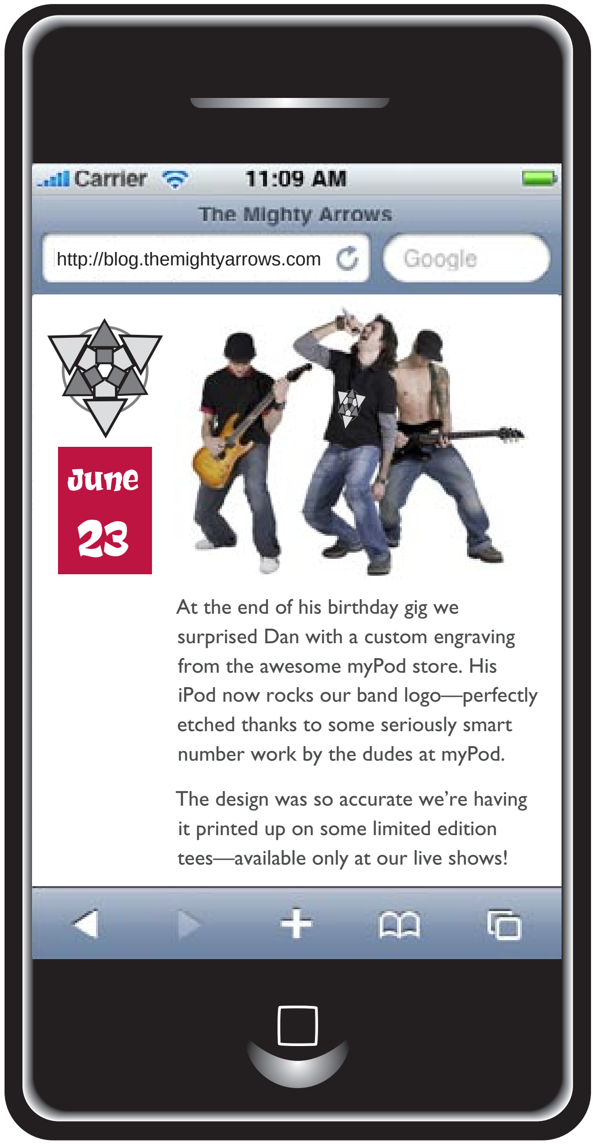
The length with a ratio of 8 is 2.4cm, so scale $= \frac{2.4\text{cm}}{8} = 0.3\text{cm}$

Then multiply all the other ratios by 0.3:
$a = 6 \times 0.3 = 1.8\text{ cm}$
$b = 4 \times 0.3 = 1.2\text{cm}$
$c = 3 \times 0.3 = 0.9\text{cm}$



Your new design ROCKS!

Check out the band's latest blog entry on your brand new iPhone—your myPod bonus for getting that extra order to print all those T-shirts!



Oh, yeah, you got backstage tickets, too!



Your Geometry Toolbox

You've got Chapter 2 under your belt and now you've added Similarity and Congruence to your toolbox. For a complete list of tool tips in the book, visit www.headfirstlabs.com/geometry.

Similar shapes have equal angles.

Similar shapes have the same ratios between side lengths.

You can spot similar triangles from two equal angles.

Actually they have three angles that are equal, but as soon as you spot two you know that the third is gonna match, too.

Congruent shapes are similar and the same size.

Congruent shapes have angles that are equal, just like similar shapes, but they're also the same scale.

All squares and circles are similar.



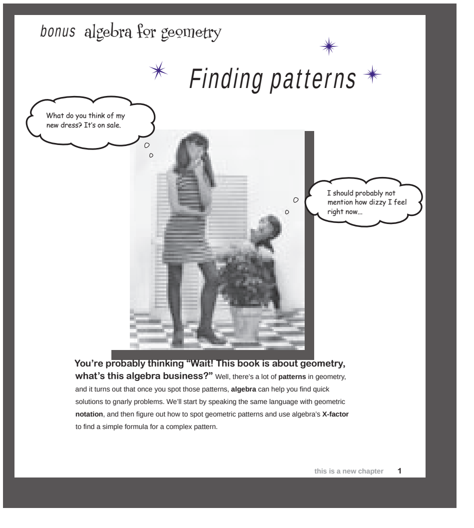
Letters and numbers colliding? Oh, no! It must be ALGEBRA!

But DON'T TURN BACK!

Instead, check out our FREE (and pain free) downloadable bonus chapter on algebra and geometry here:

http://www.headfirstlabs.com/geometry

...and find out how YOU can harness the power of algebra to solve 215 geometry problems in one go.



bonus algebra for geometry

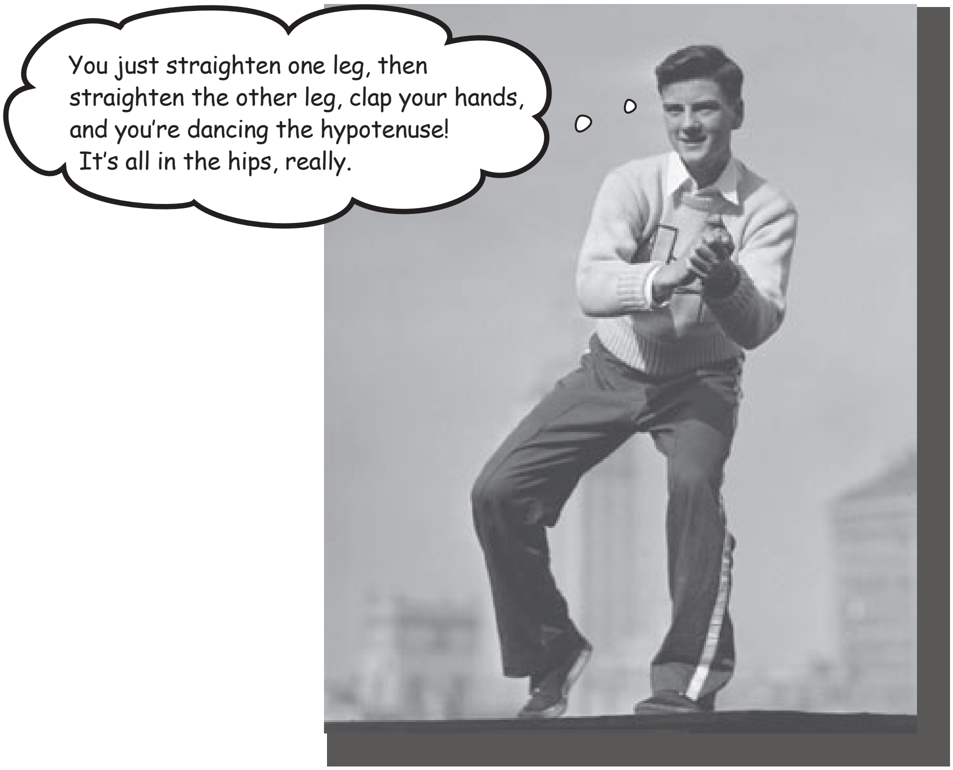
Finding patterns

You're probably thinking "Wait! This book is about geometry, what's this algebra business?" Well, there's a lot of **patterns** in geometry, and it turns out that once you spot those patterns, **algebra** can help you find quick solutions to gnarly problems. We'll start by speaking the same language with geometric **notation**, and then figure out how to spot geometric patterns and use algebra's **X-factor** to find a simple formula for a complex pattern.

this is a new chapter 1

3 the pythagorean theorem

All the right angles



Sometimes, you really need to get things straight.

Ever tried to eat at a wobbly table? Annoying, isn't it? There is an alternative to shoving screwed-up paper under the table leg though: use the Pythagorean Theorem to make sure your designs are **dead straight** and not just *quite straight*. Once you know this pattern, you'll be able to **spot and create right angles** that are **perfect every time.** Nobody likes to pick spagetti out of their lap, and with the **Pythagorean Theorem**, you don't have to.

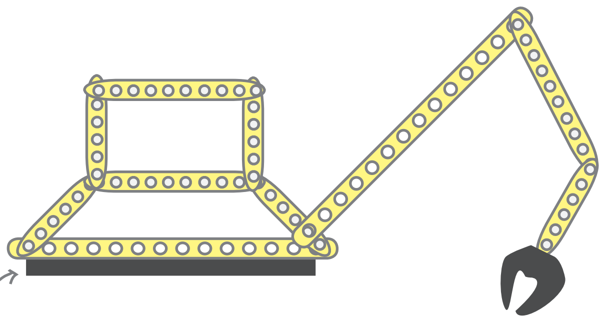
Giant construction-kit skate ramps

Sam is seriously into skating, and she funds her skate habit by building street courses—a jungle of ramps and rails where skaters can kickflip and 5-0 themselves silly.

But building the ramps takes up time (and money) that Sam would rather spend on actually skating. But last week, while babysitting her nephew, Sam had the best idea she's ever had: ***use quick assembly standard sized pieces to build the skate ramps.***

...and skateboards, and shoes and hoodies....

With giant construction kit parts, instead of making all our ramps from scratch, we could use standard size pieces that assemble together quickly. It'll be faster and cheaper!



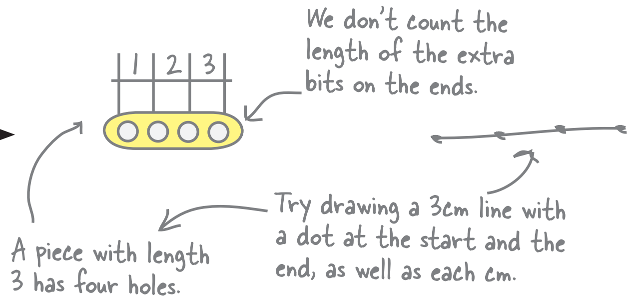
The inspiration for Sam's genius plan was a kids' construction kit.



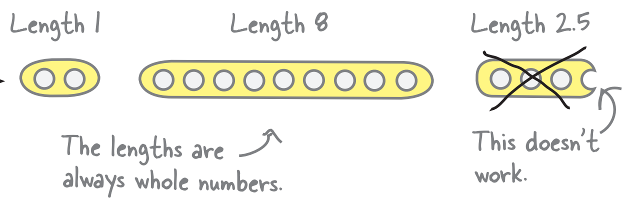
Standard-sized-quick-assembly-what?!?

Sam's found a supplier for giant construction kit parts. Kwik-klik makes parts which are just like construction kit toys you might have seen or played with when you were younger, but on a much bigger scale.

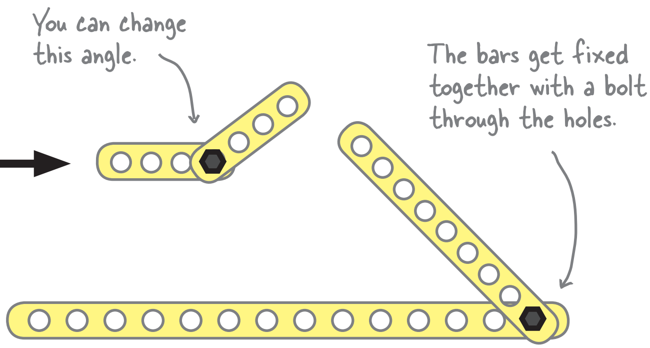
- **The pieces have holes in them at evenly spaced intervals.** Because there's a hole on either end, each piece has one more hole than the length it represents.



- **The parts come in tons of sizes, from size 1 upward, but always as whole numbers.** You can't have a size "two and a half" part.



- **To build stuff, you just bolt the parts together through lined-up holes.** The pieces can pivot to any angle around the bolt, so you can swing them into any position you need.



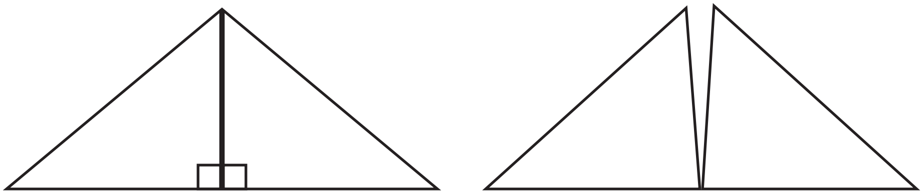
So easy, anyone can do it! But Sam's first ramp has a major problem...

The ramps must have perpendicular uprights

The "upright" is the vertical part of the ramp.

Sam needs the ramps to have true vertical uprights, **perpendicular** (90º) to the bases, so that she can put them back to back or against a wall without a nasty gap to get your wheels caught in.

Lines or objects that are perpendicular meet or cross at a right angle.

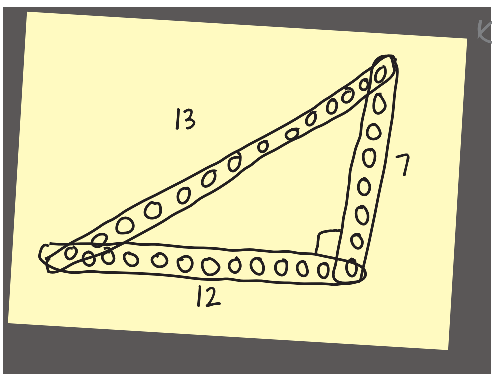


Perpendicular uprights are perfectly vertical and let the ramps fit together.

Slanted uprights create wheel-trapping gaps.

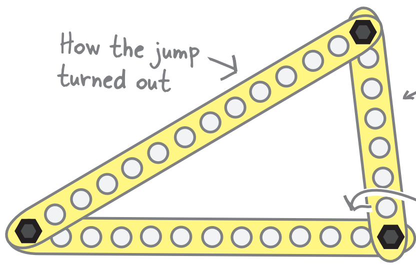
But Sam's first prototype is not squaring up

Even though it looked good on paper, now Sam's built her first ramp and…well, it's just plain wonky.



What Sam planned to build

How the jump turned out



This one isn't perfectly vertical.

And this angle isn't a right angle.



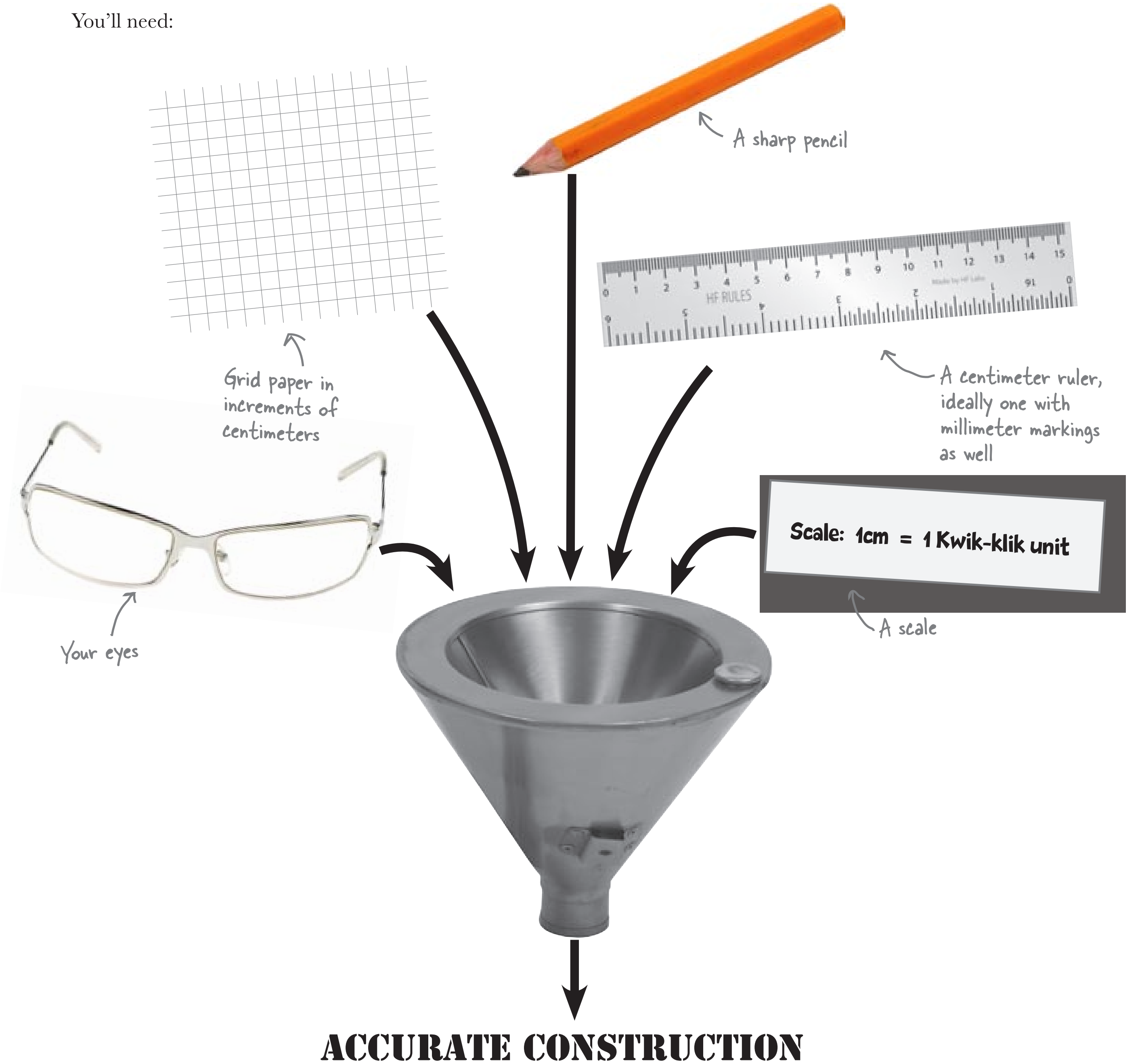
BRAIN POWER

Could you use a pencil, a ruler and some paper to check out whether a ramp design will give you a perfectly vertical upright ***before*** you build it?

You can use accurate construction to test ramp designs on paper

Accurate construction is different from making a sketch.

You'll need:



Hello? Didn't you spend chapters 1 & 2 going on about how we couldn't trust a drawing, couldn't just measure it, blah, blah, blah?



True. But making an accurate construction is different.

Sharpen your pencil

How is making your own accurate drawing different from just measuring a sketch or diagram you're given? Write out your answer in words below.

Sharpen your pencil Solution

How is making your own accurate drawing different from just measuring a sketch or diagram you're given?

If you're drawing the diagram yourself you can keep it in proportion, and you can get the angles right, too. You can use a set square or a protractor, or gridded paper, to make sure lines that are supposed to be perpendicular are drawn at right angles.

If you draw a line 3cm and another line 6cm—measured with a good ruler—you know for sure that the first line you drew is half the length of the other.

When you're given a sketch you don't know whether the person who drew it used a ruler and protractor to make the drawing accurate, or just did it roughly.

Your answer might be worded differently—that's OK, it's the thought that counts....

Right, so if we know we've kept to the same scale—like using a line exactly 3cm long for a size 3 Kwik-klik piece—we can use drawing to kind of simulate the ramp?



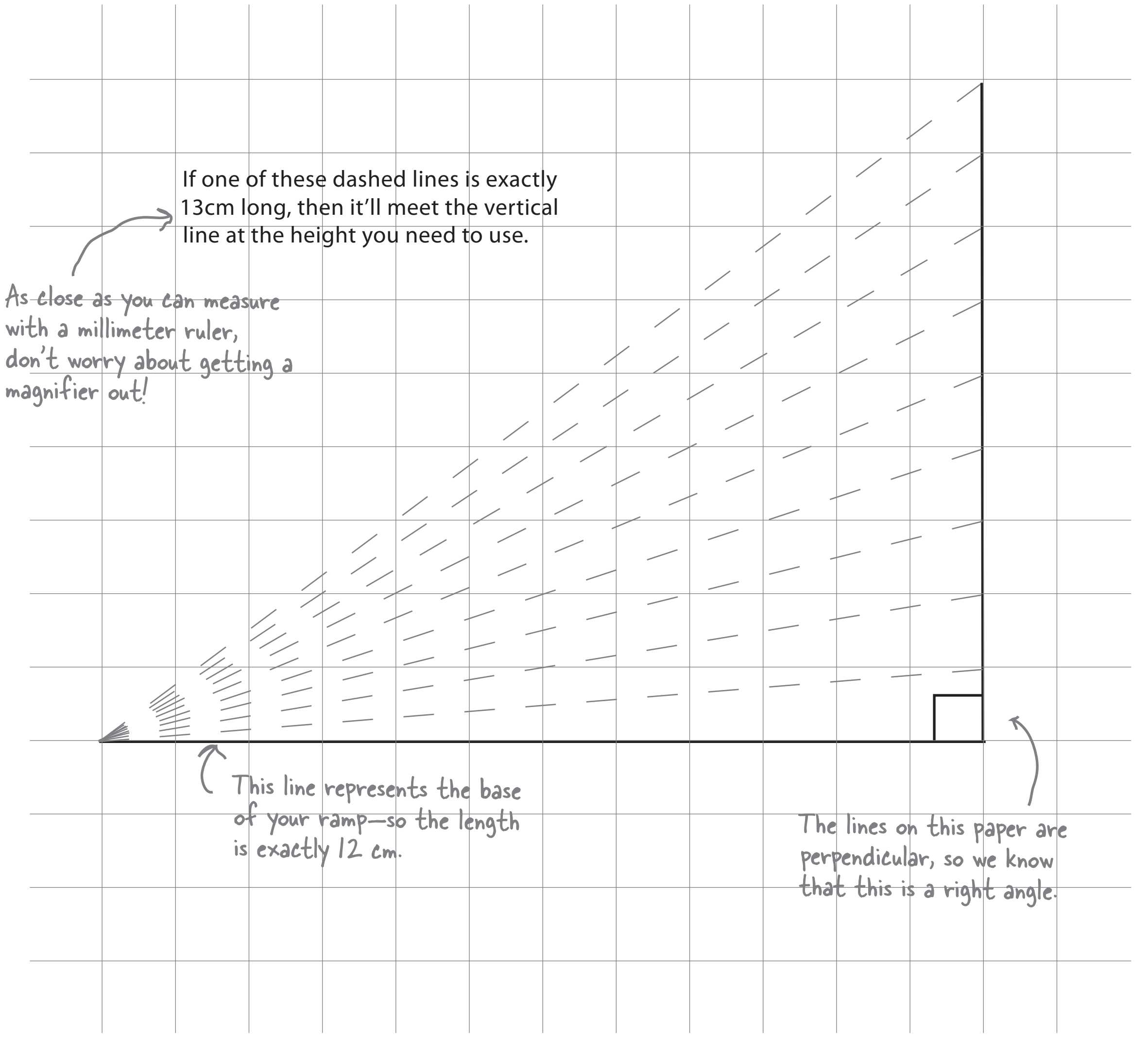
Yup. If you can't draw it with a right angle, then you can't build it with a right angle, either!

Centimeter grid paper has horizontal and vertical lines which are perfectly perpendicular—making it extra useful for drawing shapes with right angles like our ramps need to be. Just try it out....

Sharpen your pencil and ruler

Use accurate construction to find what length the vertical piece needs to be in order for the ramp to have a perpendicular upright.

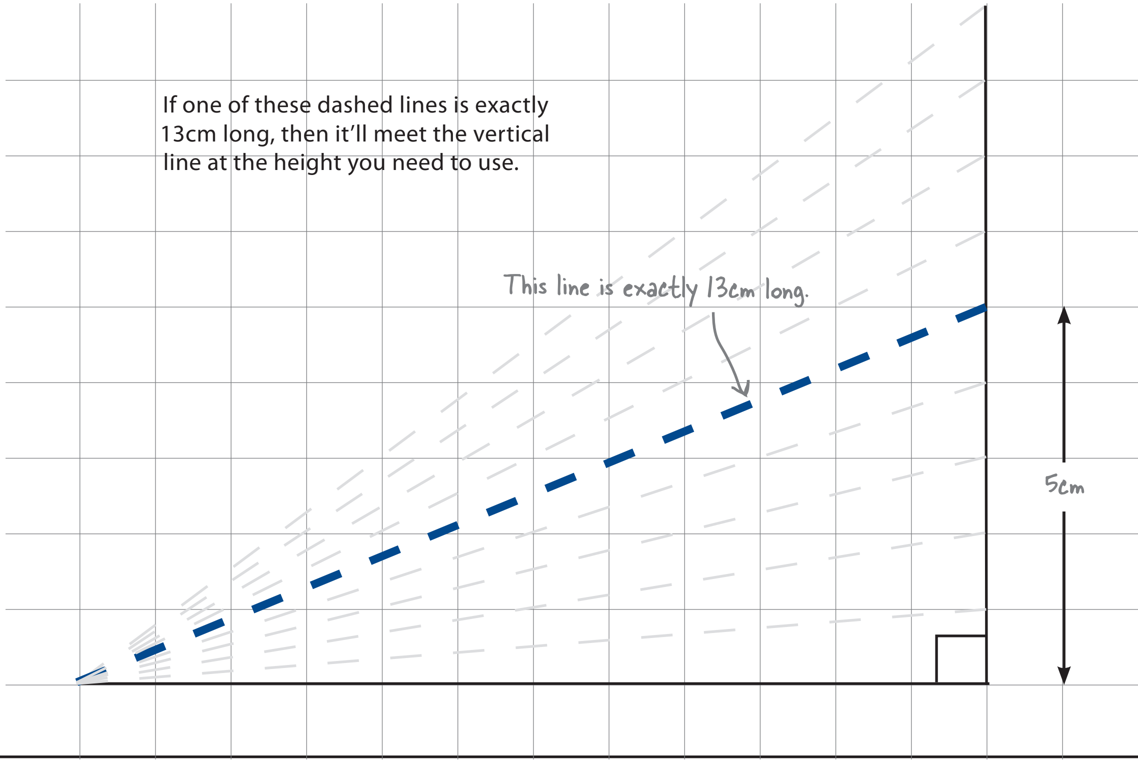
Using a scale of 1cm to 1 Kwik-klik unit (so a piece of length 2 would be drawn as 2cm), use your ruler to find the part that fits.



Sharpen your pencil and ruler solution

Use accurate construction to find what length the vertical pole needs to be in order for the ramp to have a perpendicular upright.

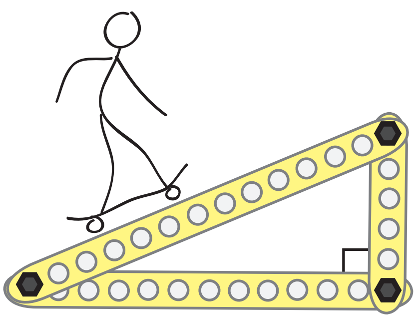
Using a scale of 1cm to 1 Kwik-klik unit (so a part of length 2 would be drawn as 2cm), use your ruler to find the part that fits.



The size 5 upright gives us a perfect vertical

The parts with lengths 12, 13, and 5 make a perfect ramp with a right angle between the horizontal and vertical parts.

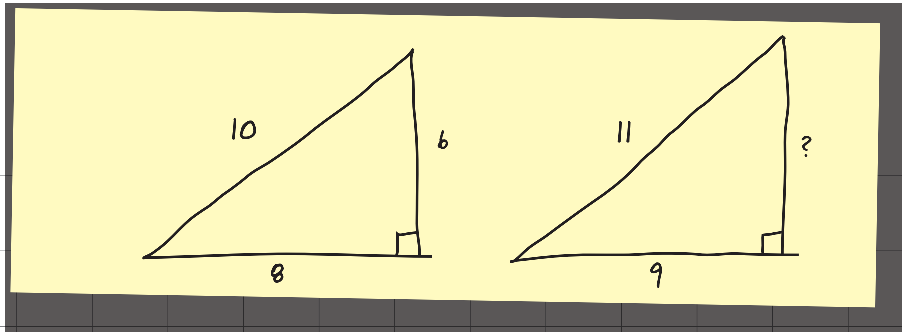
Of course what we just drew was a scaled version of our final ramp—nobody wants a skate ramp 5cm high, but a ramp that's 5 kwik-klik units high is plenty big enough.



Sharpen your pencil and ruler

Sam's sketched up another two jumps which she *hopes* can be built from the Kwik-klik parts.

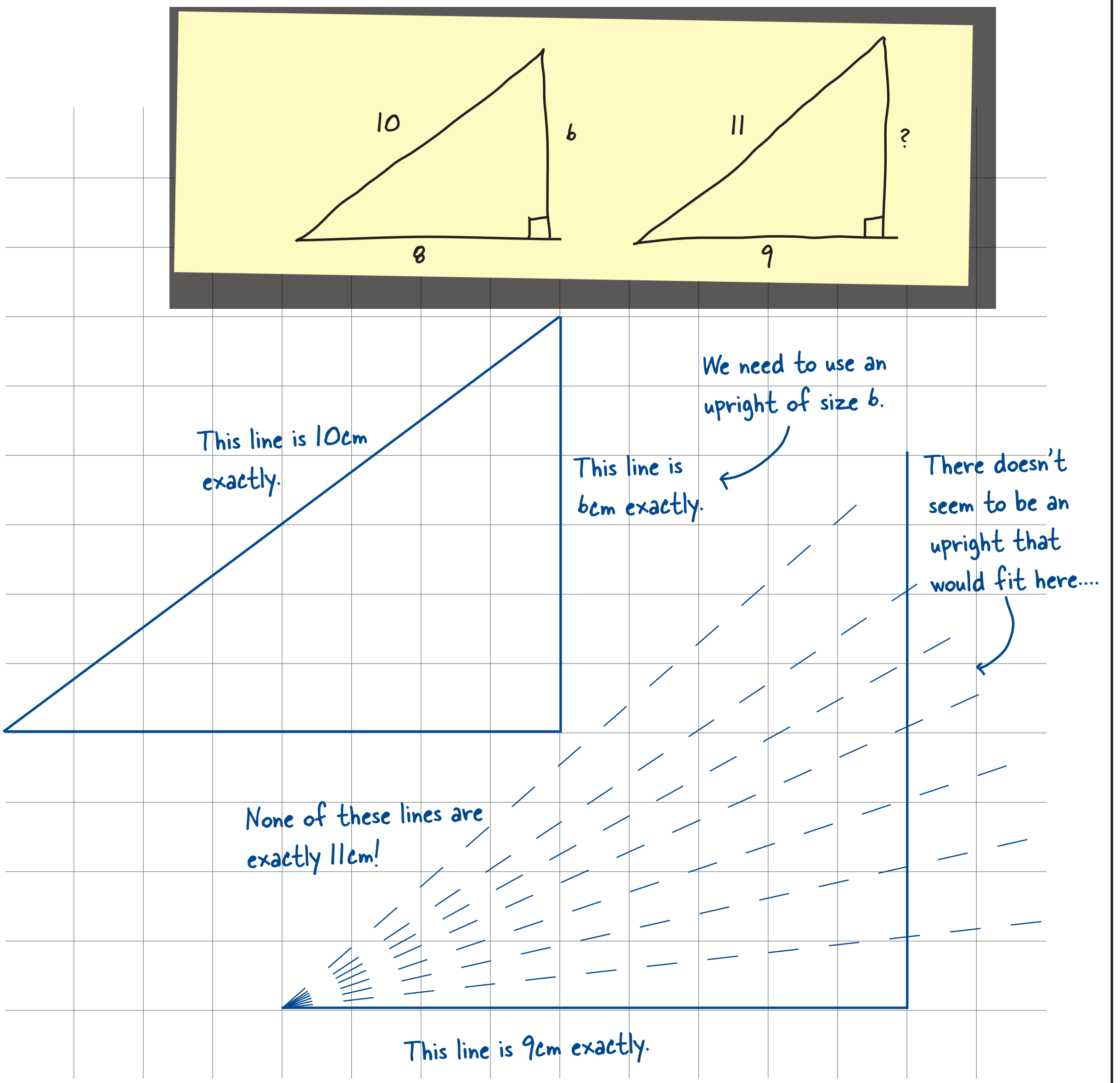
Use the same pencil and ruler construction technique to find out whether there's a part that creates a perfect perpendicular ramp for each of them.



Sharpen your pencil and ruler solution

Sam's sketched up another two jumps which she *hopes* can be built from the Kwik-klik parts.

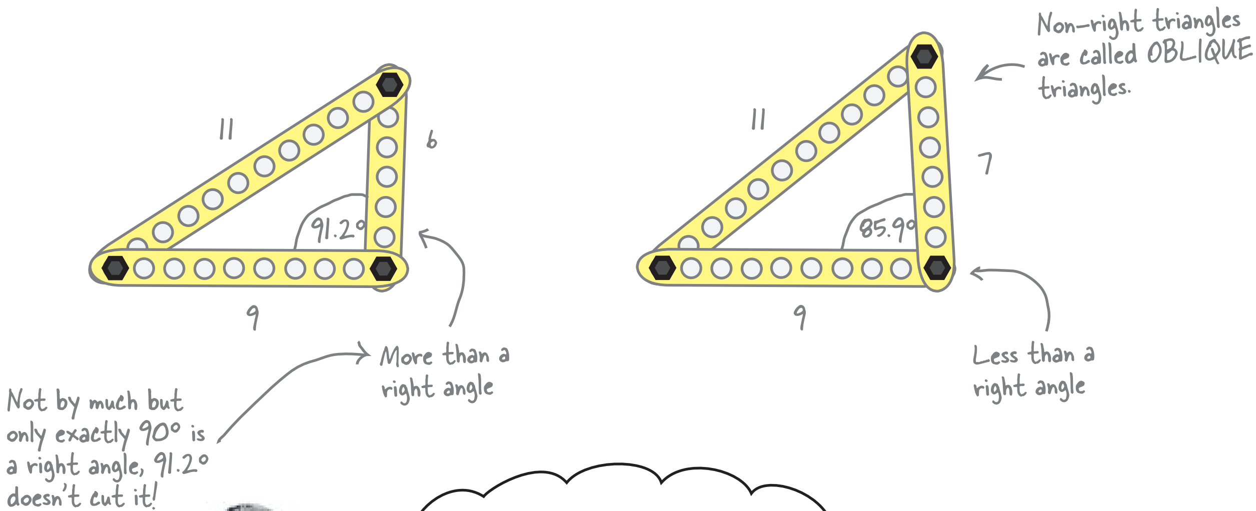
Use the same pencil and ruler construction technique to find out *whether* there's a part that creates a perfect perpendicular ramp for each of them.

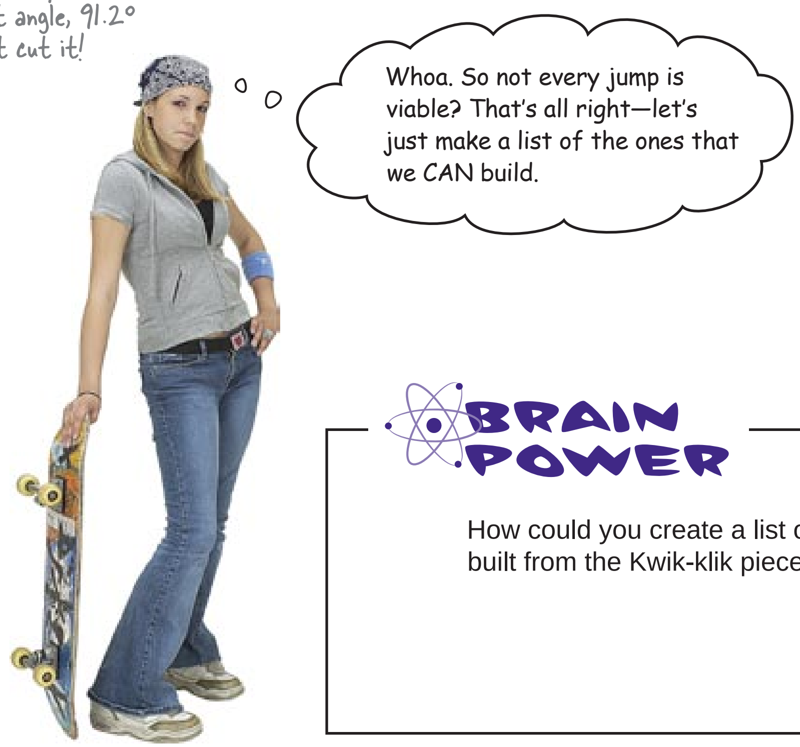


Not all lengths make a right triangle

There isn't a length that can complete Sam's design for an 11 - 9 - ? jump and give a perfectly vertical upright. Size 7 is too long and size 6 is too short. It seems some lengths can make right triangles and others can't.

Kwik-klik doesn't make a "six and a bit" size piece so you can only build jumps with whole number lengths.





BRAIN POWER

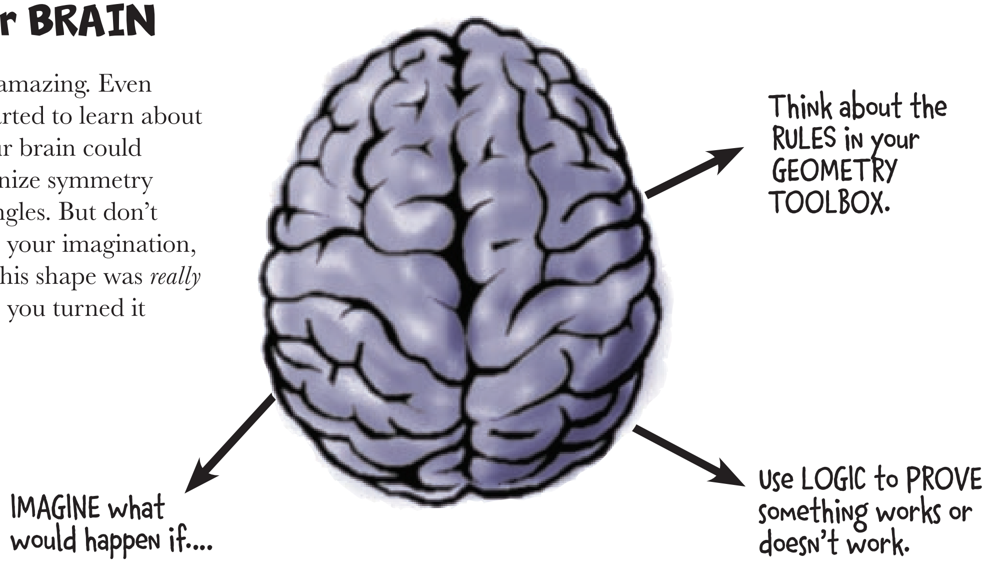
How could you create a list of jumps that can be built from the Kwik-klik pieces?

You can explore a geometry problem in different ways

There's more than one way to investigate a geometry problem, and the choice of which one to go for is often about how your own brain works rather than one being "better."

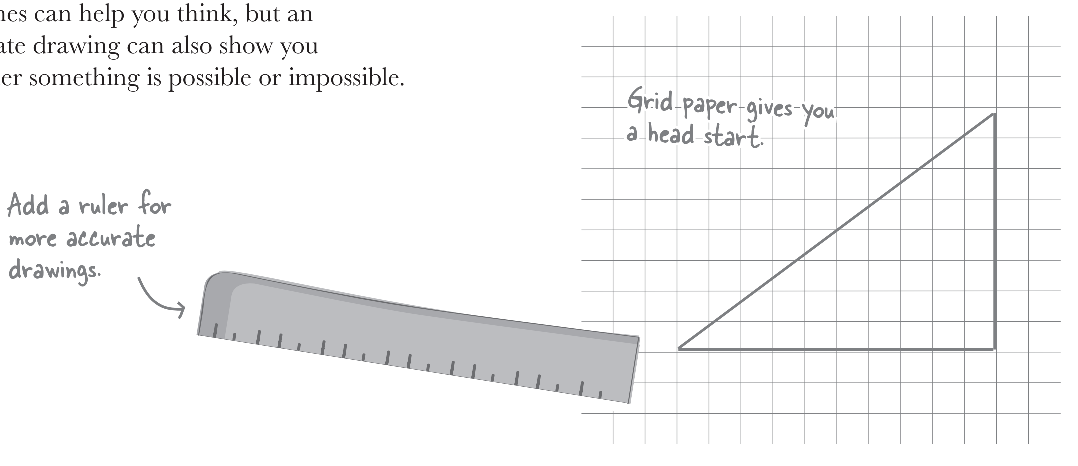
1 Use your BRAIN

Your brain is amazing. Even before you started to learn about geometry, your brain could already recognize symmetry and special angles. But don't just think, use your imagination, too. What if this shape was *really* BIG? What if you turned it upside down?



2 Use a PENCIL and PAPER

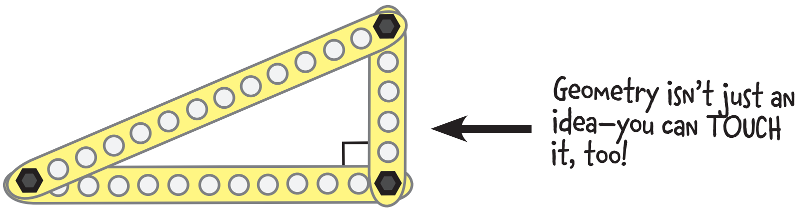
Sketches can help you think, but an accurate drawing can also show you whether something is possible or impossible.



3 Use the REAL WORLD

Geometry isn't just important for your grades, it's what engineers and scientists rely on to make buildings stand up and cars go around corners.

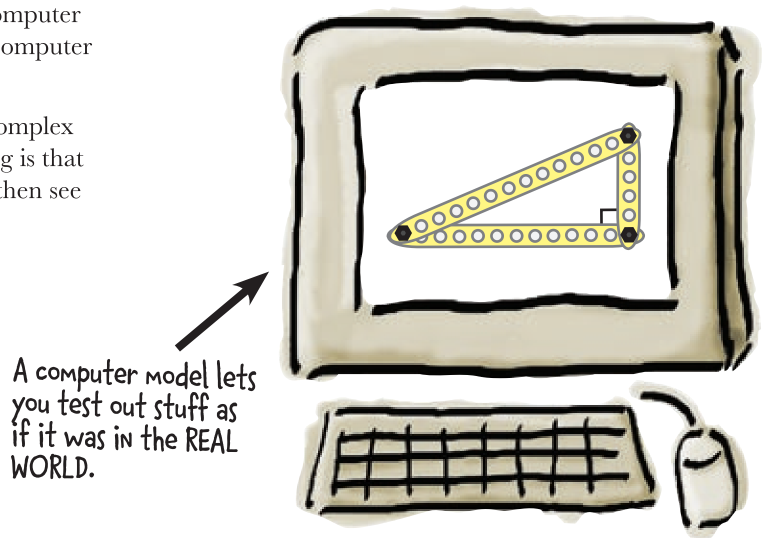
Use kids' toys, a scrap piece of wood, or make models out of cardboard to put your theories to the test.



4 Use a COMPUTER MODEL

If you've ever played a video or computer game then you've already used a computer model of some geometry.

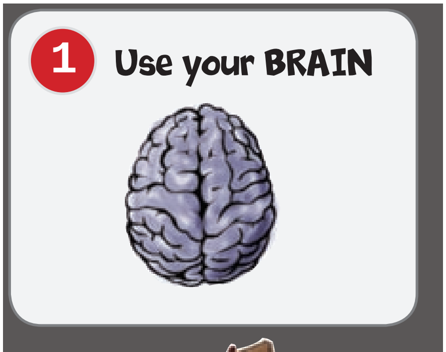
Whether the model is simple, or complex (like a racing game), the great thing is that you get to change something and then see what impact your change makes.

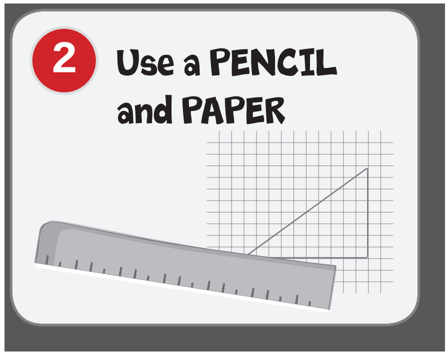


So—what's your preference? Do you think it matters which exploration technique you use to find the jumps you can build?

In geometry, the rules are the rules

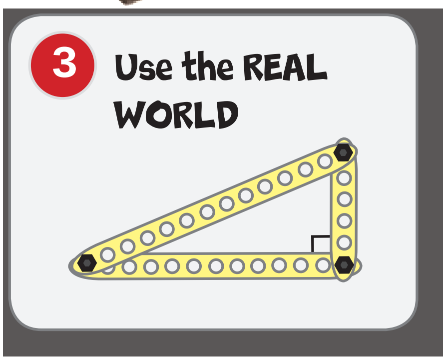
In geometry there's no "i before e *except* after c" stuff. The rules are the rules. Everything in your Geometry Toolbox applies whether you think, draw, touch, look, play, or test your way to an answer.

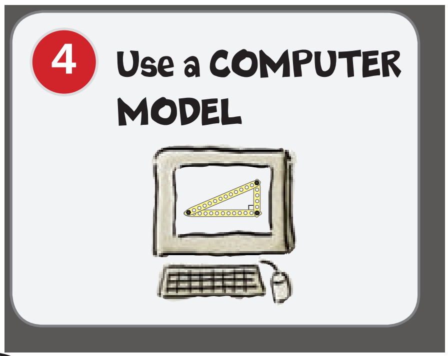






Your GEOMETRY TOOLBOX works with ALL of these methods.







So no matter which of these methods I use, I'll get the same answers?

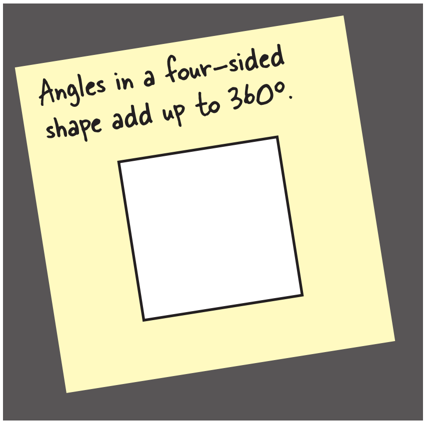
Yes, you'll get the same results however you approach it.

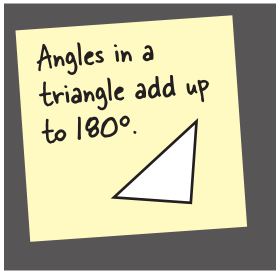
Geometry is nice like that! For practical reasons, we'll be using your brain and a pencil and paper for this chapter, but if you want to make use of the other methods as well then that's cool, too.

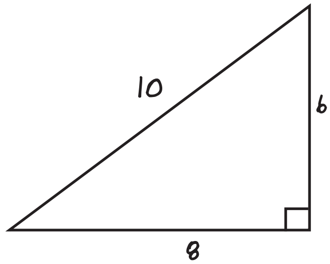


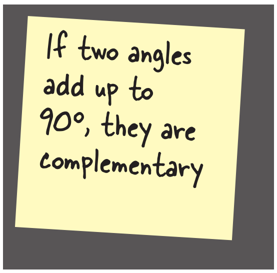
You've already worked out that you can build a ramp from Kwik-klik parts, using lengths 10, 8, and 6.

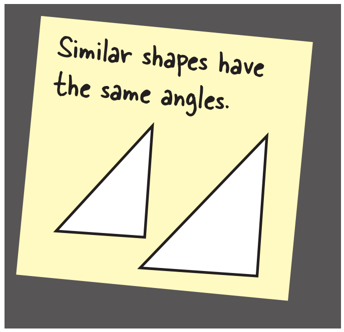
Which two rules from your Geometry Toolbox can help you quickly find some other lengths that would also work?

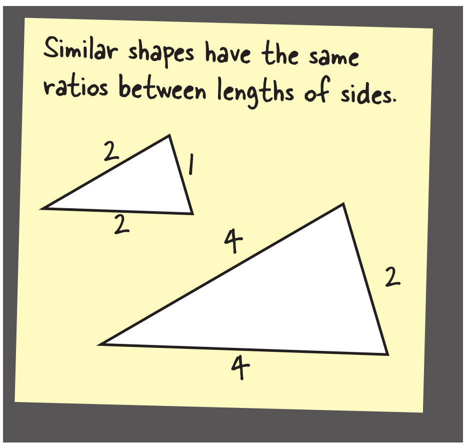


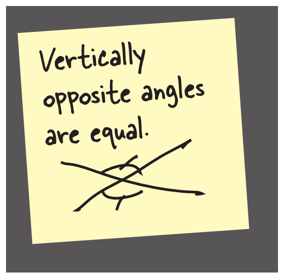












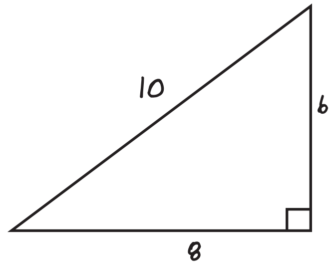
Exercise Solution

You've already worked out that you can build a ramp from Kwik-klik parts, using lengths 10, 8, and 6.

Which two rules from your Geometry Toolbox can help you quickly find some other lengths that would also work?

Angles in a four-sided shape add up to 360°.

Angles in a triangle add up to 180°.



If two angles add up to 90°, they are complementary.

Similar shapes have the same angles.

Similar shapes have the same angles, so any ramp SIMILAR to the 10–8–6 ramp will also have a right angle to make the upright perfectly straight.

Similar shapes have the same ratios between lengths of sides.

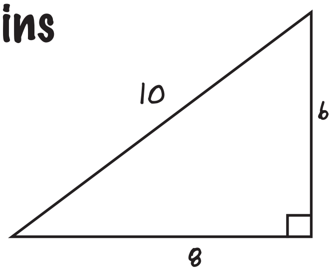
Similar shapes have the same ratios between the lengths of their sides, so if we choose lengths with the same ratios as 6:8:10, then they'll definitely build straight ramps.

Vertically opposite angles are equal.

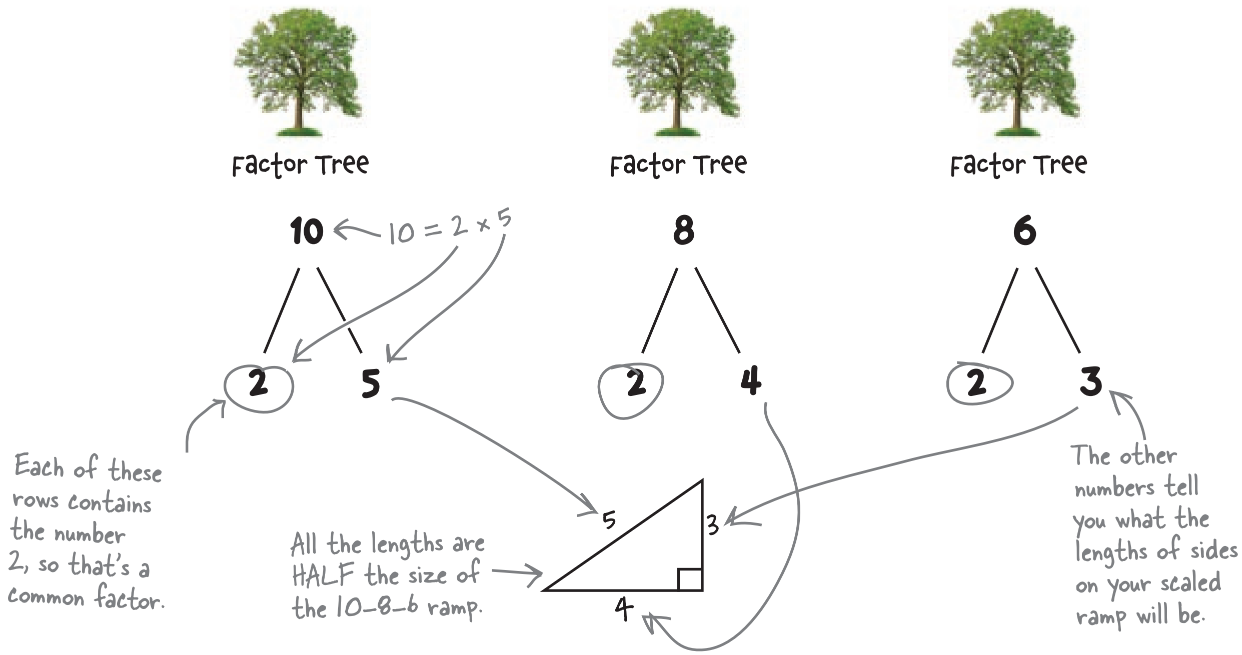
Any good jump has some similar scaled cousins

When you scale the ramp—making it bigger or smaller—none of the angles change, so it stays a good ramp with a right angle.

To find whole number lengths for the smaller-scaled sizes, look for a common factor in your current lengths. That's easiest to do by using a set of factor trees.



Use factor trees to find whole-number miniatures



Sharpen your pencil

What's the next largest ramp you can build that is similar to the 10-8-6 design?

Sharpen your pencil Solution

What's the next largest ramp you can build that is similar to the 10-8-6 design?

The lengths 15, 12, and 9 have the same ratios as 10-8-6 and as 5-4-3.

$5 \times 3 = 15$

$4 \times 3 = 12$

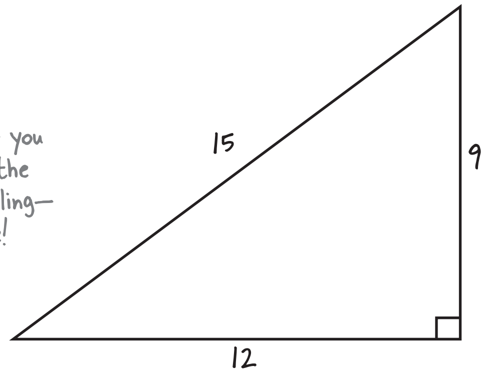
$3 \times 3 = 9$

$10 \times 1.5 = 15$

$8 \times 1.5 = 12$

$6 \times 1.5 = 9$

It doesn't matter whether you used the original ramp or the mini version to do your scaling—it works out just the same!



there are no Dumb Questions

Q: What if I wanted a ramp even smaller? Can I just keep doing more factor trees?

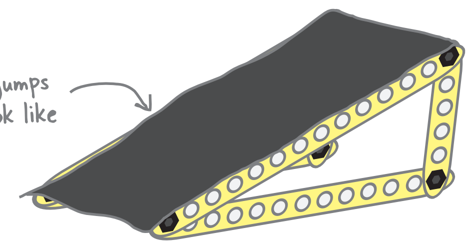
A: The Kwik-klik units don't come in half sizes—there isn't a 1.5 length, so you'd quickly run out of parts, but assuming you weren't just talking about building it with the kit parts, you still only need to do your factor trees until one of the numbers on the bottom is a prime number—that means it can't be divided by anything except itself and one. Then to make a really small ramp you'd multiply those factors by a fraction.

Q: How can we skate on a 2D ramp? Isn't this gonna be more like a rail that you can slide on?

A: What we're actually representing is the side of the ramp. There would be two of these side triangles the same, with a panel connected to the sloping beam on each triangle. This is a 3D problem which has a 2D solution.

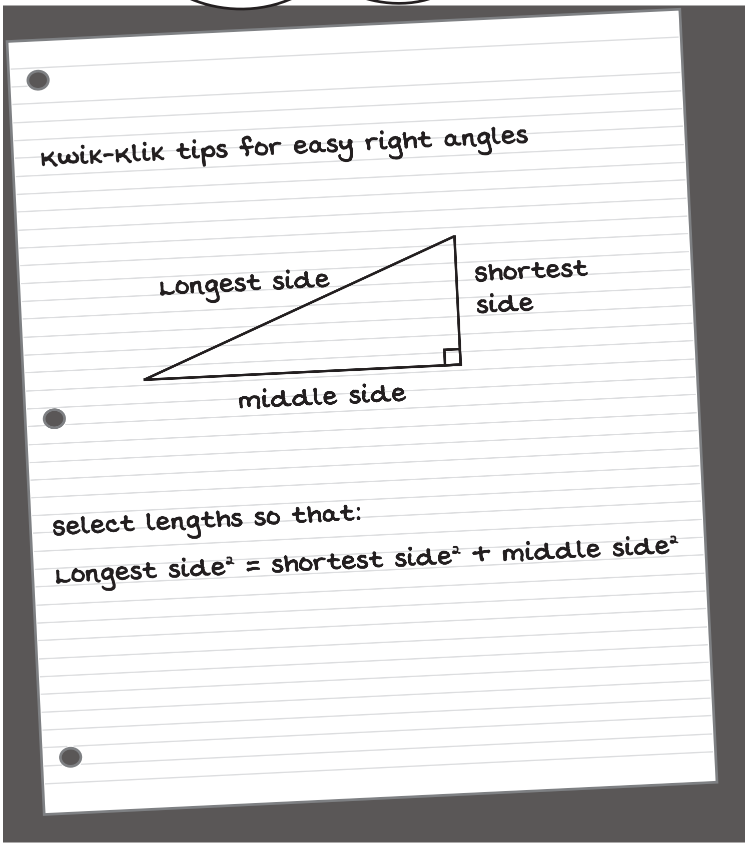
If you're interested in exploring 3D problems further, come and catch up with Sam in *Head First 3D Geometry*.

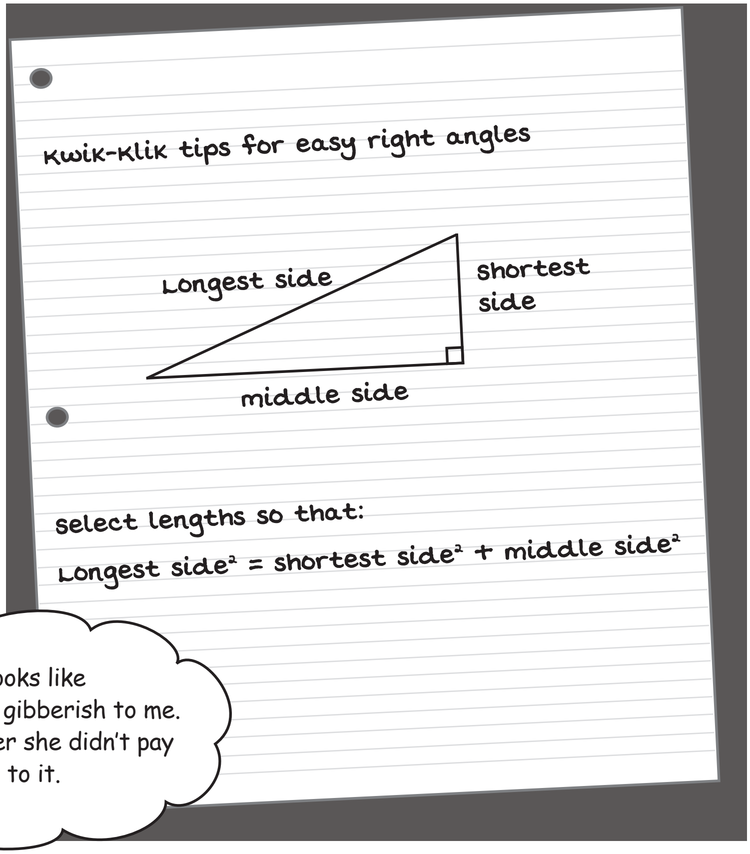
What the jumps actually look like in 3D





So how do we know which sets of ratios can give us a right triangle? The dude at the store gave me this slip of paper with some odd stuff on it—tips for building right angles or something.... I didn't really pay it any attention—what do you think it means?







Frank: But it must mean something. I mean, nobody goes to the trouble of writing something down unless it's useful.

Jim: True. But how would you use it? And why would squaring the side lengths have anything to do with the angle of the triangle?

Joe: Oh…those twos are for squaring! Yeah—no way that would work.

Frank: OK, don't freak out, but if you just try it, like for the 3-4-5 ramp design…it works out perfect.

Jim: What? Are you sure you got your numbers right?

Frank: Yeah—I'm sure. Longest side is 5, and 5 squared is 25. Shortest side is 3, and 3 squared is 9, and the middle side is 4, and 4 squared is 16. So—add up the middle and shortest sides squared—9 plus 16—and you get.…

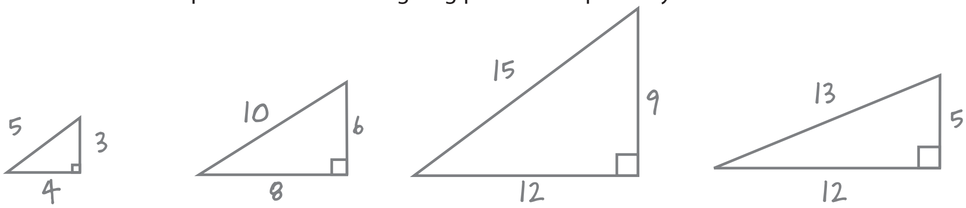
Joe: 25. The same as the square of the longest side. That has got to be a coincidence.

Frank: There's only one way to find out—let's check the others.…

Geometry Detective

Here's that tip again: longest-side squared = shortest-side squared + middle-side squared seems to work for at least one ramp design.

Based on the four jumps you've successfully designed so far, complete the table to discover whether there really is a secret pattern behind designing perfect ramps every time.



Triangle	3-4-5	6-8-10	5-12-13	9-12-15
Shortest-side's length	3			
Middle-side's length	4			
Longest-side's length	5			
Shortest-side squared	9			
Middle-side squared	16			
Longest-side squared	25			
Shortest-side squared + longest-side squared	9 + 16 = 25			

Add these two together

Now, isn't THAT kind of freaky....

The lengths of the sides are linked by a pattern

Did you find it? For the four right triangles we tested, it seems like the square of the length of the longest side is equal to the squares of the other two sides added together.

Triangle	3-4-5	6-8-10	5-12-13	9-12-15
Shortest-side's length	3	6	5	9
Middle-side's length	4	8	12	12
Longest-side's length	5	10	13	15
Shortest-side squared	9	36	25	81
Middle-side squared	16	64	144	144
Longest-side squared	25	100	169	225
Shortest-side squared + middle-side squared	9 + 16 = 25	36 + 64 = 100	25 + 144 = 169	81 + 144 = 225

This row + this row = this row!



True. Can we really trust this pattern?

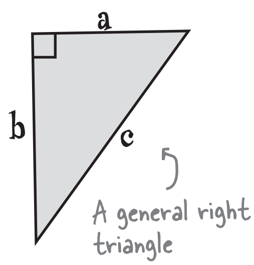
The numbers we've tested so far seem pretty conclusive, but do we definitely know that this pattern is going to work for all the possible right-angled jumps we might need?

And how do the squares relate to the triangles? Let's investigate this pattern in a more general way.



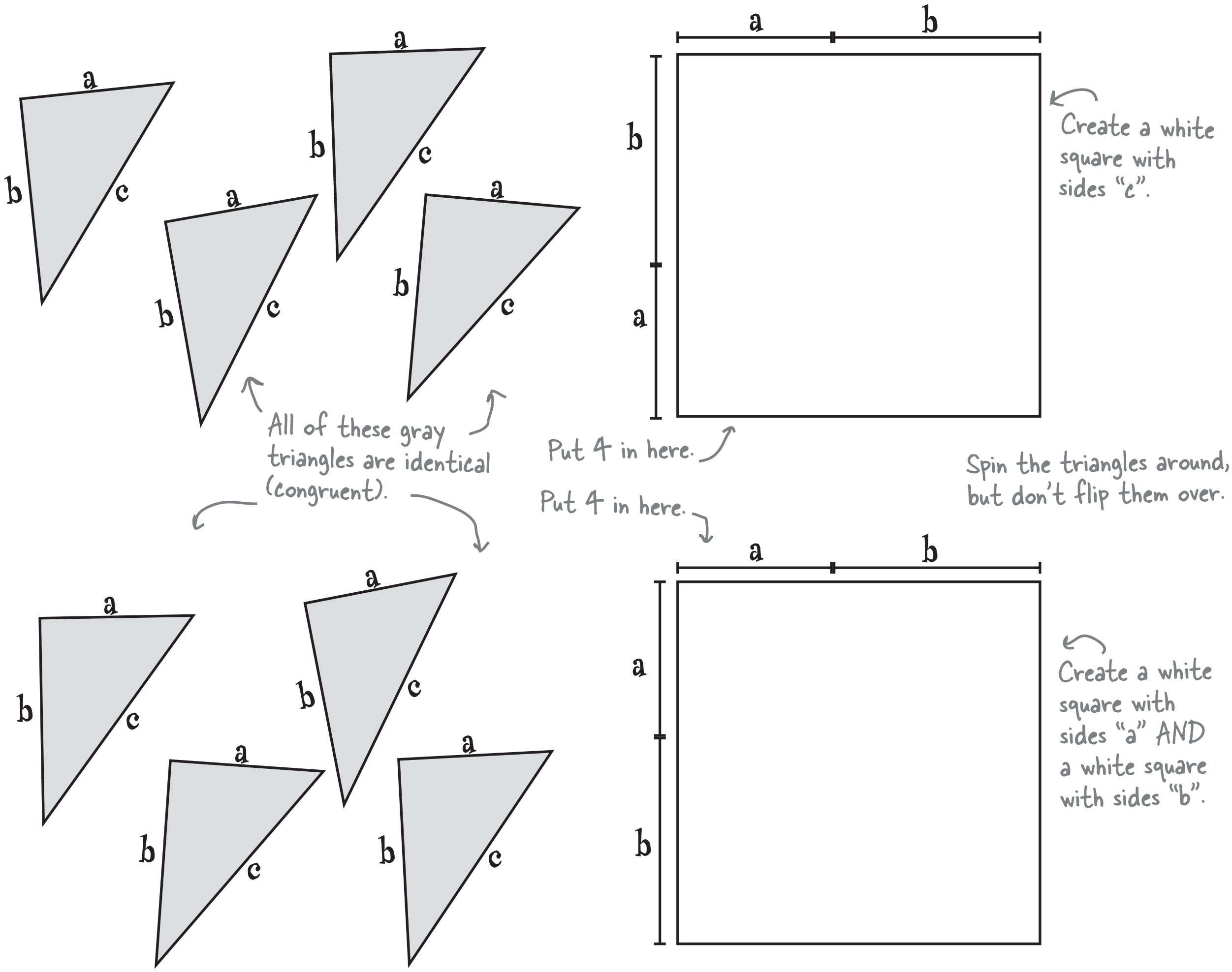
Geometry Investigation Magnets

Let's experiment with a general right triangle. The sides of the triangle can be a, b, and c—where c is the longest side. Below are two large squares, each of which has side length a+b.



Can you arrange the gray triangles inside the squares so that in one box you are left with a square with side length "c" and in the other box you are left with two squares—one with side length "a" and one with side length "b"? Make sure to use four triangles in each box.

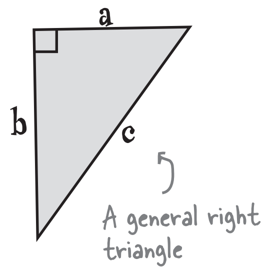
What do you know about the white area left in each box? What does this tell you about how the pattern you found might work for a right triangle with sides a, b, and c?





Geometry Investigation Magnets Solution

Let's experiment with a general right triangle. The sides of the triangle can be a, b, and c—where c is the longest side. Below are two large squares, each of which has side length a+b.

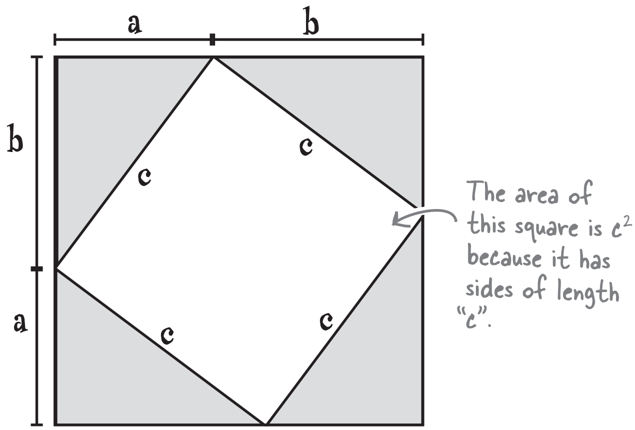


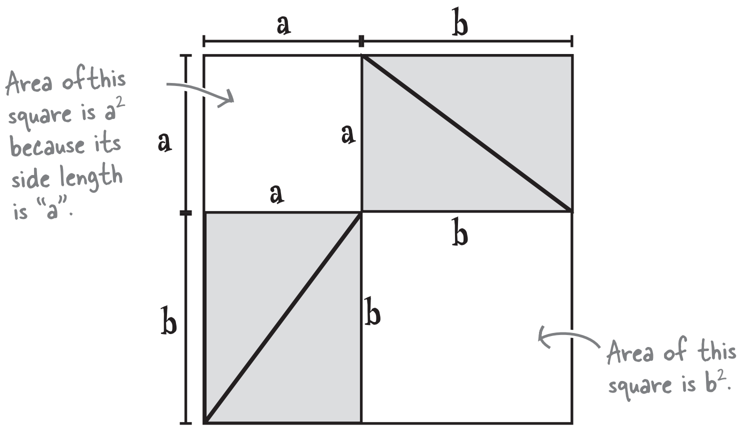
Can you arrange the gray triangles inside the squares so that in one box you are left with a square with side length "c" and in the other box you are left with two squares—one with side length "a" and one with side length "b"? Make sure to use four triangles in each box.

What do you know about the white area left in each box? What does this tell you about how the pattern you found might work for a right triangle with sides a, b, and c?

The gray triangles are all congruent, so the gray area we've created in each box must be equal. This means that the leftover area in white must also be equal.

For a right triangle with sides a, b, and c, the square of c is equal to the squares of a and b added together.



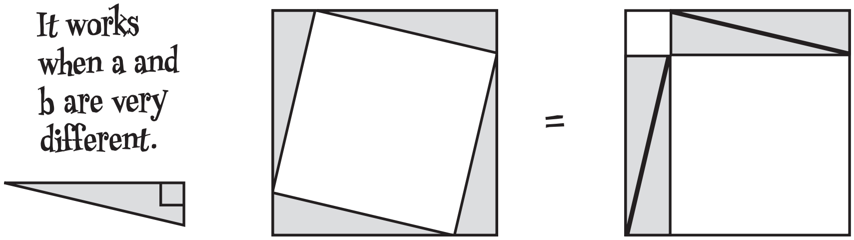




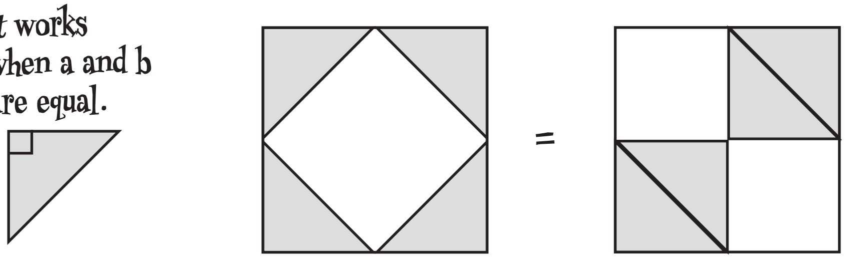
So this pattern should work for any right triangle, because those white areas will always be equal?

Yes. No matter what your right triangle looks like, this should always work.

It works when a and b are very different.



It works when a and b are equal.



BRAIN BARBELL

You've got a skate course to design! Drawing triangles and squares might be a good way to investigate the pattern, but it's probably not the most useful way to capture it for reuse. How could you capture this pattern:

1. As words? ....

....

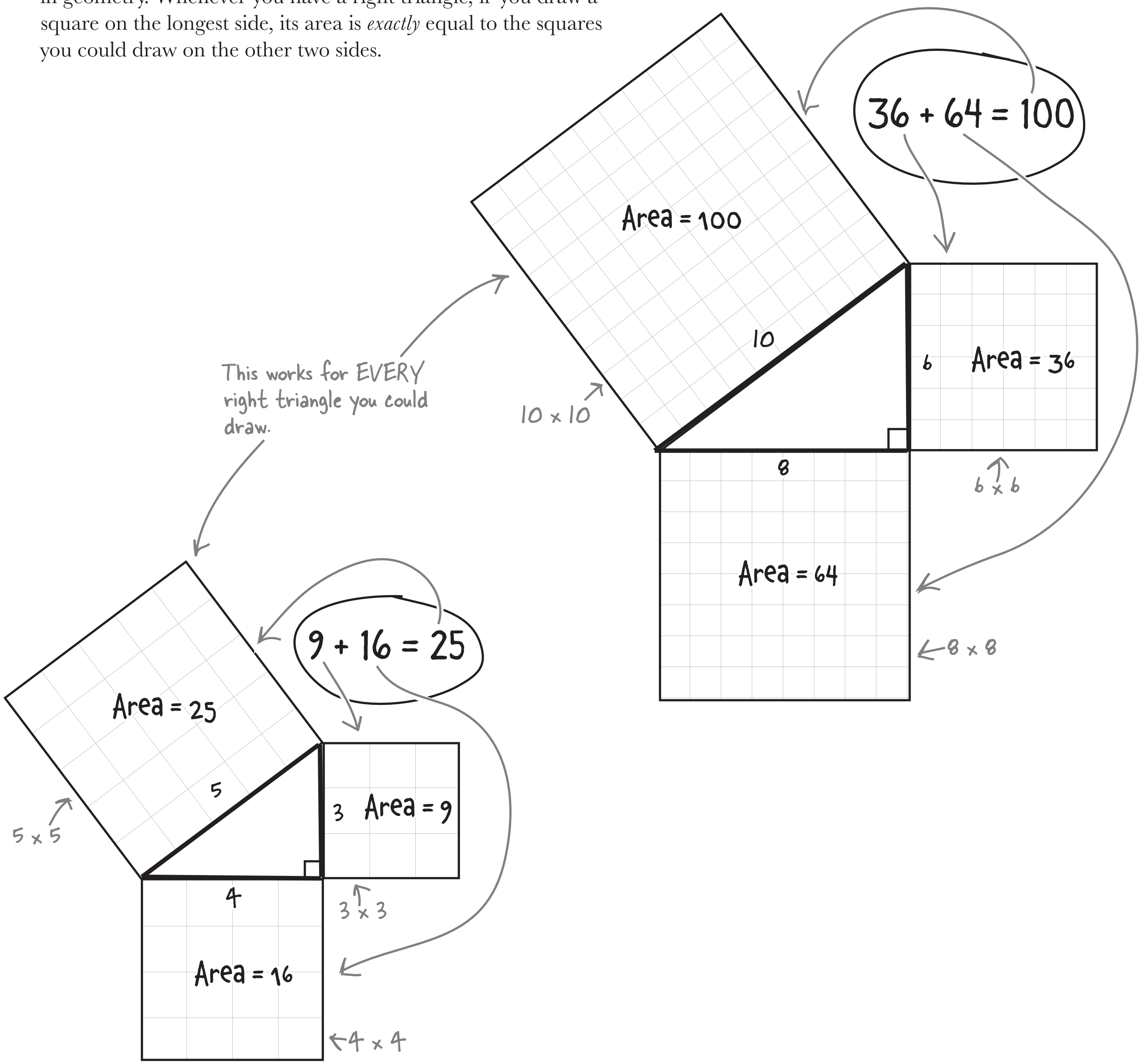
2. Using algebra (use the letters a, b, and c like in the magnets exercise)

....

The square of the longest side is equal to the squares of the other two sides added together

Your answer might be in words or letters.

This has to be one of the most amazing patterns you'll discover in geometry. Whenever you have a right triangle, if you draw a square on the longest side, its area is *exactly* equal to the squares you could draw on the other two sides.

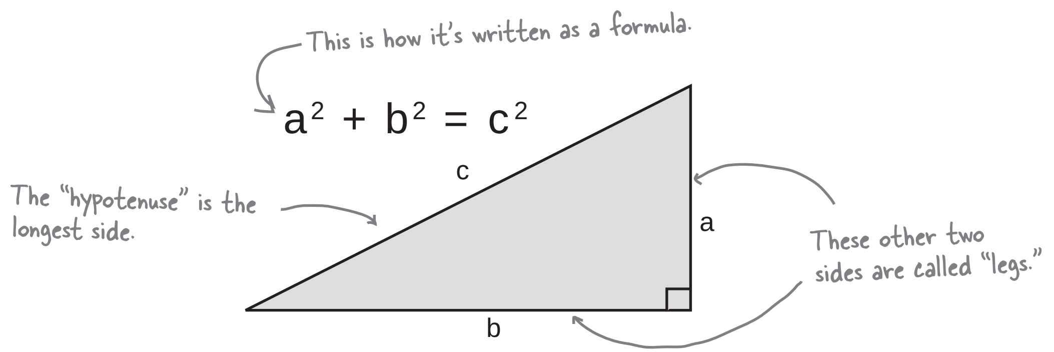


The Pythagorean Theorem: $a^2 + b^2 = c^2$

This pattern is known as ***The Pythagorean Theorem***. Using it you can find out whether the corner opposite the longest side of a triangle is acute, obtuse, or a right angle. In words, the theorem is usually written:

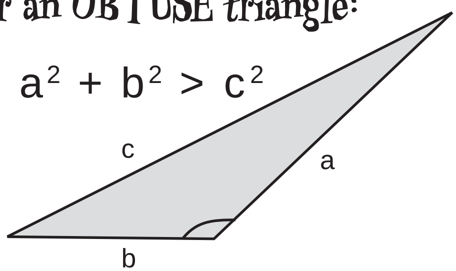
Author's note: In British English, it's known as Pythagoras' Theorem.

The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.



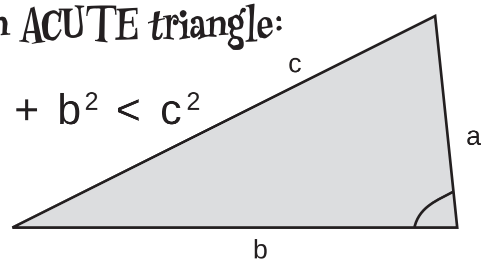
For an OBTUSE triangle:

$$a^2 + b^2 > c^2$$

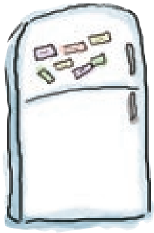


For an ACUTE triangle:

$$a^2 + b^2 < c^2$$



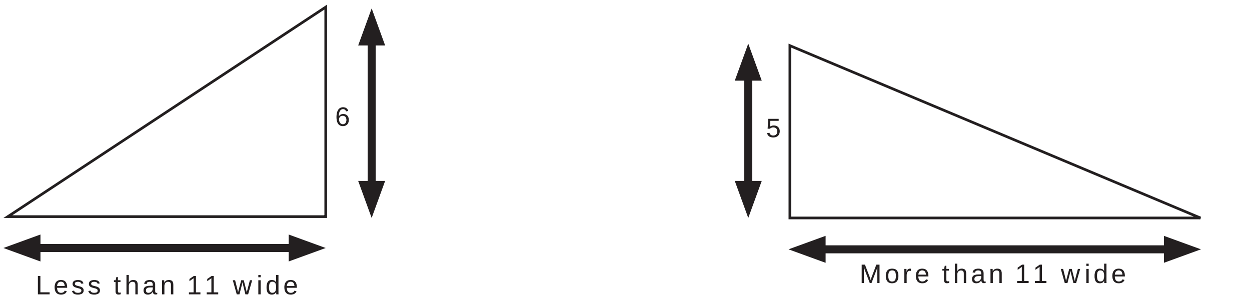


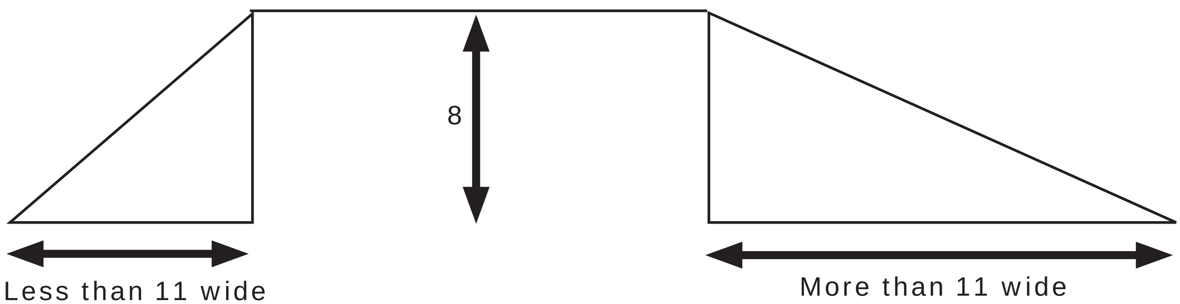


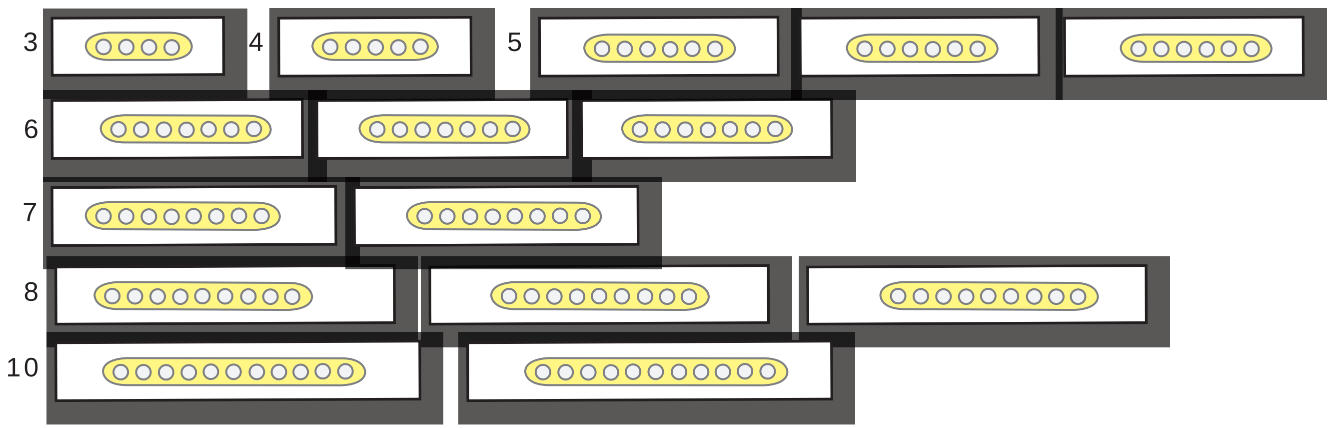
Skate Ramp Magnets

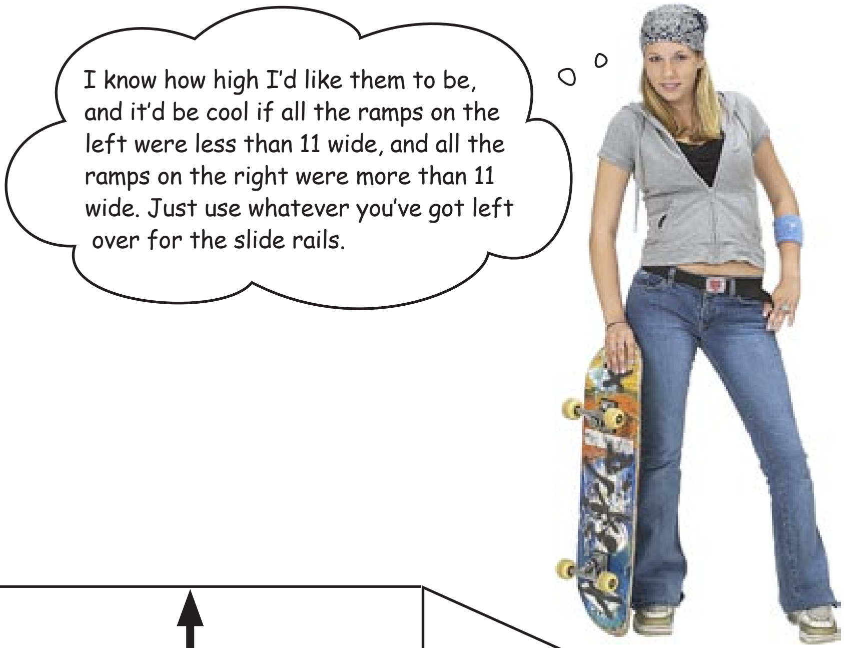
Sam's sketched out a rough design for the competition course. She wants three different sets of ramps for the skaters to demo their skills on—one with a gap to jump and two with joining rails to slide on.

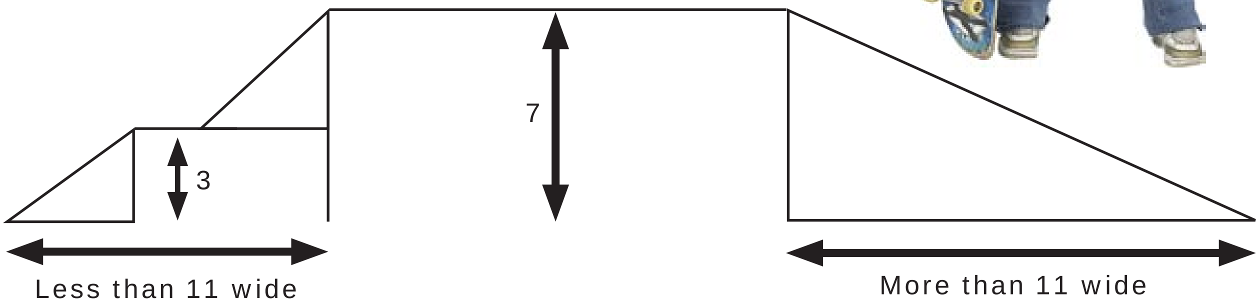
Using the Pythagorean Theorem and the construction kit magnets, work out where each part needs to go to complete the course design. Use each part exactly once.

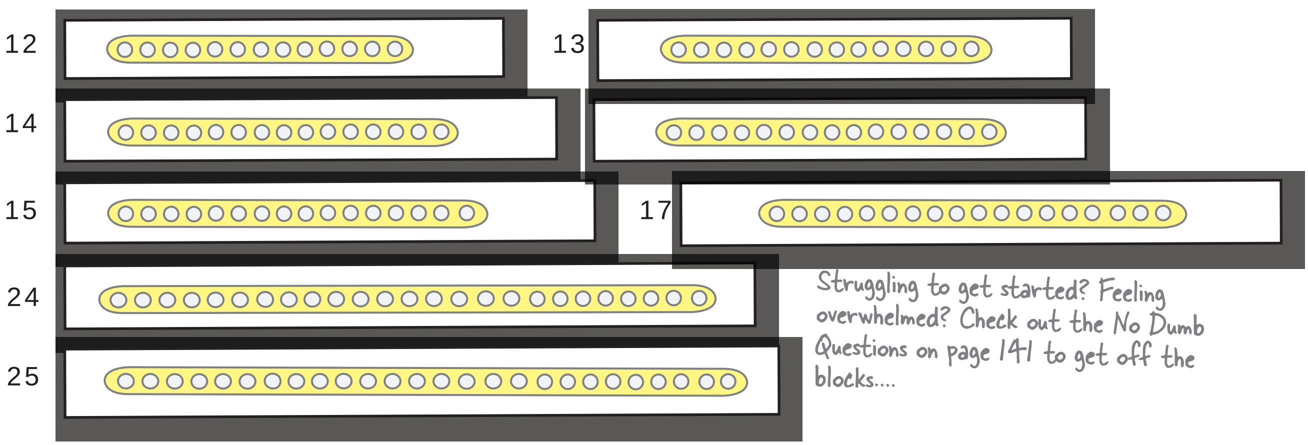




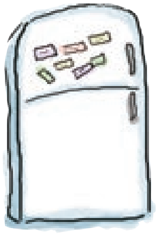








Struggling to get started? Feeling overwhelmed? Check out the No Dumb Questions on page 141 to get off the blocks....

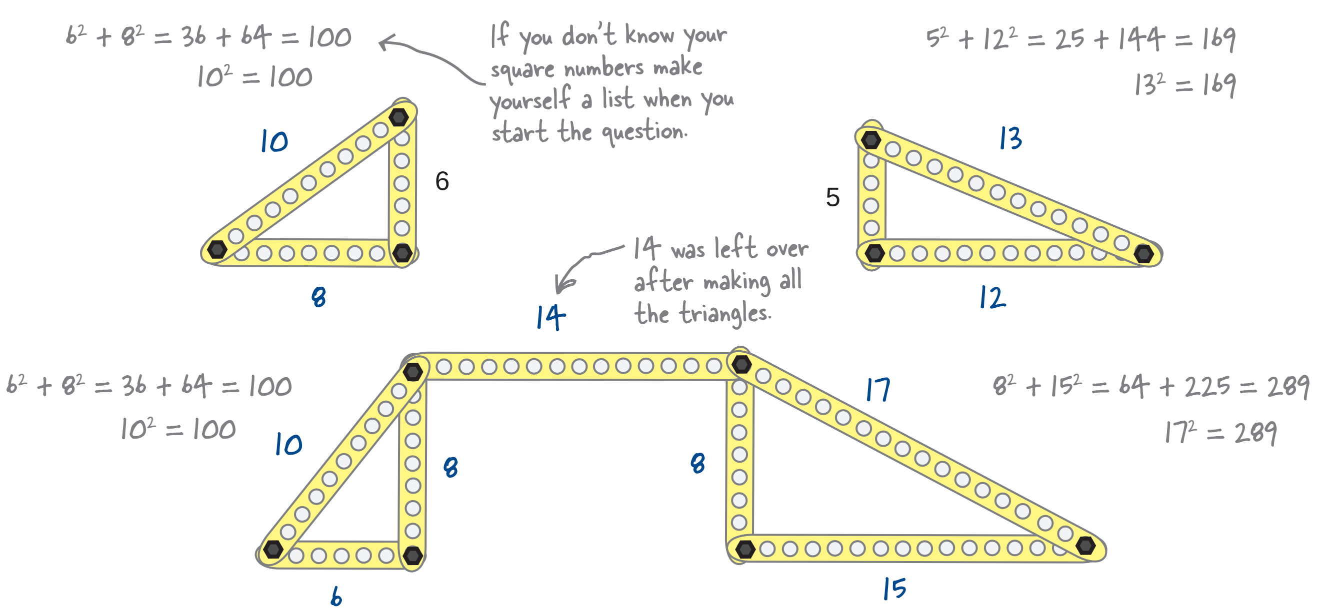


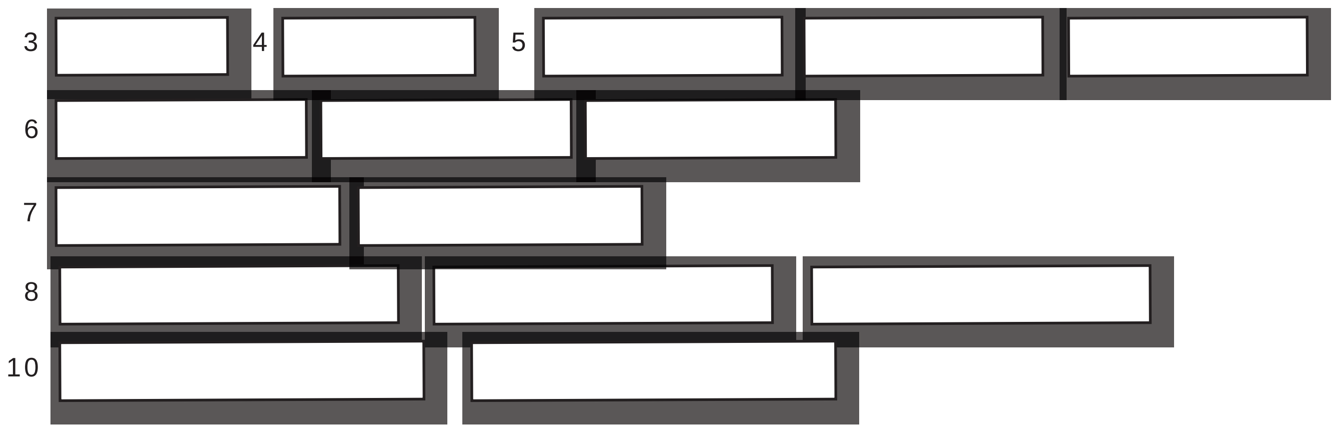
Skate Ramp Magnets Solution

Sam's sketched out a rough design for the competition course. She wants three different sets of ramps for the skaters to demo their skills on—one with a gap to jump and two with joining rails to slide on.

Using the Pythagorean Theorem and the construction kit magnets, work out where each part needs to go to complete the course design. Use each part exactly once.

Solving this problem is mostly a matter of trial and error—checking to see where you can find lengths which fit the Pythagorean Theorem: Longest side squared = sum of other two sides squared.





Don't worry if you didn't do it this way—but it's a good tip to get yourself going.

The key to finding sets of lengths that work for right triangles is to remember that the Pythagorean pattern is all about combining square numbers.

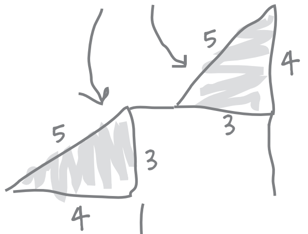
So—you could start by writing down each length, and its square, and then look to see whether the difference between any of your square numbers is the same as another square number you've written down.

When you've got one side length you even know which difference you're looking for: e.g., short side 8 = difference 64.

Lengths	Squares
3	9
4	16
5	25
6	36
7	49
8	64
10	100
12	144
13	169
14	196
15	225
17	289
24	576
25	625

Difference between 576 and 625 is 49

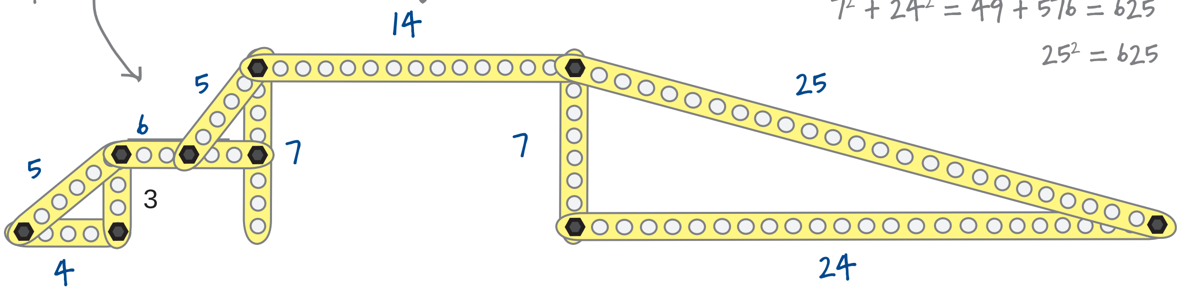
This part here is tricky—you might have recognized that you need to use two of those useful 3–4–5 triangles.

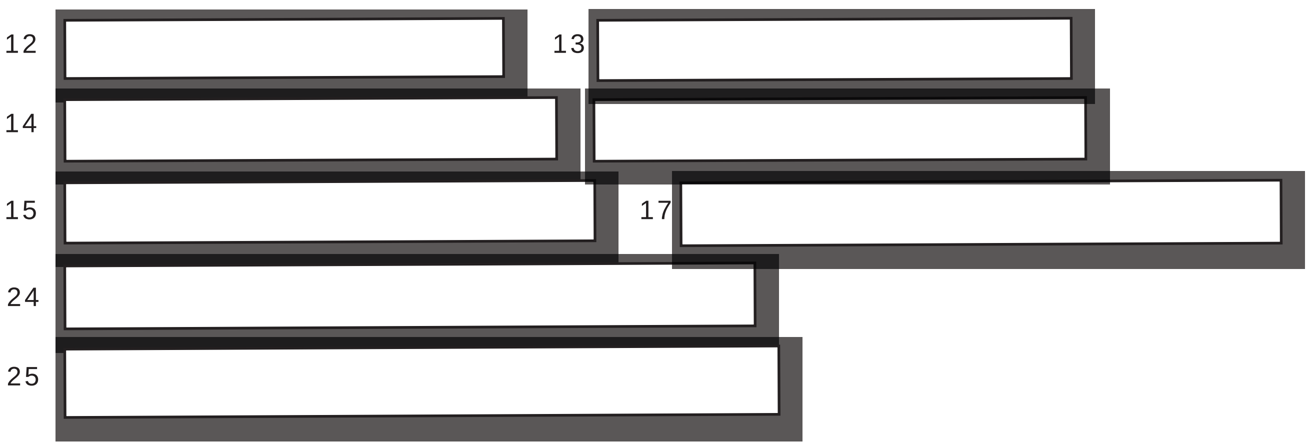


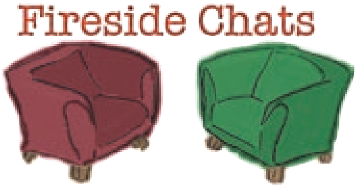
14 was left over after making all the triangles.

$7^2 + 24^2 = 49 + 576 = 625$

$25^2 = 625$







Tonight's talk: **Integer solutions vs non-integer solutions for right triangles**

Integer solutions:

I guess we're quite a famous family. Most people can recognize at least one or two of us. 3-4-5 gets the most attention but that's not the whole range.

Non-integer solutions:

Range schmange. We don't have a range—think of a number, and we can find you a solution. We non-integer solutions are flexible. Any size, any place, you can still have a right angle.

Integer solutions:

Yeah but you totally need a calculator—nobody knows the square roots of weird numbers by heart… and who carries one of those around? We integer solutions are convenient—especially if you can remember them!

Non-integer solutions:

Remember? People forget we even exist—never mind learning specific versions of us…you lot are so precious.

Integer solutions:

Us—precious? You're the one with all those high-maintenance decimal points, baby.…

Non-integer solutions:

Oh, I knew you'd bring that up. Yeah, yeah, we're detailed—OK? But cell phones and computers have calculators, and then you don't have to remember a thing—just use us where you need. And don't forget you can always write a root as a root.…

Integer solutions:

That's true. I always think you look better that way actually.…

Using Kwik-klik skate ramps is definitely the right angle!

By finding right triangles with whole number ratios between the lengths of the sides, you've managed to build a whole load of different ramps from the Kwik-klik standard parts. It's a fast, cheap, and smart way to build a skate course, and you're in a great position to reap the rewards.



BULLET POINTS

- To find the longest side, square the other two sides, add those up, and then find the square root.
- To find a short side, square the other two sides and subtract one from the other. Make it a positive number and then find the square root.
- Finding missing sides using the Pythagorean Theorem only works for right triangles.

there are no Dumb Questions

Q: But WHY? How come it adds up like that?

A: That is a very good question and one which mathematicians and philosophers have struggled with for centuries. Unfortunately there's not always good "why" reasons in geometry—stuff just IS. The good news is that it's very reliable…so while understanding why the Pythagorean Theorem works is beyond most of us, at least we can make good use of it.

Q: Why is it called The Pythagorean Theorem? Why not something more meaningful and easier to spell like "Right triangle check" theorem?

A: Pythagoras was a Greek guy. He was first to write down the theorem. That isn't to say that nobody had noticed it before, but he gets the props. Shame he wasn't called something easier to spell!

Q: I tried to use the Pythagorean Theorem to find a missing short side, and I was stuck trying to find the square root of a negative number. What should I do?

A: You've probably got your legs and your hypotenuse mixed up. Try to redo your subtraction the other way round. This should give you a positive number, and you'll be able to find the square root.

Q: So is "c" always the hypotenuse?

A: The Pythagorean Theorem uses "c" for the hypotenuse and "a" and "b" for the legs. Of course we know that algebra is just a tool for describing a pattern, and in this case, the pattern is what's important: **the longest side squared equals the other two sides squared.** So, if you get mixed up about your a, b, and c, think about what the pattern behind that formula is, and go from there.

Q: Do all triangles have a hypotenuse? What if I have two long sides of equal length and one short side? Which is the hypotenuse?

A: Only right triangles officially have a hypotenuse. And if you've got two sides equally long, and one shorter one, your triangle can't have a right angle, because it would fail the $c^2 = a^2 + b^2$ test, whichever of your longer sides you decided was "c". But don't think you can't use the Pythagorean Theorem for triangles that don't have a right angle…it's just takes a little more thought to apply it. (More on that in a minute.)



From: info@xtremeaction.com
Subject: $3000 for some TV work
Date: 4 July 2009 11:22:14
To: sam@skatedesignsunlimited.com

Hi Sam,

I'm the executive producer of the popular TV show *Xtreme Action* and one of the skaters we regularly film with has told me about your exciting competition. Apparently there's a real buzz going round—people are saying you're pushing the ramp design way beyond the normal level, and that you've got some cool new technology that means that the courses are portable?

We can see huge potential in what you're doing, and we'd like to film your next competition and feature it on our show. We'll be able to offer you a fee of $3,000 dollars per competition in return for the filming rights.

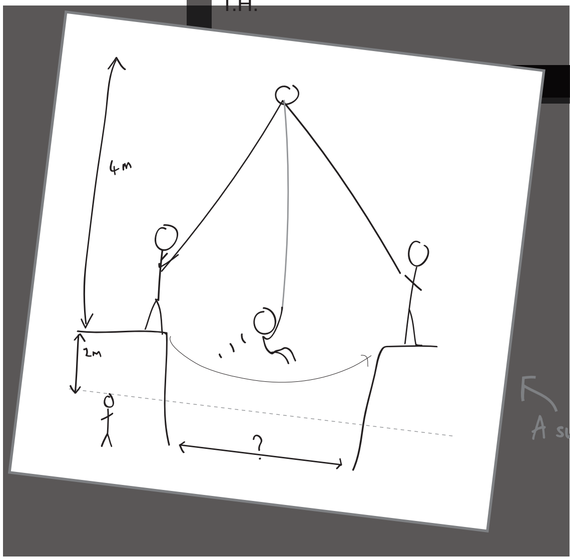
We'd also like you to add a huge rope swing in the middle of the course, can you handle the design?

We're touring the show and at the first venue the platforms are gonna be 2m above head height (allowing for someone real tall), and the clearance from there to the ceiling where we'll fix the swing is 4m. How far apart can we place the blocks without anyone swinging straight into the floor?

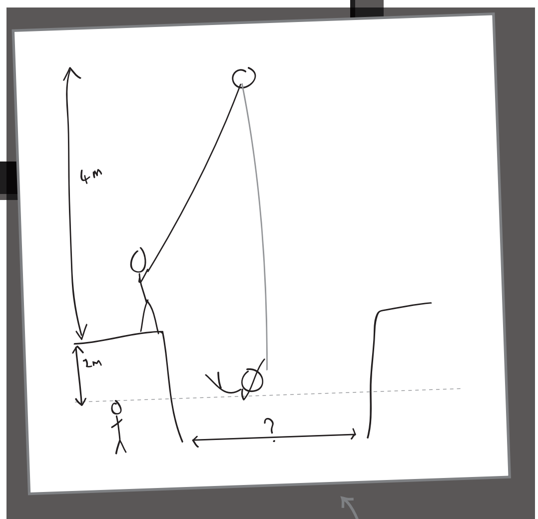
I've attached a couple of sketches,

Ride on!

T.H.



A successful swing

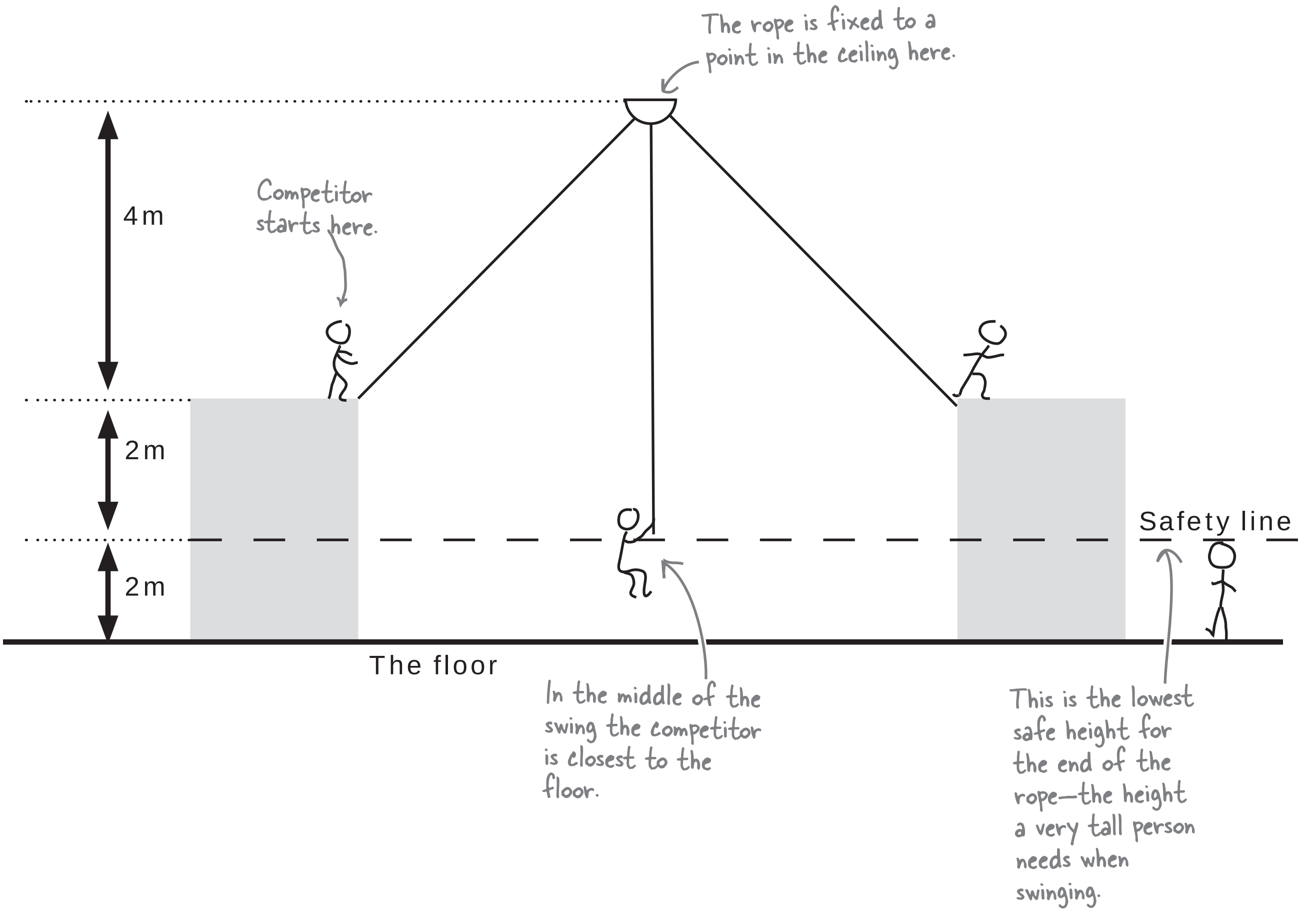


Ouch—better avoid the face plant!

The TV company wants you to design them a rope swing, so that they get maximum excitement and minimum lawsuits. Are they right to be worried about accidents if the blocks at either side of the swing are too far apart?

A longer rope swings further and lower

The wider the gap between the platforms, the more exciting the swing will be—but there's a catch. The TV company doesn't just want the widest swing, they want the widest possible swing without people smashing into the floor!





Sharpen your pencil

What's the longest the rope can be without going below the safety line?

there are no Dumb Questions

Q: The Pythagorean Theorem formula ($c^2 = a^2 + b^2$) looks like it's for finding the hypotenuse (c). But sometimes we're finding a length of a short side. How do I know what I'm supposed to be finding and how to find it?

A: You can rearrange the formula to focus on one of the short sides (legs). Like $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$, but the most reliable thing to do is to focus on the meaning behind the pattern. If you're looking for the hypotenuse remember that the squares of the two shortest sides add up to make the square of the longest side.

If you're finding a short side then you need to think of the pattern like this: the difference between the square of the longest side and another side is the square of the remaining side.

If you're stuck trying to design the skate course on page 133, then try this to get you started.

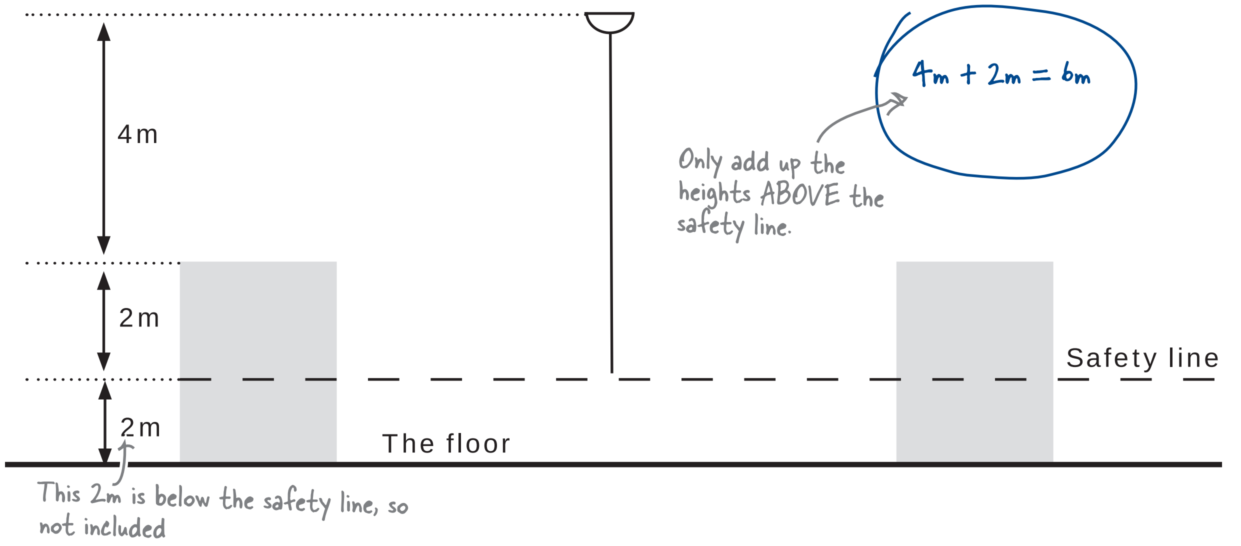
Q: OK, that's not so bad when I'm given the lengths of two sides and I have to find the other side's length. But what about when I'm only given one side and I have to find integer values that complete a right triangle? The formula can't give me the answer because I don't have enough values!

A: Again, think about the pattern behind the formula. If you've got a load of possible values—or if you know the value is within a range (like "less than 20")—then here's a trick you can rely on to find the answers: just remember that this pattern is about square numbers. (1, 4, 9, 16, 25, 36, 49, 64, 81, 100,…etc).

Write down all the values you've been given to choose between, and then write down their squares. Then compare pairs of numbers and see whether they add up to a square number, or whether the difference between them is a square number.

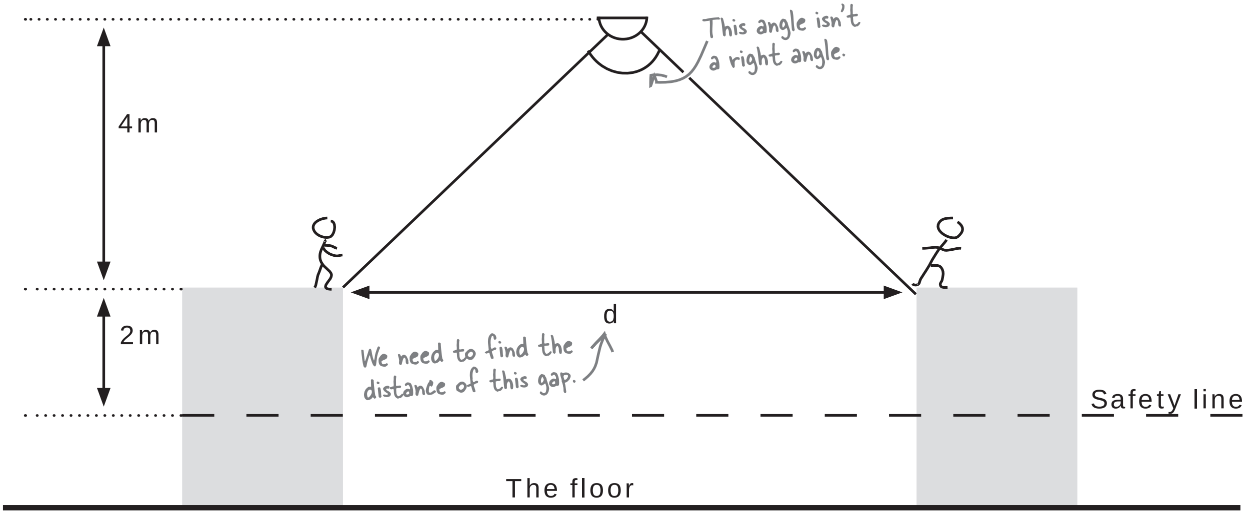
Sharpen your pencil Solution

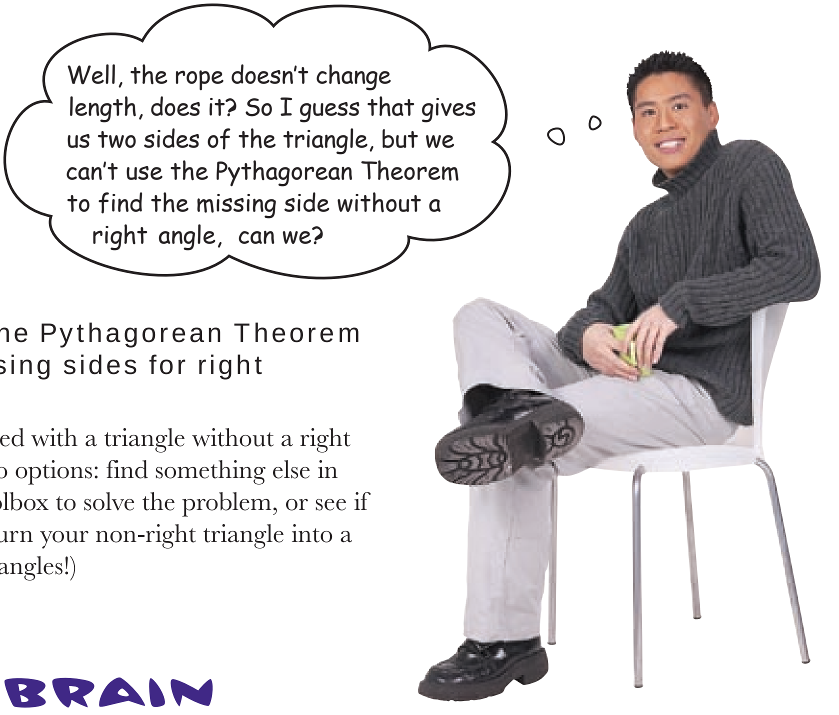
What's the longest the rope can be without going below the safety line?



So, how far can you swing on a six-meter rope?

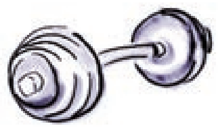
The gap between the platforms is the base of a triangle, with the rope making up two of the sides joined at the point where the rope is fixed at the top. So the distance of the gap is the same as the length of the base of the triangle.





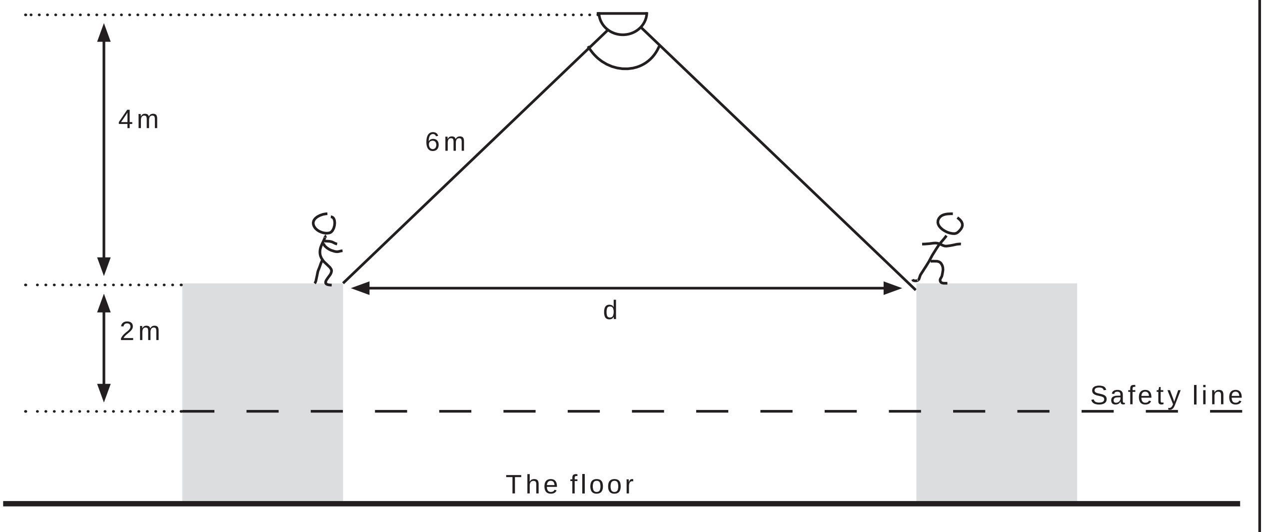
That's right—the Pythagorean Theorem only finds missing sides for right triangles.

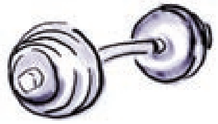
So, when you're faced with a triangle without a right angle you've got two options: find something else in your Geometry Toolbox to solve the problem, or see if you can somehow turn your non-right triangle into a right triangle (or triangles!)



BRAIN BARBELL

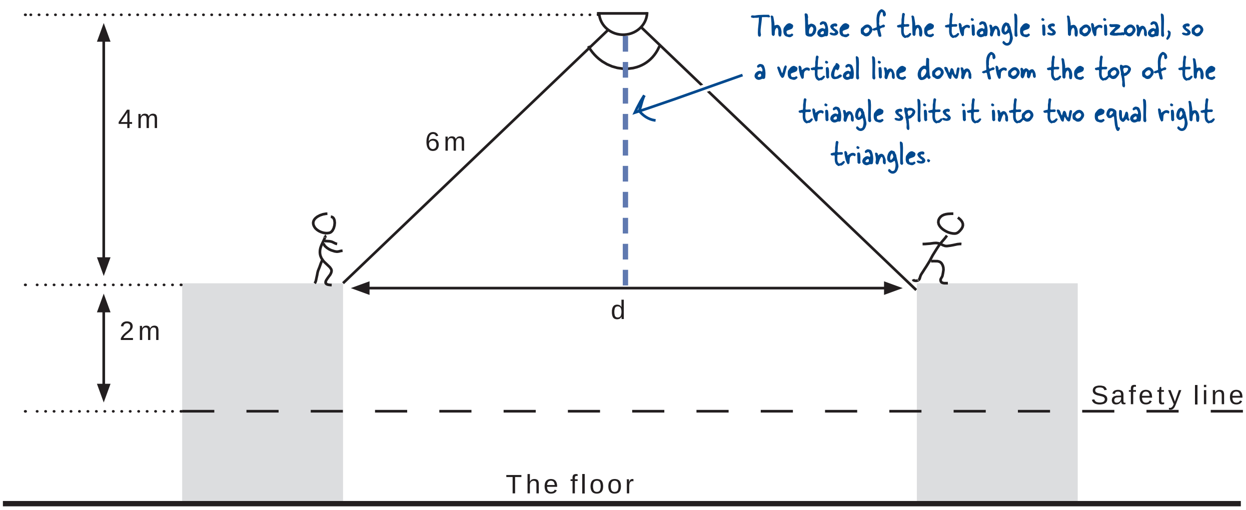
How can you make the swing problem into a right triangle problem so that you can use the Pythagorean Theorem to find length d?





BRAIN BARBELL SOLUTION

How can you make the swing problem into a right triangle problem so that you can use the Pythagorean Theorem to find d?

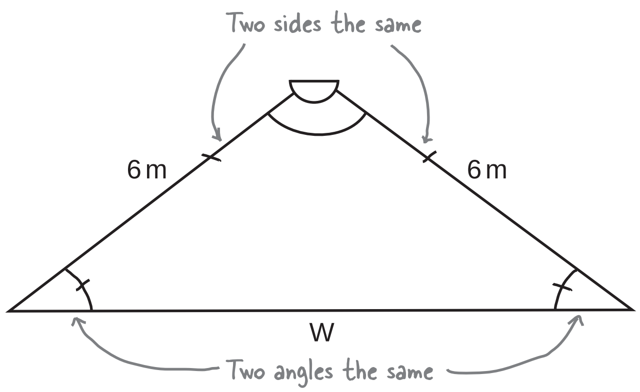


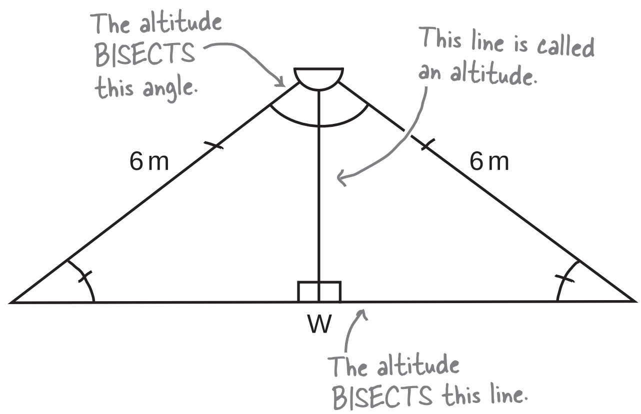
You can split an isoceles triangle into two congruent right triangles

You can split a triangle into two right triangles by drawing an ***altitude***—a line which joins the top of the triangle to the base, perpendicular to the base. An isoceles triangle has two sides and two angles the same, so the two triangles created are congruent.

Bisect

Chop into two equal parts.







BE the Rope Swing

Your job is to play like you're the 6m rope. Use the Pythagorean Theorem to work out how far across a gap someone could swing on you.

there are no Dumb Questions

Q: I get that the rope length doesn't change, so the sides are equal, but how did we know that the bottom two angles are equal?

A: An isoceles triangle has two sides equal but also two angles equal—always. So—if you see that the sides are the same you know the angles are the same, and the other way around.

Q: How did we know that the altitude would be vertical?

A: The altitude is always perpendicular to the base that it's drawn on, so if that base is horizontal (as it is in this case), then the altitude must be vertical.

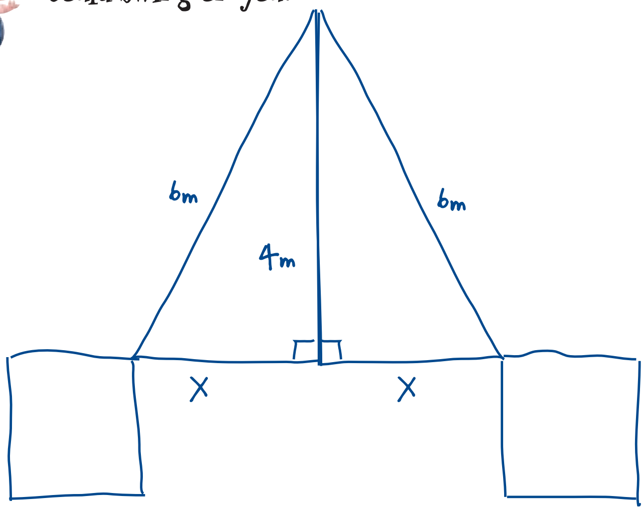
Q: So, which triangle does the altitude belong to?

A: Both! The altitude is the shared side of the two identical (congruent) triangles it creates in this case. So, it belongs to both of them.



BE the Rope Swing Solution

Your job is to play like you're the 6m rope. Use the Pythagorean Theorem to work out how far across a gap someone could swing on you.



Using the Pythagorean Thereom:

$6^2 = 4^2 + X^2$

$36 = 16 + X^2$

$36 - 16 = X^2$

$\sqrt{20} = X^2$

$20 = X$

$4.47 = X$

Total distance you can swing $= 2X = 2 \times 4.47 = 8.94$ meters

there are no Dumb Questions

Q: Wait a second—weren't we looking for integer solutions? How come 8.94 meters is OK?

A: The Kwik-klik parts only came in integer lengths, so we needed integer solutions for our ramps, but for the rope swing a non-integer solution is fine. We can put the blocks 8 meters and 94 centimeters apart.

Q: Right. So, generally, is this Pythagorean thing for finding integer solutions or not?

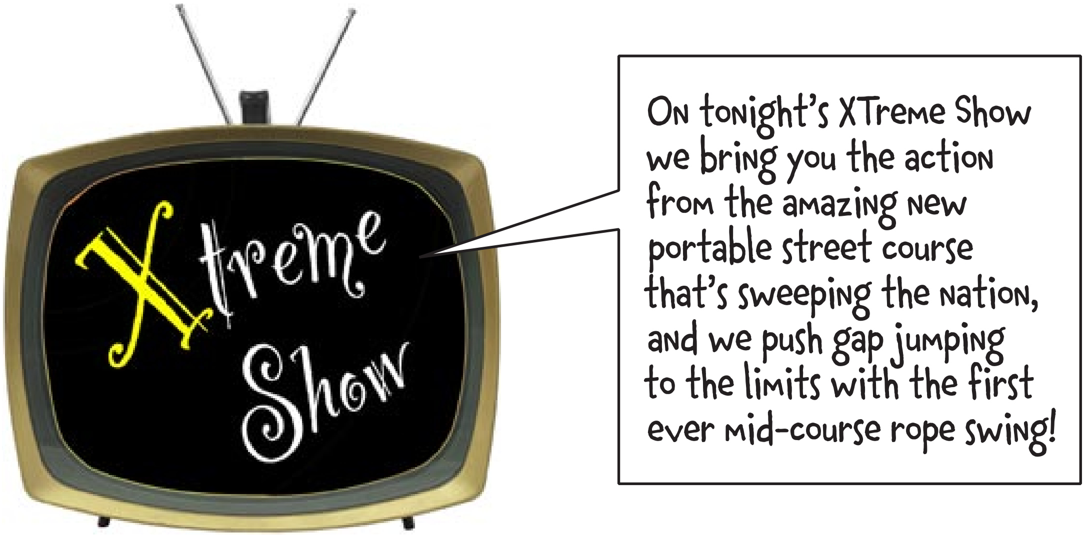
A: You'll find that a lot of geometry problems center around the "Pythagorean triples." You can think of them as *Super Triangles*—right triangles with integer side lengths. You've already found some—3,4,5 and 5,12,13 are two of the most important.

Q: Super Triangles...cool! Do I need to learn the actual values or just know that they exist?

A: You can always use the Pythagorean Theorem to find them, but that's gonna be pretty slow...so if you're good at remembering stuff then learning the first three or four (in the toolbox at the end of this chapter) can be a real time saver.

Your rope swing is perfect

The design is exactly right, so your swing goes straight into action on the first show of the series.







Your Geometry Toolbox

You've got Chapter 3 under your belt and now you've added the Pythagorean Theorem to your tool box. For a complete list of tooltips in the book, head over to www.headfirstlabs.com/geometry.

Super Triangles

(Also known as Pythagorean triples)

3 – 4 – 5

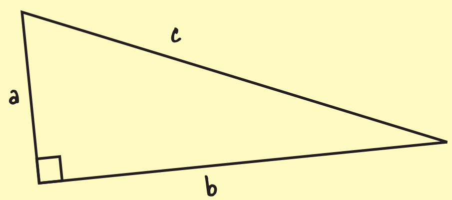
5 – 12 – 13

7 – 24 – 25

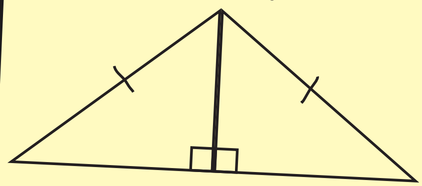
8 – 15 – 17

The Pythagorean Theorem

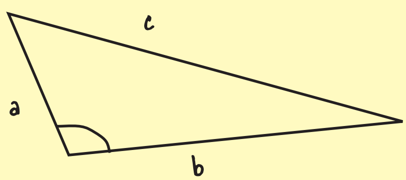
$a^2 + b^2 = c^2$



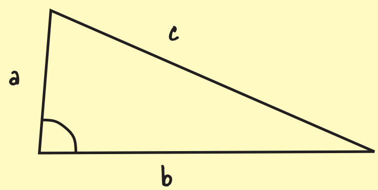
The altitude of an isoceles triangle splits it into two congruent right triangles.



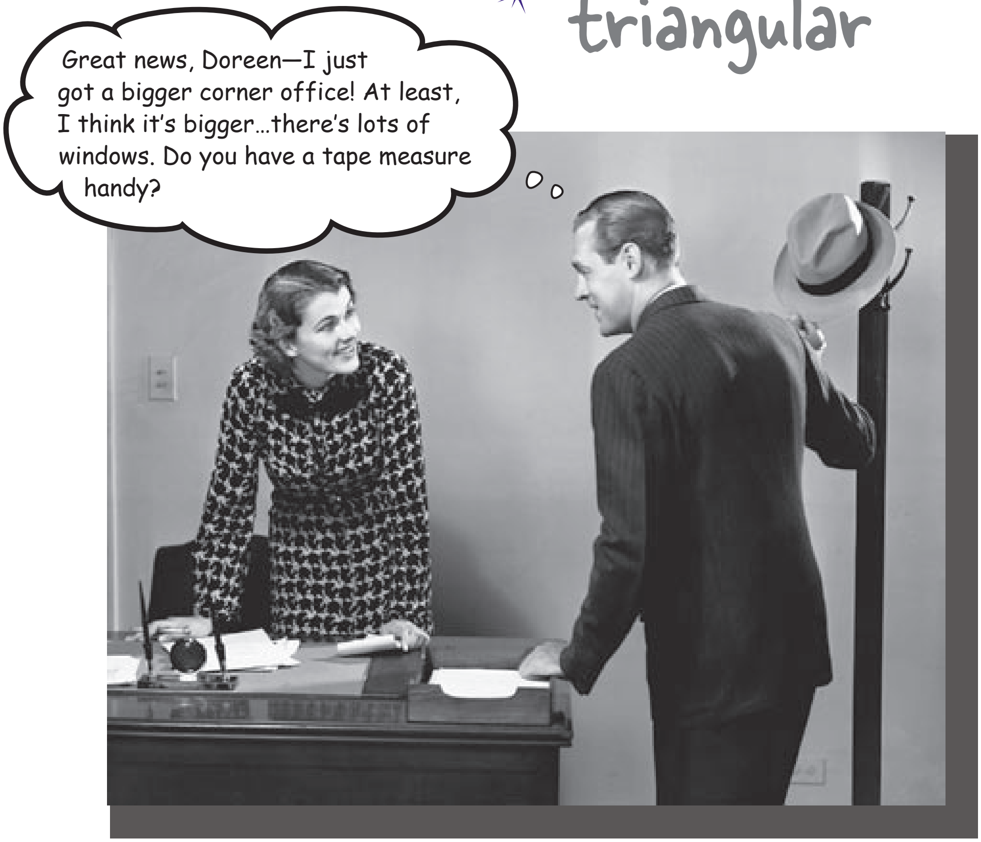
If $a^2 + b^2 < c^2$, then the triangle is obtuse.



If $a^2 + b^2 > c^2$, then the triangle is acute.



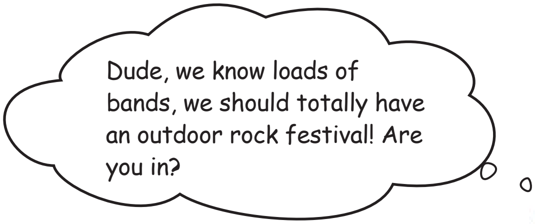
Between a rock show and a ~~hard~~ triangular place



Ever had that sinking feeling that you've made a bad decision?

In the real world, *choices can be complex*, and wrong decisions can cost you **money** and **time**. Many solutions aren't always straightforward: even in geometry, bigger doesn't always mean better—it might not even mean longer. ***So what should you do?*** The good news is that you can *combine your triangle tools* to **make great decisions** even when it seems like you don't have the right information to answer the question.

Everybody loves organizing a rock festival





Your buddies have big plans for a local rock festival, but these things don't just organize themselves. There's a venue to choose, security fencing to sort out, a sound system to select.…

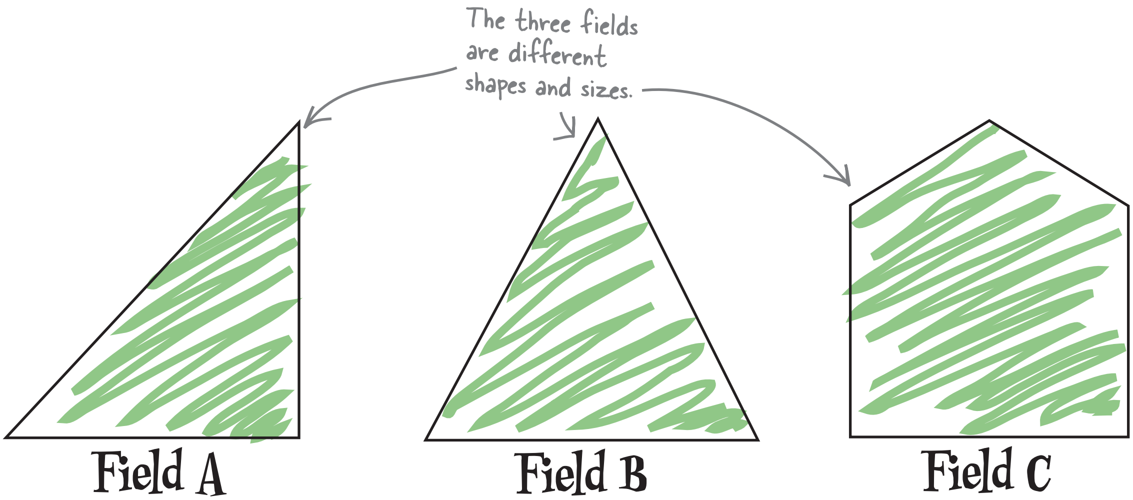
Think you can handle it?



First we need to pick a venue

There are three local fields where this sort of event can take place, and your choice of venue is absolutely critical. It determines how many people can come to your festival and what it will cost to make sure they all pay to get in.

Here are the three possible venues:



Sharpen your pencil

What do you need to find out about each potential venue to figure out the following? Just describe these in normal words—don't worry about geometry jargon.

a) The fencing costs: ....

We don't want people coming in for free!

b) The capacity: ....

The number of people who can come to the festival

Sharpen your pencil Solution

What do you need to find out about each potential venue to figure out the following? Just describe these in normal words—don't worry about geometry jargon.

a) The fencing costs: The length of the edges

If you wrote "perimeter," that's correct, too.

we don't want people coming in for free!

b) The capacity The area—how much space there is

number of people who can come to the festival

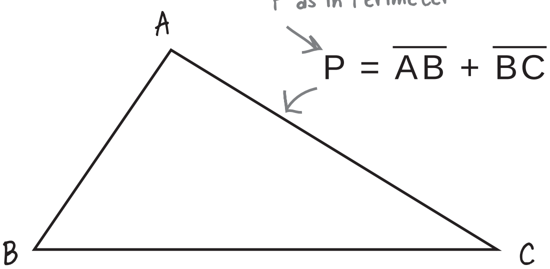
The perimeter is the total length of the sides of a shape

If you add up the lengths of all of the sides, or edges, of a shape then you've got the perimeter. The exact calculation you need to do depends on how many sides your shape has.

P as in Perimeter

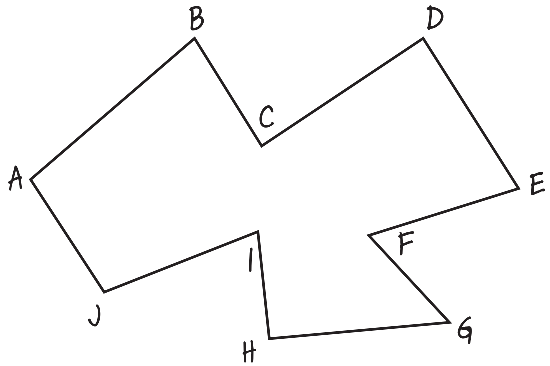
$$P = \overline{AB} + \overline{BC} + \overline{CA}$$

These line notations mean the length of the line segment between those points.

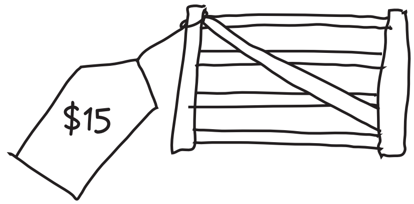


$$P = \overline{AB} + \overline{BC} + \overline{CD} + \overline{DE} + \overline{EF} + \overline{FG} + \overline{GH} + \overline{HI} + \overline{IJ} + \overline{JA}$$

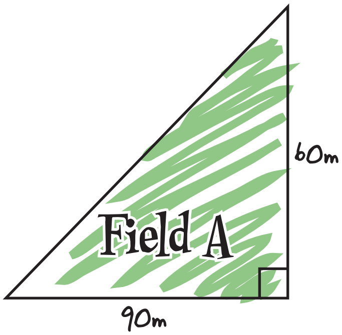
Works for weird shapes, too!

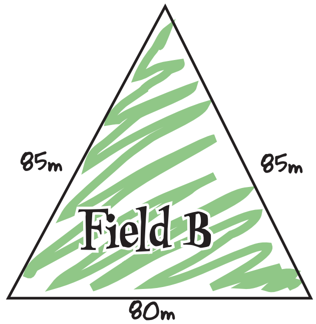


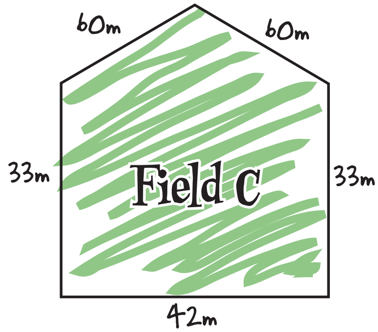
Fencing costs money



To make sure that only the people who've bought a ticket for your festival can come in, you're going to need to set up a security fence around the perimeter. And that's not cheap! The cost of setting up the security fence will be different for each potential venue.









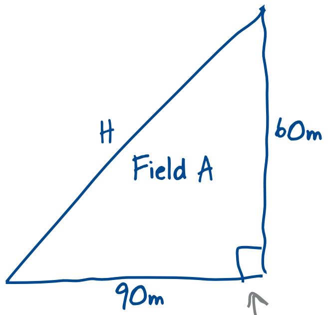
Work out what it would cost to put up a security fence around each venue.
(Fencing costs $15 per meter—work by rounding UP to a whole meter.)

Use a calculator because the numbers get a bit ugly.



Exercise Solution

Work out what it would cost to put up a security fence around each venue. (Fencing costs $15 per meter—work by rounding UP to a whole meter.)



Field A is a right triangle.

For field A we need to use the Pythagorean Theorem to find the missing side (the hypotenuse).

$H^2 = 60^2 + 90^2$

$H^2 = 3600 + 8100$

$H^2 = 11700$

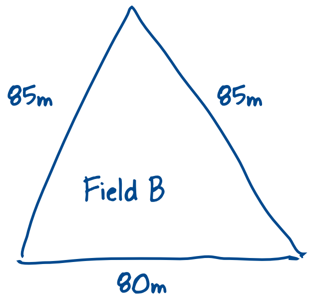
$H = \sqrt{11700}$

$H = 108.16 = 109\text{m}$ ← Rounded up to the next meter—can't buy a bit of a fence panel!

Perimeter, $P = 60 + 90 + 109$

$P = 259\text{m}$ ← Don't forget your units.

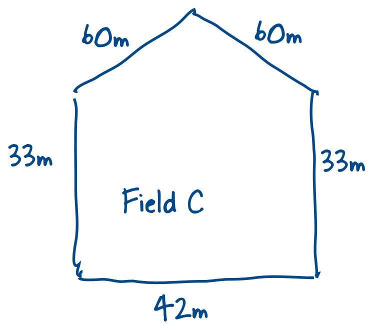
Cost $= 259 \times 15 = \$3885$



Perimeter, $P = 80 + 85 + 85$

$P = 250\text{m}$

Cost $= 250 \times 15 = \$3750$



Perimeter, $P = 60 + 60 + 33 + 33 + 42$

$P = 228\text{m}$

Cost $= 228 \times 15 = \$3420$ ← This one is the cheapest.

Does a bigger perimeter mean a bigger area?



Chris: No way! We should go with the most expensive.

Ben: Why?

Chris: Well, the one with the longest fence has to be the biggest area, and that means more ticket sales.

Tom: You think?

Chris: Makes sense to me! We should use field A for sure.

Tom: But field C is better value! It saves us over 400 bucks.

Ben: I really don't think we know which is best yet… don't you think we need to work out how many people each can hold before we decide?

BRAIN POWER

What approach would **you** take?

How many people can each venue hold?

Your buddies are sure the festival will be a sellout, and at $10 a ticket every extra person counts. Each person needs 2 square meters of space in the festival field—so how many people can each venue hold?

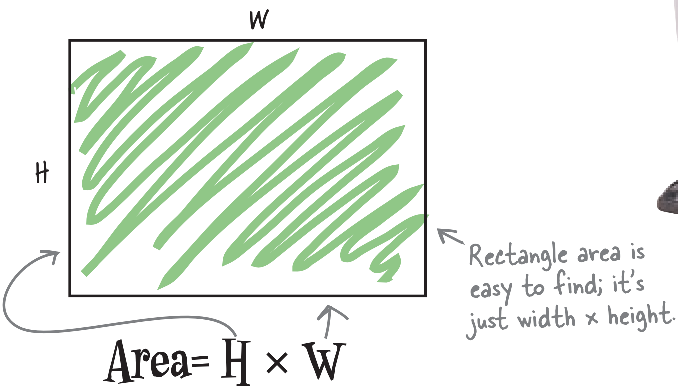


Well, area of a rectangle is width times height. But none of these are rectangular...so we must need to do something different....



That's right.

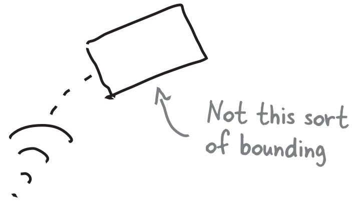
The rectangle area formula isn't a bad place to start, though.



So how does a triangle relate to a rectangle?

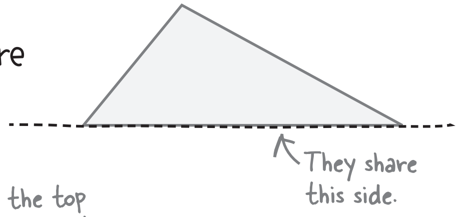
A triangle fits inside a bounding rectangle

A bounding rectangle is a box that fits tightly around the triangle.

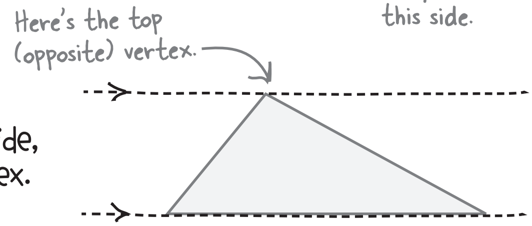


To draw a bounding rectangle:

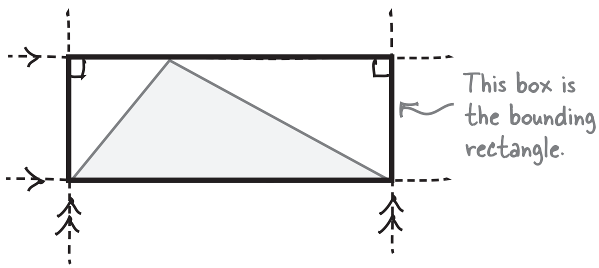
1. Pick a side of your triangle to share with the bounding rectangle.



2. Draw a line parallel to this side, touching the opposite vertex.



3. Draw two perpendicular lines, one at each of the other vertices.

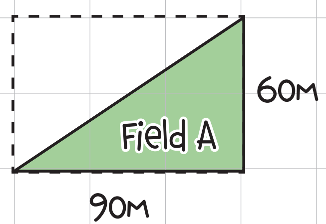


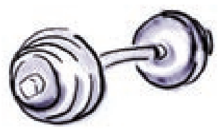


BRAIN BARBELL

How does the shaded area of triangular field A compare with the area of the dashed bounding rectangle surrounding it?

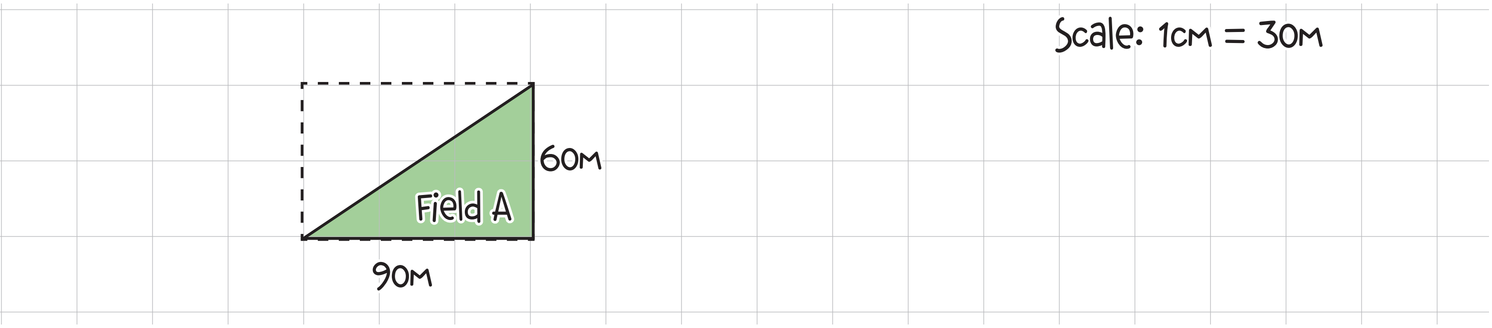
Scale: 1cm = 30m





Brain Barbell

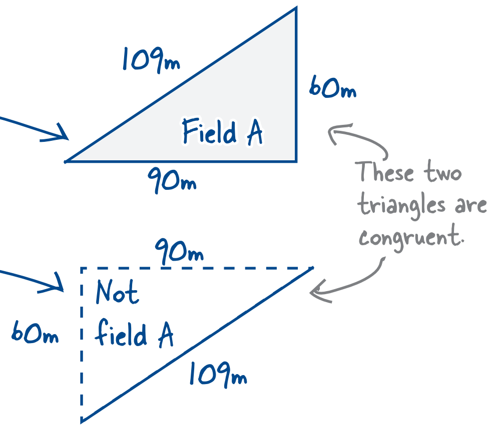
How does the shaded area of triangular field A compare with the area of the dashed bounding rectangle surrounding it?



There are two different ways to think about this question....

1. There's a shaded triangle representing the part of the rectangle which is the field.

   And there's an unshaded triangle representing the part of the rectangle which isn't the field.



If the triangles are congruent, then they must both have the same area, so they share the rectangle area equally—half each.

Both methods give you the same answer: half.

2. You might have recognized visually that each bit of shaded cm square grid has matching unshaded bit. Counting up the shaded and unshaded squares:

As they are both the same, the rectangle is shared equally—half each.

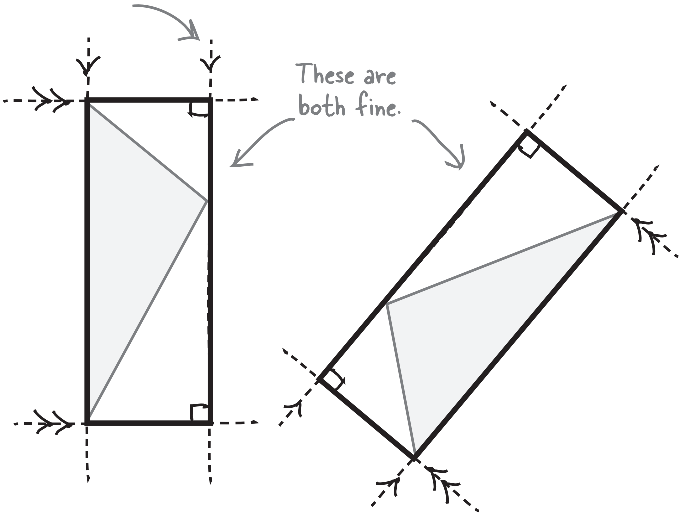


THE AREA OF THE TRIANGLE IS HALF THE AREA OF THE BOUNDING RECTANGLE.

there are no Dumb Questions

Q: What if I've got a triangle which doesn't have a horizonal side? How do I draw a bounding rectangle?

A: Use a vertical side if you have one because it does make things easier, but if you don't have one, then any side will do.

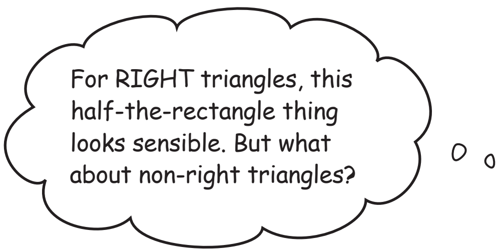


Q: We've talked about sides and angles being the same in congruent triangles, but how do you know that congruent triangles have the same area, too?

A: With a bit of flipping and rotation you can always place congruent triangles on top of one another, and they fit perfectly—so they must have the same area.

Q: What if I don't know the size of the bounding rectangle?

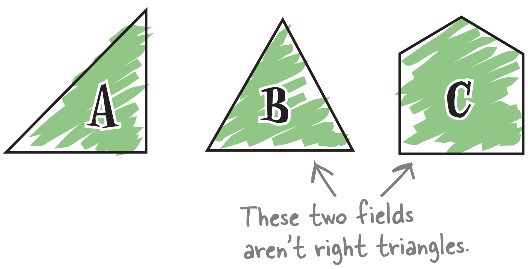
A: Good point. We're going to cover some ways to find that out.





Good question—two of our fields are not right triangles.

So what can we do?

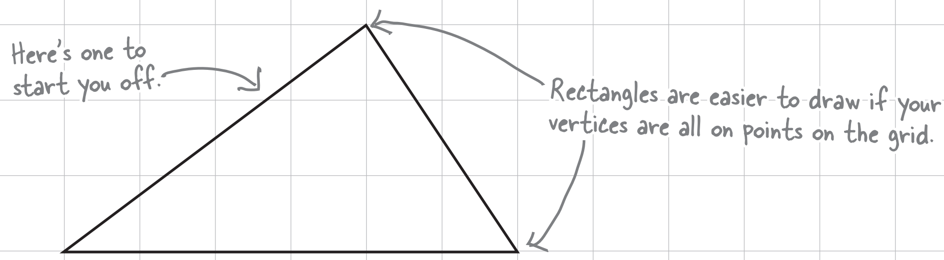




Geometry Detective

Using a ruler, draw some non-right triangles. Use a horizontal or vertical grid line for one side (not two) of each triangle, and make the side that's on the grid line the longest side.

Draw a bounding rectangle around each triangle. How does the triangle area relate to the rectangle area?



Draw at least three more triangles on these pages



Need a 10-minute time-out?

If you're feeling kind of fuzzy about how your triangle areas relate to rectangle areas try having a 10-minute brain-break, then look again before you flip the page.



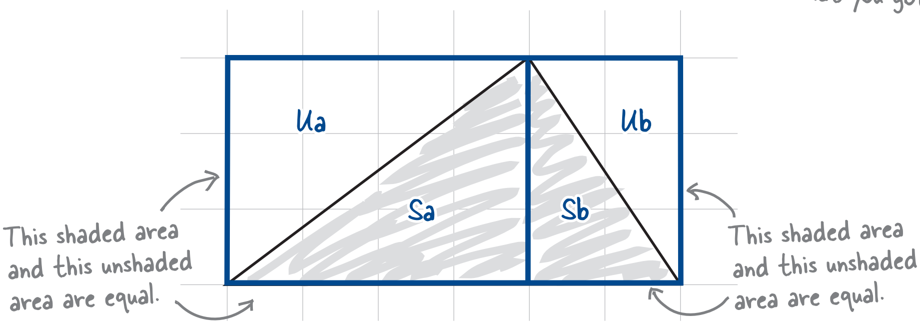
Geometry Detective Solution

Using a ruler, draw some non-right triangles. Use a horizontal or vertical grid line for one side (not two) of each triangle.

Draw a bounding rectangle around each triangle. How does the triangle area relate to the rectangle area?

It's easiest to see the relationship if you split the triangle into two right triangles.

You probably didn't write this up like this—what matters is what you got as the answer.



Shaded part Sa & unshaded part Ua are congruent
So area of shaded part Sa = Area of unshaded part Ua

U and S just stand for Unshaded and Shaded.

Shaded part Sb & unshaded part Ub are congruent
So area of shaded part Sb = Area of unshaded part Ub

Total shaded area = Sa + Sb = Ua + Ub

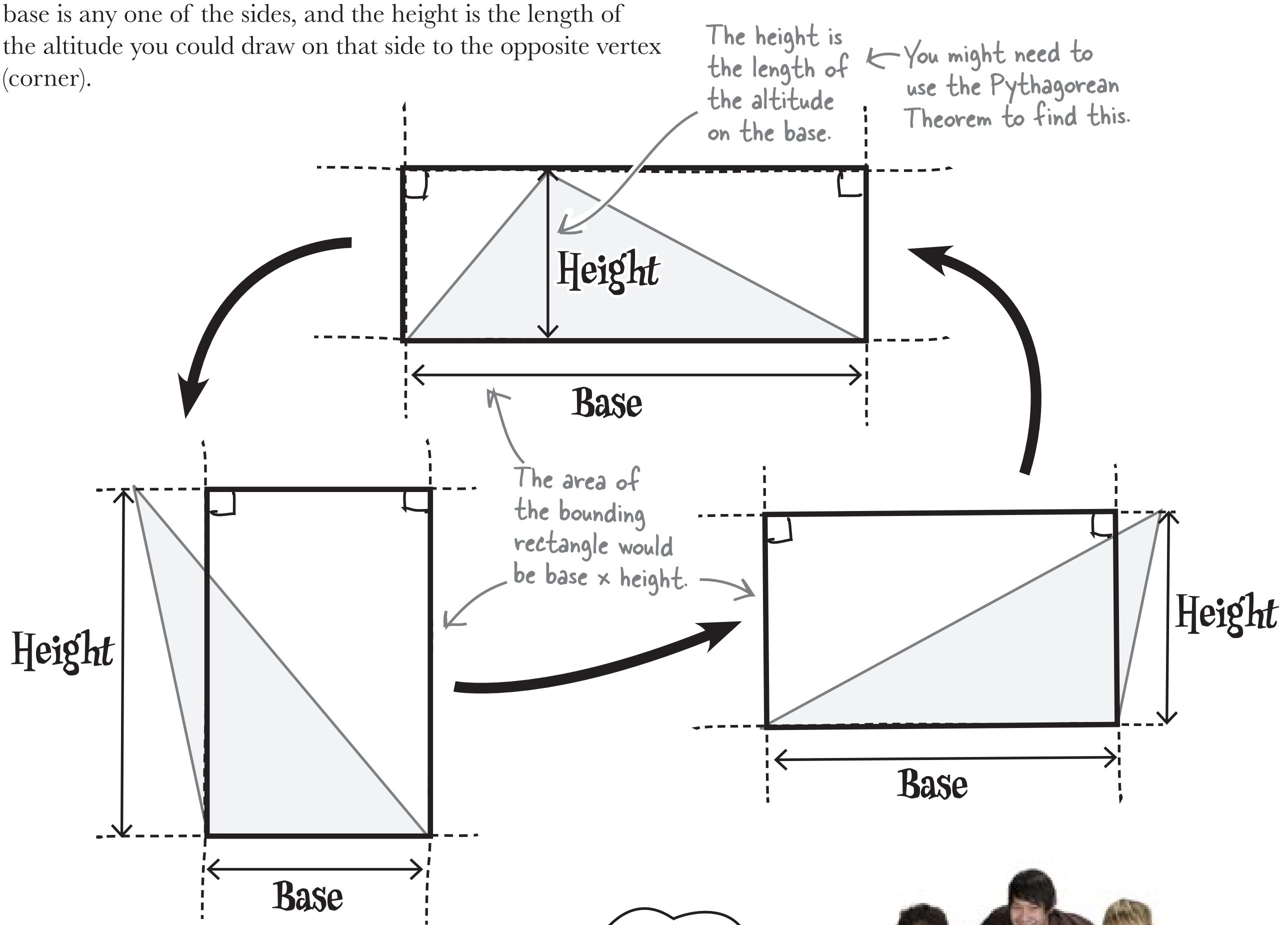
So—even for non-right triangles:

AREA = 1/2 THE AREA OF THE BOUNDING RECTANGLE.

This is the key thing. If you thought this—even if you didn't write it down—then you're golden.

The area of a triangle = 1/2 base x height

The area of a triangle is half the area of the bounding rectangle you could draw around it. It's usually written and found using the formula "half of base times height." The base is any one of the sides, and the height is the length of the altitude you could draw on that side to the opposite vertex (corner).



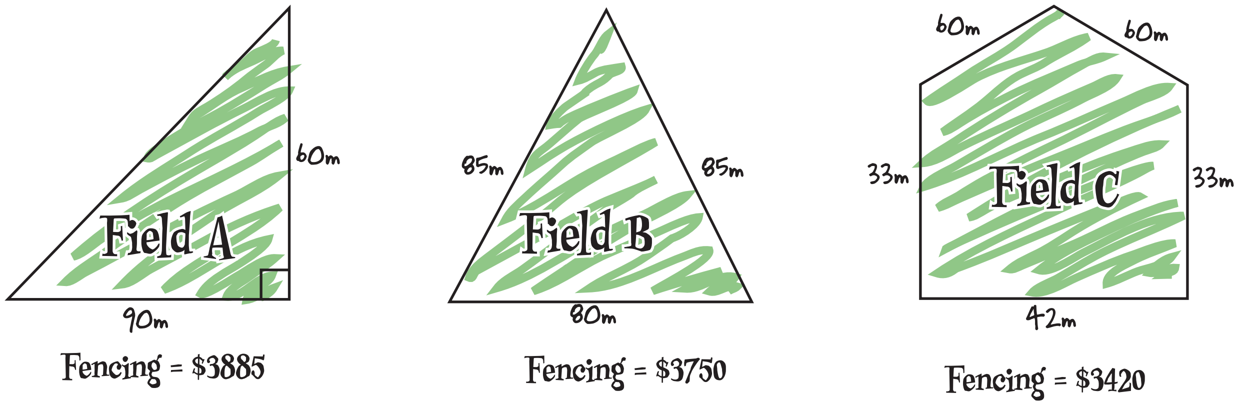


Long Exercise

Which venue gives you the most money to spend on the festival (and split the profits)?

Don't forget that you've got fencing costs. Assume it's a sellout and that each person needs 2 square meters of area.





So—which venue do you choose?

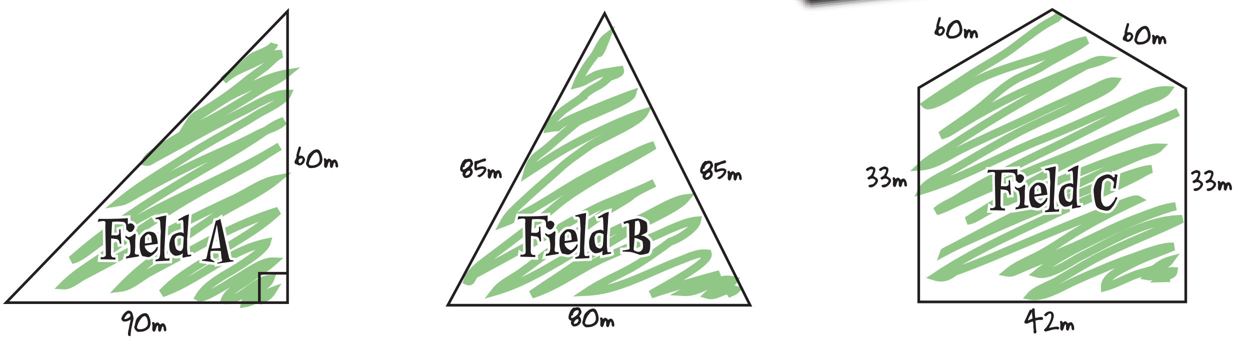


Long Exercise Solution

Which venue gives you the most money to spend on the festival (and split the profits!)?

Don't forget that you've got fencing costs. Assume it's a sellout and that each person needs 2 square meters of area.





Fencing = $3885

Fencing = $3750

Fencing = $3420

(A) Area = 1/2 × 90 × 60 ← Area of triangle = 1/2 base × height

= 2700 sq meters

People = $\frac{2700}{2}$ = 1350 ← 2 square meters per person

Ticket money = 1350 × 10 = $13500 ← $10 per ticket

Cost of fencing = $3885 ← Worked out earlier

Remaining money = 13500 – 3885 = $9615

(B) First we need to find the height, using the Pythagorean Theorem:

$H^2 = 85^2 - 40^2$

$H = \sqrt{7225 - 1600} = \sqrt{5625} = 75m$

Area = 1/2 x 80 x 75

= 3000 sq meters

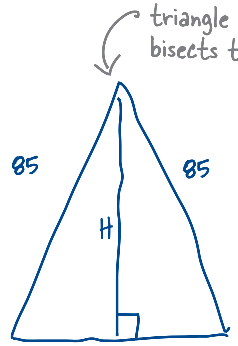
People = $\frac{3000}{2}$ = 1500

Ticket money = 1500 x 10 = $15000

Cost of fencing = $3750

Remaining money = 15000 - 3750 = $11250

For an isosceles triangle the altitude bisects the base.



(C) This shape is like a rectangle with a triangle on the top.

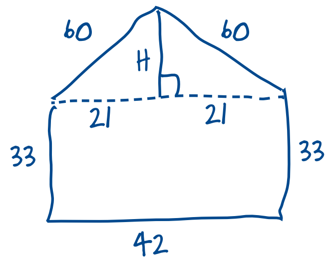
Area of the rectangle bit = 42 x 33 = 1386 sq meters

For the triangle we need to find the height—using the Pythagorean Theorem:

$H^2 = 60^2 - 21^2$

$H = \sqrt{3600 - 441} = \sqrt{3159} = 56.2m$

Triangle Area = 1/2 x 42 x 56.2 = 1180 sq meters



Total Area = rectangle area + triangle area = 1386 + 1180 = 2566 sq meters

People = $\frac{2566}{2}$ = 1283

Ticket money = 1283 x 10 = $12830

Cost of fencing = $3420

Remaining money = 12830 - 3420 = $9410

So—which venue do you choose?

Field B is the best!

You've got $11,250 to spend

After working out the capacity (number of people you can sell tickets to) and the fencing costs for each potential venue, it turns out that Field B is the best—it gives you $11,250 to spend on the festival.

Field A

Ticket money = 1350 x 10 = $13500
Cost of fencing = $3885
Remaining money = 13500 - 3885 = $9615

This field gives you the most bucks.

Field B

Ticket money = 1500 x 10 = $15000
Cost of fencing = $3750
Remaining money = 15000 - 3750 = $11250

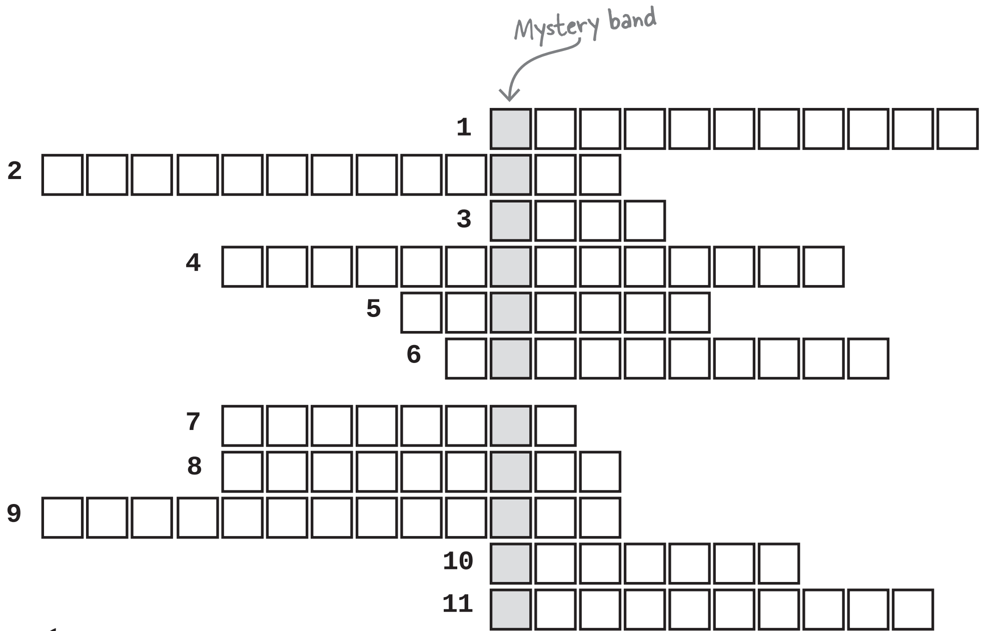
Field C

Ticket money = 1283 x 10 = $12830
Cost of fencing = $3420
Remaining money = 12830 - 3420 = $9410



TriangleGrid

Stop the press! Your buddies have secretly lined up a headline act for the festival. Solve the grid puzzle to be the first to know who the mystery band is.



Clues

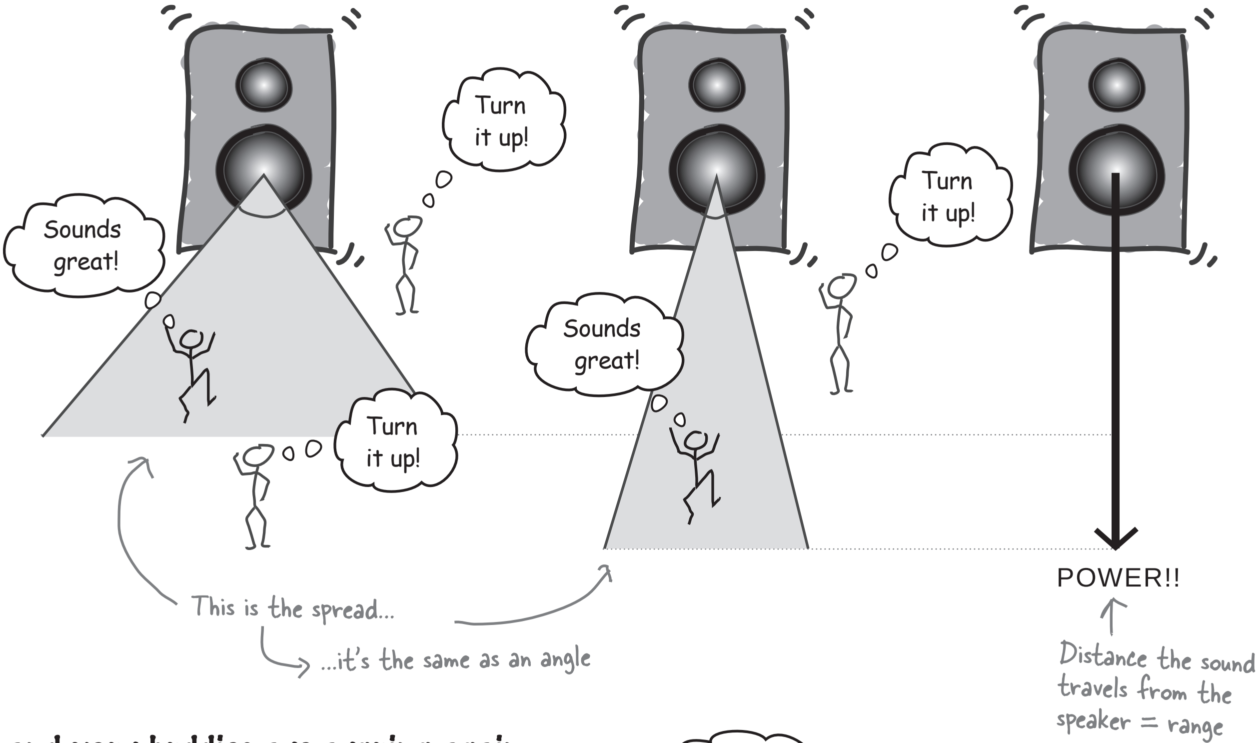
1. You use this theorem to find missing sides of right triangles.
2. Two angles adding up to 90º are known as this.
3. The month before August.
4. When lines cross, they create equal and opposite pairs of these.
5. If shapes have equal angles but are different sizes they are this.
6. To find common factors you can use one of these.
7. The height of a triangle is the length of this.
8. If triangles are similar and the same size then they are this.
9. Two angles adding up to 180º are know as this.
10. 3-4-5 and 5-12-13 are examples of Pythagorean ....
11. The longest side of a right triangle is called this.

Next up—picking the perfect speakers!



All speakers are not created equal

If you thought speakers only came in two sizes—loud and very loud—think again. The local audio store has speakers which vary by power (also called *range*)—how far the sound will travel—and by angle (also called *spread*)—how widely the sound will travel.



...and your buddies are arguing again



So what *are* you looking for in your speakers?

The audio store offers six different models of speakers, categorized by angle and range—how widely and how far the music will travel.

Power / ranges

Spread Angle	60m	100m	200m
60º	$1500	$2000	$3000
90º	$2500	$3000	$4000

Sharpen your pencil

Think about the impact that your choice of speaker has on the festival. Fill in the table to work out what you need from your speakers. (You might not fill in every box.)

If the speakers are...	Pros (good things)	Cons (bad things)
Wider than the venue		
Narrower than the venue		
Range shorter than venue		
Range longer than venue		

Sharpen your pencil Solution

Think about the impact that your choice of speaker has on the festival. Fill in the table to work out what you need from your speakers. (You might not fill in every box.)

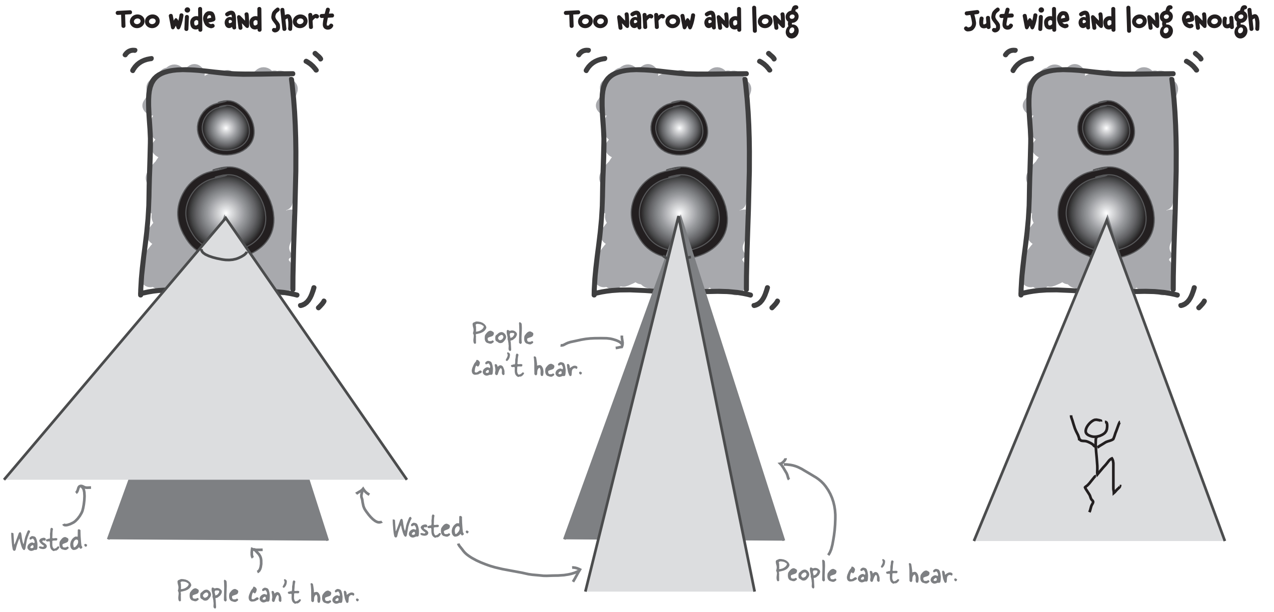
If the speakers are...	Pros (good things)	Cons (bad things)
Wider than the venue	People at sides can hear	Wasted money, more expense, noise pollution
Narrower than the venue		People at edges can't hear
Range shorter than venue		People at back can't hear
Range longer than venue	People at back can hear	Wasted money, more expense, noise pollution

The music would travel outside of the festival as well as inside.

Don't worry if your answers are worded differently, as long as you got the general ideas down.

The ideal speakers are wider and longer than the venue...but only by a little

There's no point in paying extra so that people outside the venue can hear the show, but you need to be sure that everybody inside *can* hear.



Sharpen your pencil

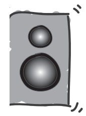
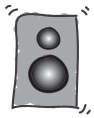
Use what you know about your venue to quickly reduce your speaker options to just two. (Cross out ones you're rejecting.)

Spread Angle	60m	100m	200m
60º	$1500	$2000	$3000
90º	$2500	$3000	$4000

Sharpen your pencil Solution

Use what you know about your venue to quickly reduce your speaker options to just two. (Cross out ones you're rejecting.)

The venue length is 75m.

Spread Angle	60m	100m	200m
 60º	~~$1500~~	$2000	~~$3000~~
 90º	~~$2500~~	$3000	~~$4000~~

Both these speakers are going to reach the back.

there are no Dumb Questions

Q: Is this for real? Are speakers really sold by angles?

A: Yup. Common sizes are 60º, 90º, and 120º. Bass speakers tend to be broad, and tweeters (for higher frequencies) are narrower, but all vary. Apparently for perfect sound quality you need to sit so that you and your two stereo speakers (left and right) make a perfect equilateral triangle. But that's only for really hardcore music geeks.

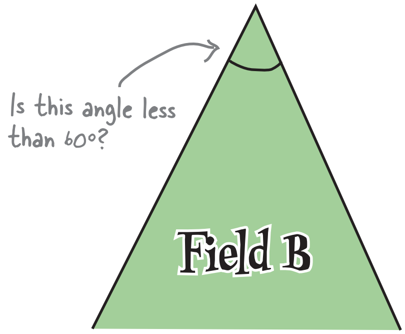
Q: And don't we need more than one speaker?

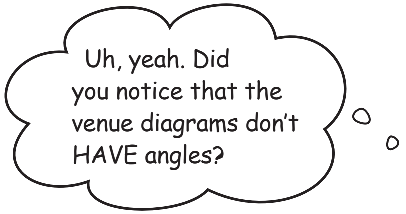
A: What you're really ordering is a speaker system, which would probably have more than one unit.

100m will do, but can you rent the 60° speaker?

The 60° speakers are way cheaper but are they going to do the job? Will everybody in the festival be able to hear or will people at the sides be straining to hear?

That all depends on the angle of your venue. The speakers project sound in the shape of an isosceles triangle. Your venue is also an isosceles triangle. If the venue angle is less than 60°, then it's all good. Because 60° are the narrowest speakers available, it doesn't really matter what that angle is, as long as it's less than 60°. If it's more than 60°, then you'll need to find an extra $1,000 for the 90° speakers.







That's true.

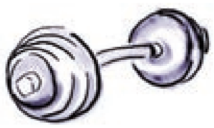
The only thing you know about the venue is the side lengths and height (you worked out the perimeter and area earlier, too, but they don't tell you the angles either). Is that enough to help you work out if the 60° speaker will do?



BRAIN BARBELL

Check any you think are true and that will help you find the angle you need (it can be more than one).

- [ ] Sides and angles change together.
- [ ] From sides you can find angles accurately.
- [ ] From sides you can sometimes find angles accurately, and sometimes find them roughly.
- [ ] Sides and angles change separately.
- [ ] Sides tell you nothing about angles.



Brain Barbell Solution

Check any you think are true and that will help you find the angle you need (it can be more than one).

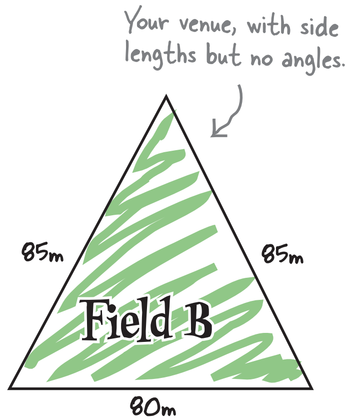
- [x] Sides and angles change together.
- [ ] From sides you can find angles accurately.
- [x] From sides you can sometimes find angles accurately, and sometimes find them roughly.
- [ ] Sides and angles change separately.
- [ ] Sides tell you nothing about angles.

Sides and height tell you a lot about angles

Imagine a triangle with three sides equal length. Do you know what the angles of that triangle would be?

The answer is upside down at the bottom of this page, but you've probably already worked it out.…





It's an equilateral triangle so the angles are each 60°



Geometry Detective

Your mission is to complete the table at the bottom of this page, by using your own hands to investigate how we can use side length and height to estimate angles.

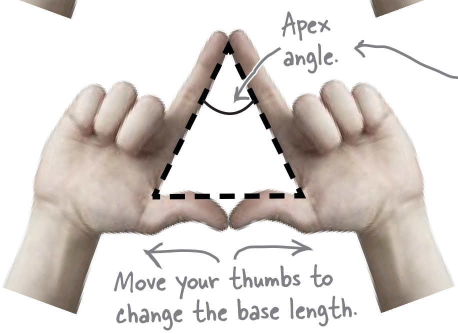


Using your forefingers and thumbs, make a triangle that is roughly equilateral.

You could cut up straws or paper strips into equal lengths and try this with those, too.

Now change the length of the base of the triangle, by moving your thumbs apart or together. Keep your fingers the same length so that you know the triangle is isosceles.

Pay attention to how changing the proportions of the triangle—the relative length of the base and sides, and the height—changes the apex angle.



Complete the table by filling in each blank using one of the following terms:

Greater than | Less than

Equal to | Nearly equal to

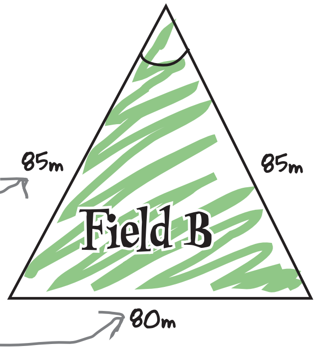
If apex angle is	Sides and base	Height and base
Almost 0º	Side .... base	Height .... base
Less than 60º	Side .... base	Height ....1/2 base
Exactly 60º	Side .... base	Height ....1/2 base
Between 60º and 90º	Side .... base	Height ....1/2 base
Exactly 90º	Side .... base	Height ....1/2 base
More than 90º	Side .... base	Height ....1/2 base
Almost 180º	Side .... 1/2 base	Height 0



Based on your table, do you think the 60º speaker will be OK?

The 60° speakers are spot on

We know that the sides of the field are longer than the base. While this doesn't allow us to say exactly what the apex angle is—23° or 57° or something else—it does tell us that the apex angle must be ***less than 60°***. That means the 60° speakers are ideal.



The side is longer than the base...

...so the triangle is in these rows.

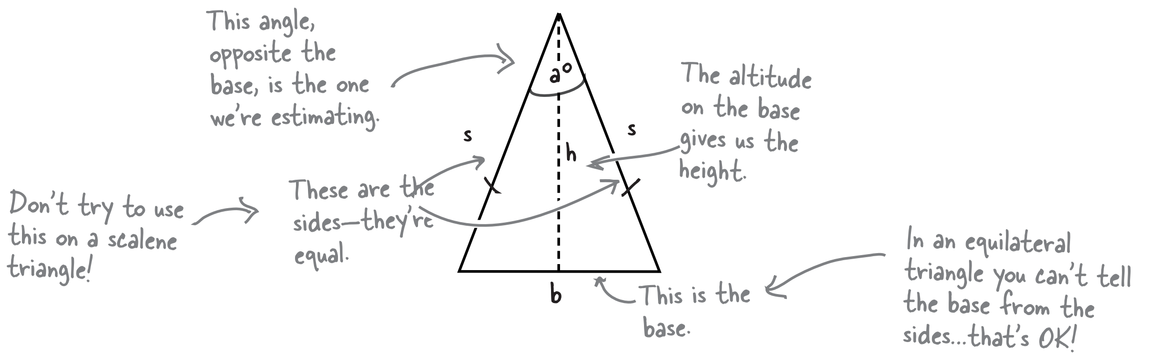
We already know everything we need to, before we even compare height and base, but we found on page 167 that the height was 75m, so that also fits with this row.

If apex angle is	Sides and base	Height and base
Almost 0°	Side Greater than base	Height Greater than base
Less than 60°	**Side Greater than base**	**Height Greater than 1/2 base**
Exactly 60°	Side Equal to base	Height Greater than 1/2 base
Between 60° and 90°	Side Less than base	Height Greater than 1/2 base
Exactly 90°	Side Less than base	Height Equal to 1/2 base
More than 90°	Side Less than base	Height Less than 1/2 base
Almost 180°	Side Nearly equal to 1/2 base	Height Nearly equal to 0

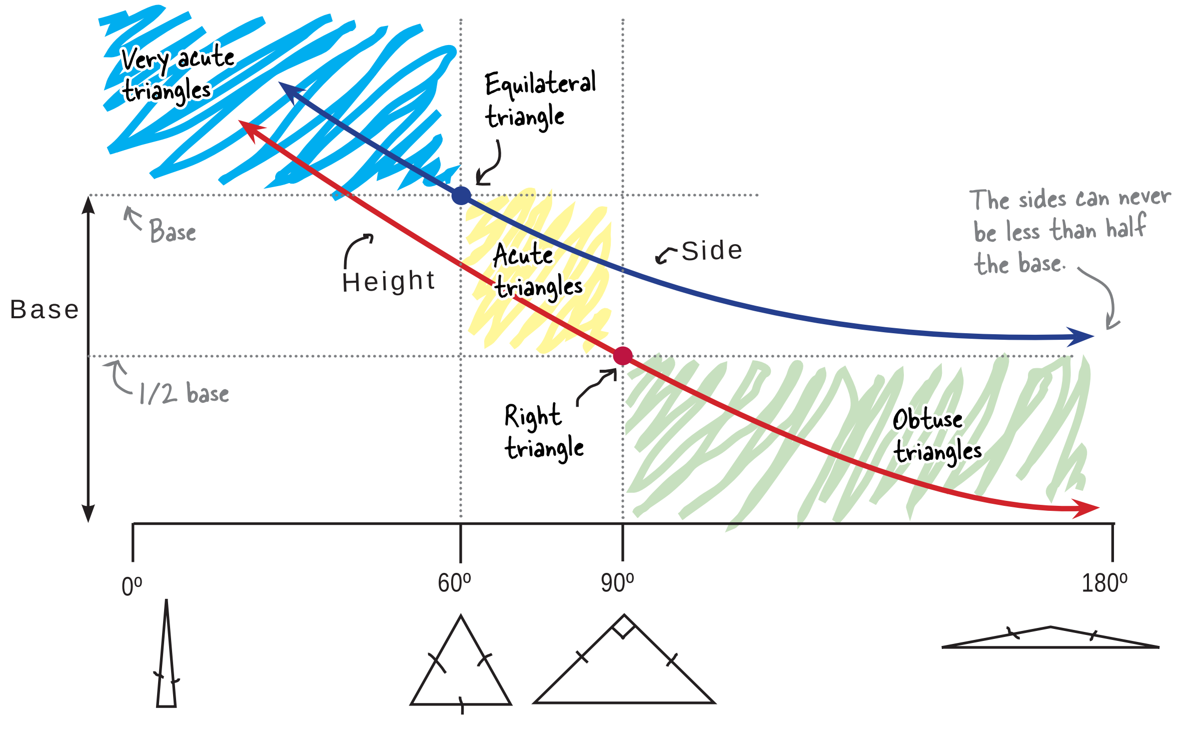
Math geeks call this "approaching"—the value gets real close to the thing it's nearly equal to, but never quite gets there.

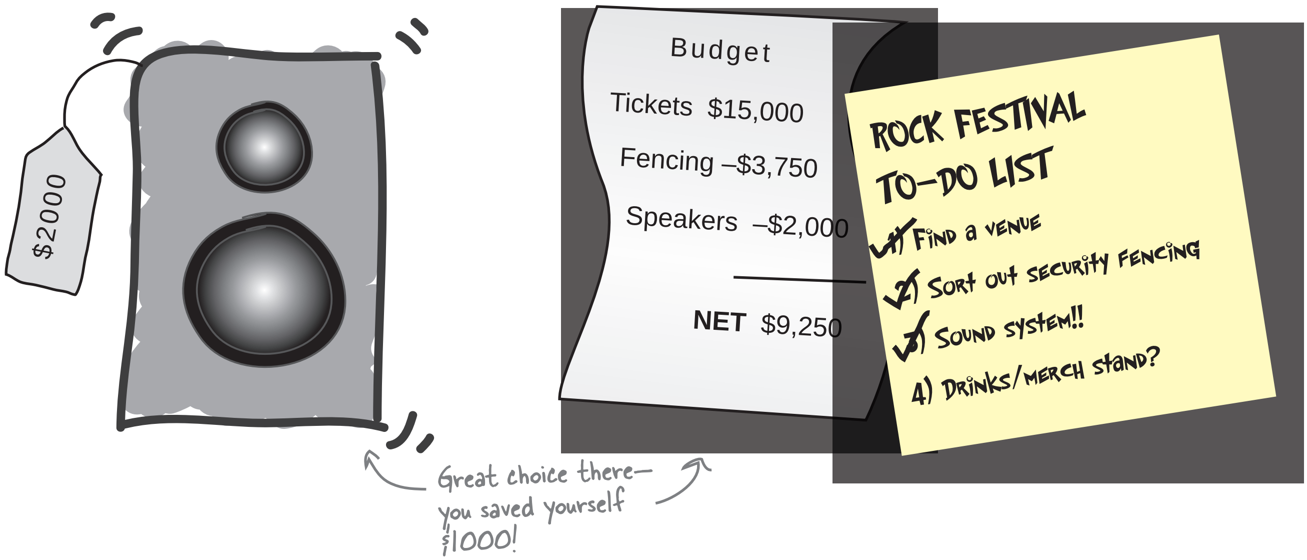
Estimating Angles Up Close

For an isoceles triangle you can use the relative values of the base, the sides, and the height to find out roughly what the apex angle (the one opposite the base) is.



Plot sides and height against base to find which zone your apex angle is in: obtuse, acute, or very acute.





there are no Dumb Questions

Q: Are you sure it's OK to be estimating angles like this? Don't we always need to work out exactly what the angle is?

A: If the problem or question requires a precise answer along the lines of "what is angle x," then you'll need to find the angle exactly, but sometimes it's possible to solve a problem just by knowing roughly what an angle is. For example, is it a right angle? Or maybe just is it acute or obtuse?

Q: And this is isosceles only, yeah?

A: Yes, this technique only works reliably for an isoceles triangle—where two angles and two sides are the same. An equilateral triangle is an isoceles triangle with an extra matching side, so it works for those, too, but then they're pretty easy to spot.

Q: What if I only know the base and the side, or the base and the height?

A: Sometimes that's all you need. Any time the base is greater than twice the height you know you've got an angle more than 90º.You really only need both for that tricky zone between 60º and 90º.

But don't worry—if you've got two out of three for the base, side, and height lengths, you can use the good old Pythagorean Theorem to find the other. (This only works for right and isoceles triangles—don't try it on a scalene triangle or you'll come unglued.)

Q: OK—but how do I remember this? I'm bound to get mixed up about the base and sides and and which is greater and less than which! I'm not a computer you know....

A: It's almost certainly easier to remember the zones on the graph than it is to remember the inequalities.

So, sketch that graph, mark what you know about the 60º and 90º triangles on it, and label the zones.

Q: Really? You think I could draw it just like that?

A: Go on...give it a shot on a scrap of paper now. It's a really time efficient way to check your answers in an exam by the way.

All that's left is to pick a spot for the drinks stall

The guys all agree that the best place to put the drinks and merchandise stall is "in the center" but they're having trouble agreeing on where the center is.…



BRAIN POWER

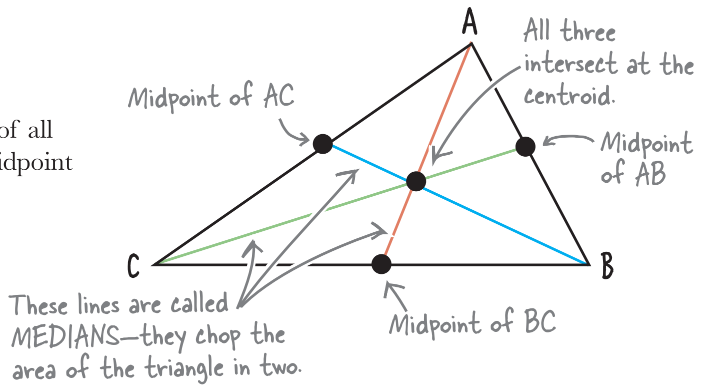
Who's right? Where do you think the center of a triangle is?

A triangle has more than one center

Believe it or not, there are four common ways of finding the "center" of a triangle.

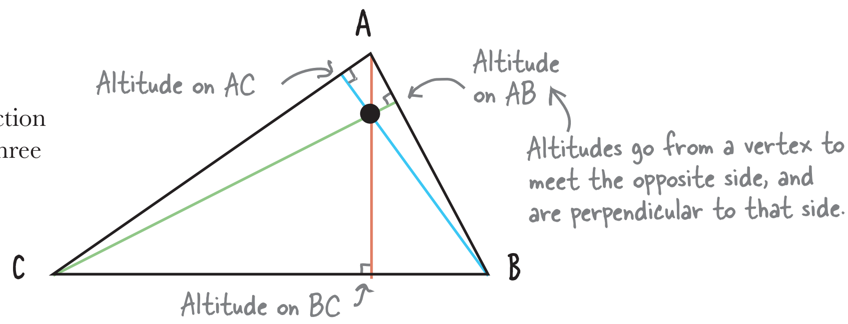
1 CENTROID

The centroid is the intersection of all three medians (lines from the midpoint of a side to the opposite vertex).



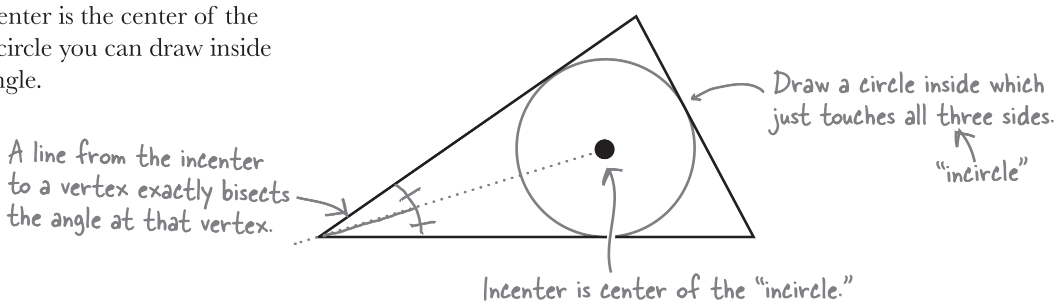
2 ORTHOCENTER

The orthocenter is the intersection of the altitudes drawn on all three sides.



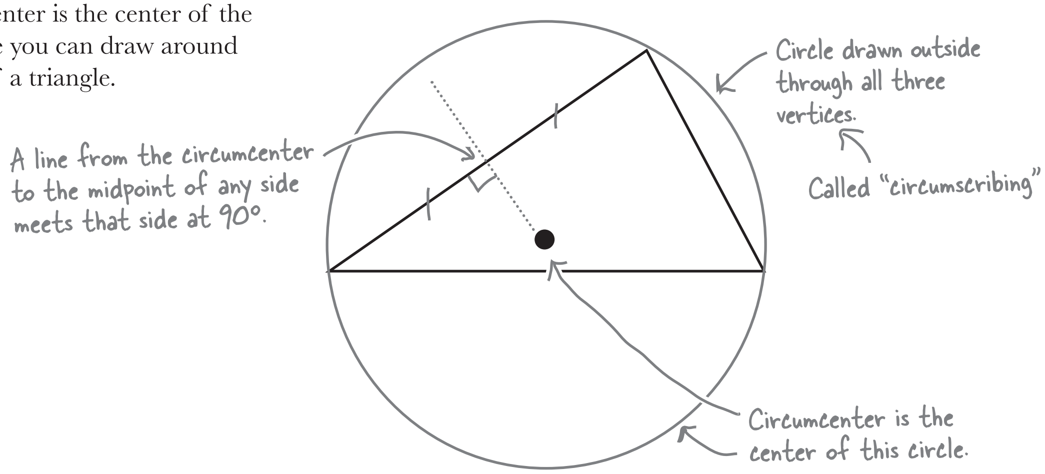
3 INCENTER

The incenter is the center of the biggest circle you can draw inside the triangle.



4 CIRCUMCENTER

The circumcenter is the center of the smallest circle you can draw around the outside of a triangle.



WHO DOES WHAT?

Which kind of center was each of your buddies describing? Pair 'em up! (You'll have one center left over.)

"The point nearest to all three fences"	Centroid
"The point nearest to all three corners"	Incenter
	Orthocenter
"The point where half the crowd is on either side, no matter which way you look"	Circumcenter

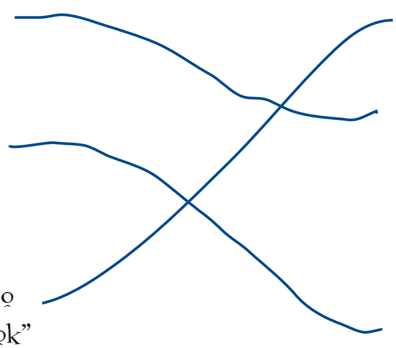
Who Does What? Solution

Which kind of center was each of your buddies describing? Pair 'em up! (You'll have a center left over.)

"The point nearest to all three fences"

"The point nearest to all three corners"

"The point where half the crowd is on either side, no matter which way you look"



Centroid

Incenter

Orthocenter

Circumcenter



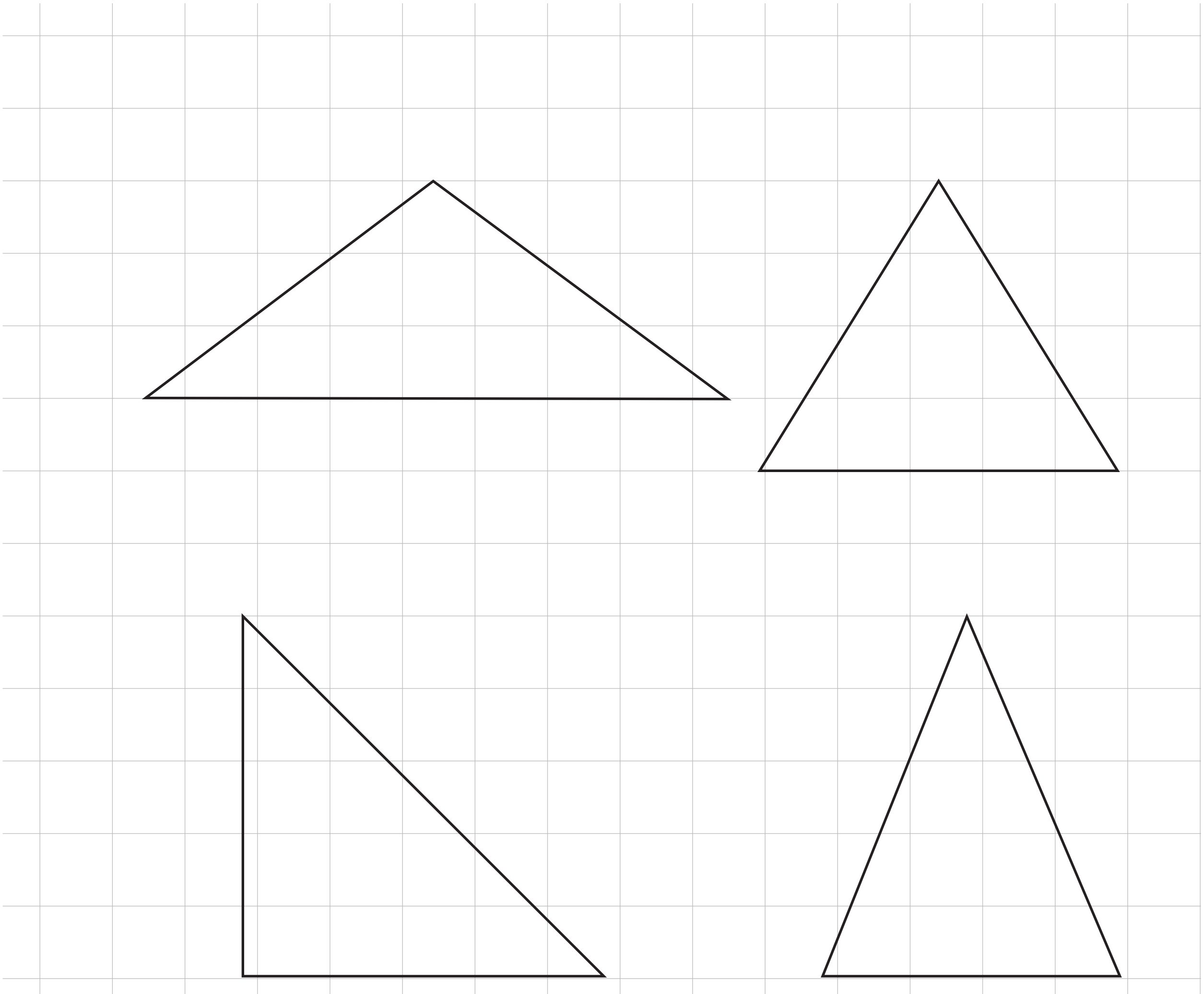
There must be a reason.

Geometry people never do ***anything*** without a reason. So, if there are different ways of defining the center it suggests that there must be times when one center is more appropriate than another, don't you think?

Geometry Detective

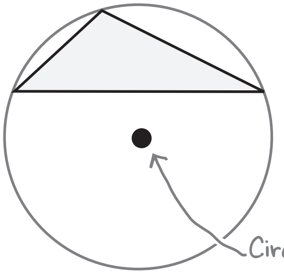
Below are four different triangles. Work out (roughly) where each of the four types of "center" is for each.

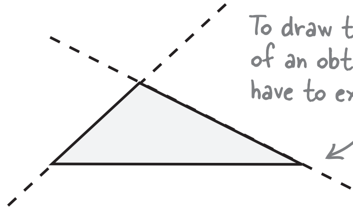
Are there any definitions of "center" that wouldn't always work for placing the drinks stall?

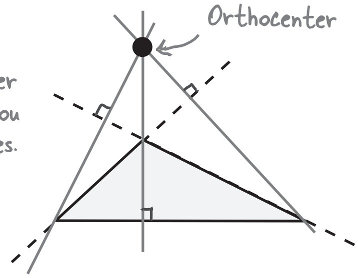


The center of a triangle can be outside the triangle

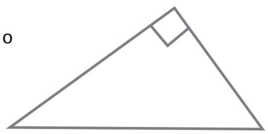
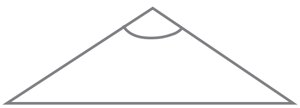
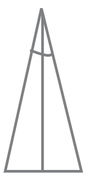
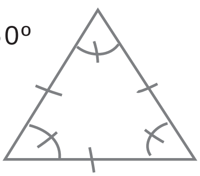
As crazy as it sounds, the circumcenter and orthocenter of a triangle can be outside the triangle. This only happens if the triangle is obtuse (one angle greater than 90º).







Triangle Centers Up Close

		ORTHOCENTER	CIRCUMCENTER
Right 90º		On the 90º vertex	On the midpoint of the hypotenuse
Obtuse		Exterior	Exterior
Acute Isoceles		Interior on altitude in line with other centers	Interior on altitude
Equilateral 60º		Same as all other centers	Same as all other centers

The centroid and incenter are always interior.

Inside the triangle

Let's put the drink stall at the centroid

Outside the venue would be a pretty weird place to put your drinks stall, so it's important to pick a center which is always interior.

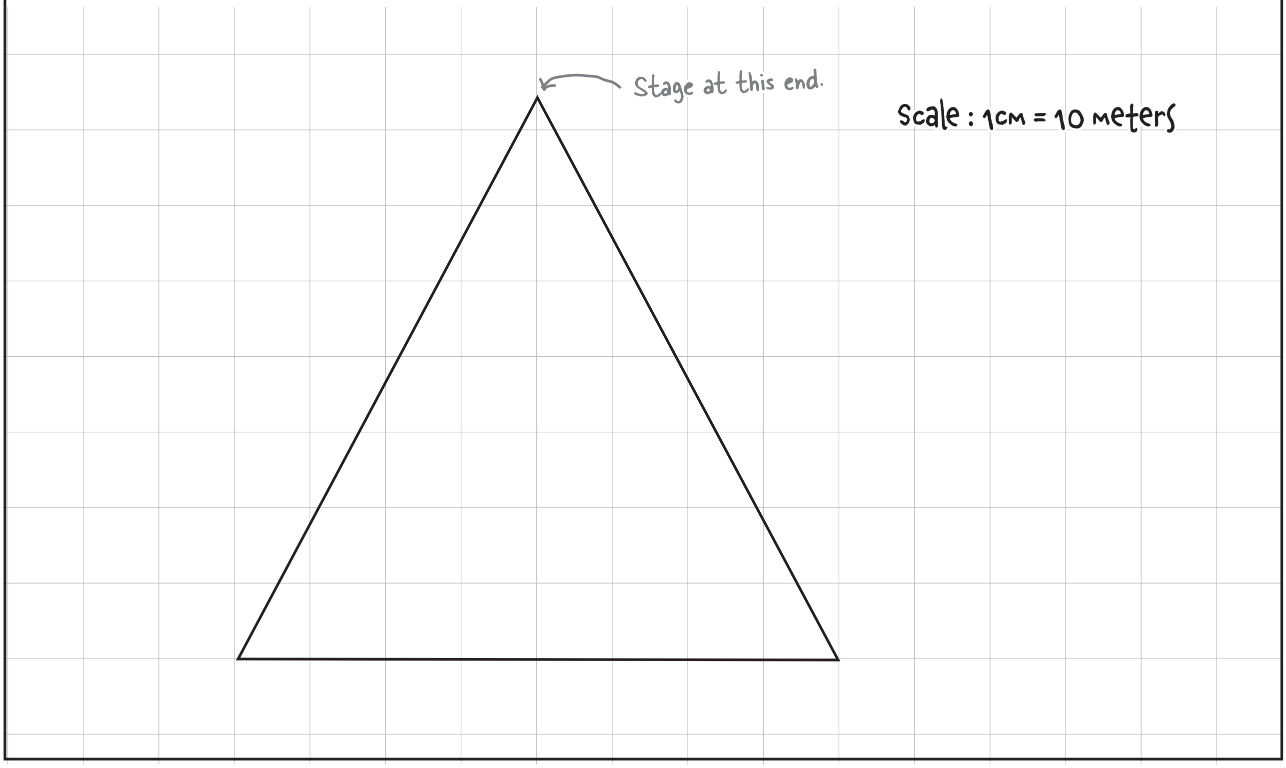
The centroid also never gets pulled particularly close to one side or vertex, whereas the incenter can end up very far from one vertex if the triangle has sides of very different lengths.

An interesting thing about the centroid is that it's where the triangle balances—this means that each median splits the triangle area into two equal halves.

Sharpen your pencil and ruler

Using accurate construction, find the position of the centroid of your field to the nearest meter.

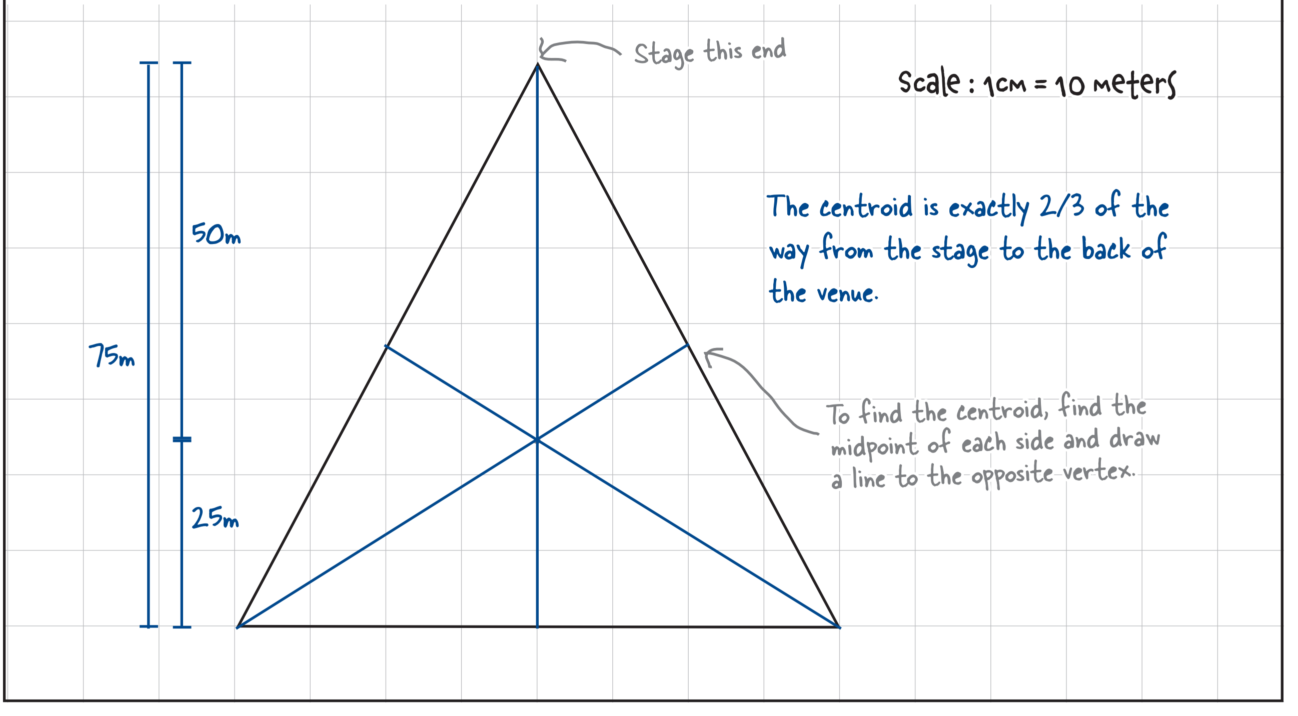
Do you notice anything interesting about the distance from the centroid to the stage?



Sharpen your pencil and ruler solution

Using accurate construction, find the position of the centroid of your field to the nearest meter.

Do you notice anything interesting about the distance from the centroid to the stage?



there are no Dumb Questions

Q: I drew an equilateral triangle and my centers were all in the same place—did I do it wrong?

A: That's perfect! All the four centers of an equilateral triangle are in the same place. It's the only type of triangle that truly has a "middle." Centers of an isoceles triangle all fall on the altitude of the base—the line you might use to find the height.

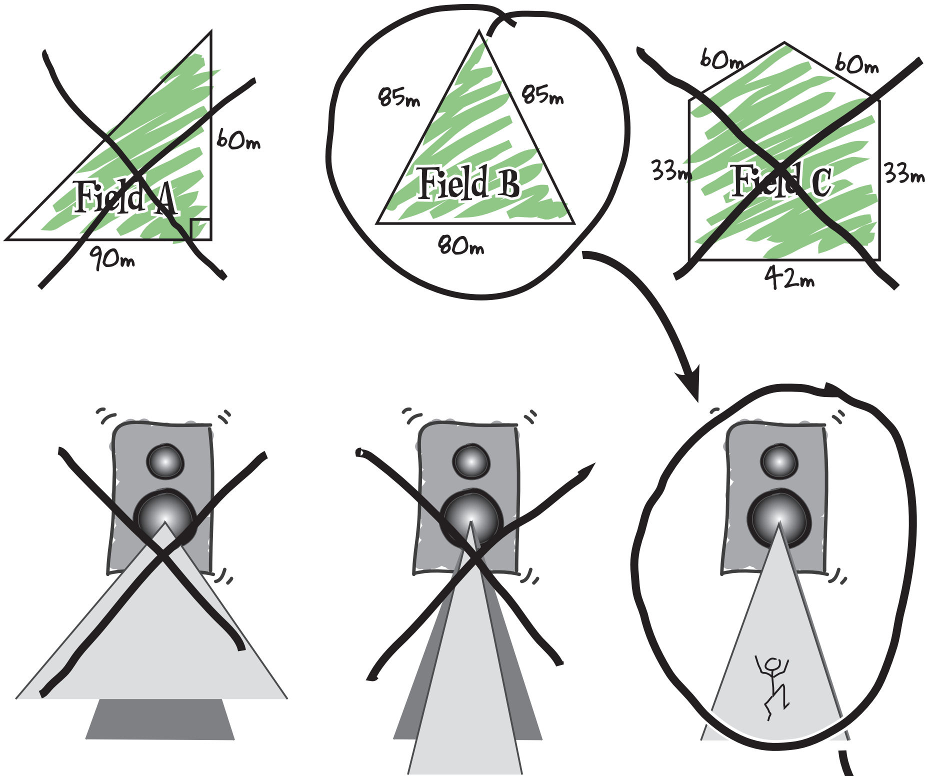
Q: Do I really need to know four different centers—won't one do?

A: There are actually over 3,500! These are the four you'll use the most, though. The centroid is used to balance a triangle in the physical world, but the other three all have uses. Each has different potential to find other things—a line from the incenter to any vertex of a triangle bisects that angle at that vertex, so you can use the incenter to find that angle if you need to.

Q: Why on earth would a center that lies outside the triangle be useful?

A: The orthocenter and circumcenter only lie outside the triangle *sometimes*. If your triangle is acute then you might have need for a center that is exactly the same distance from all three corners, or one where a line to any side meets it at a right angle. It's totally dependent on what triangle you've got and what you're trying to do with it.

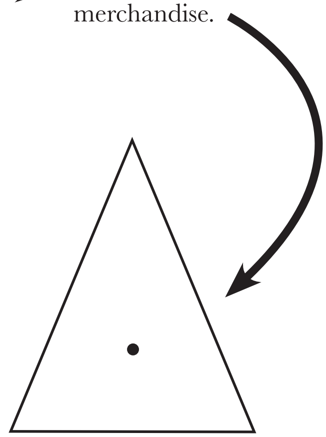
The rock festival is ready!



1. You chose a venue based on area/capacity minus the perimeter fencing costs.

2. You picked the right speakers to rent—saving yourself $1,400!

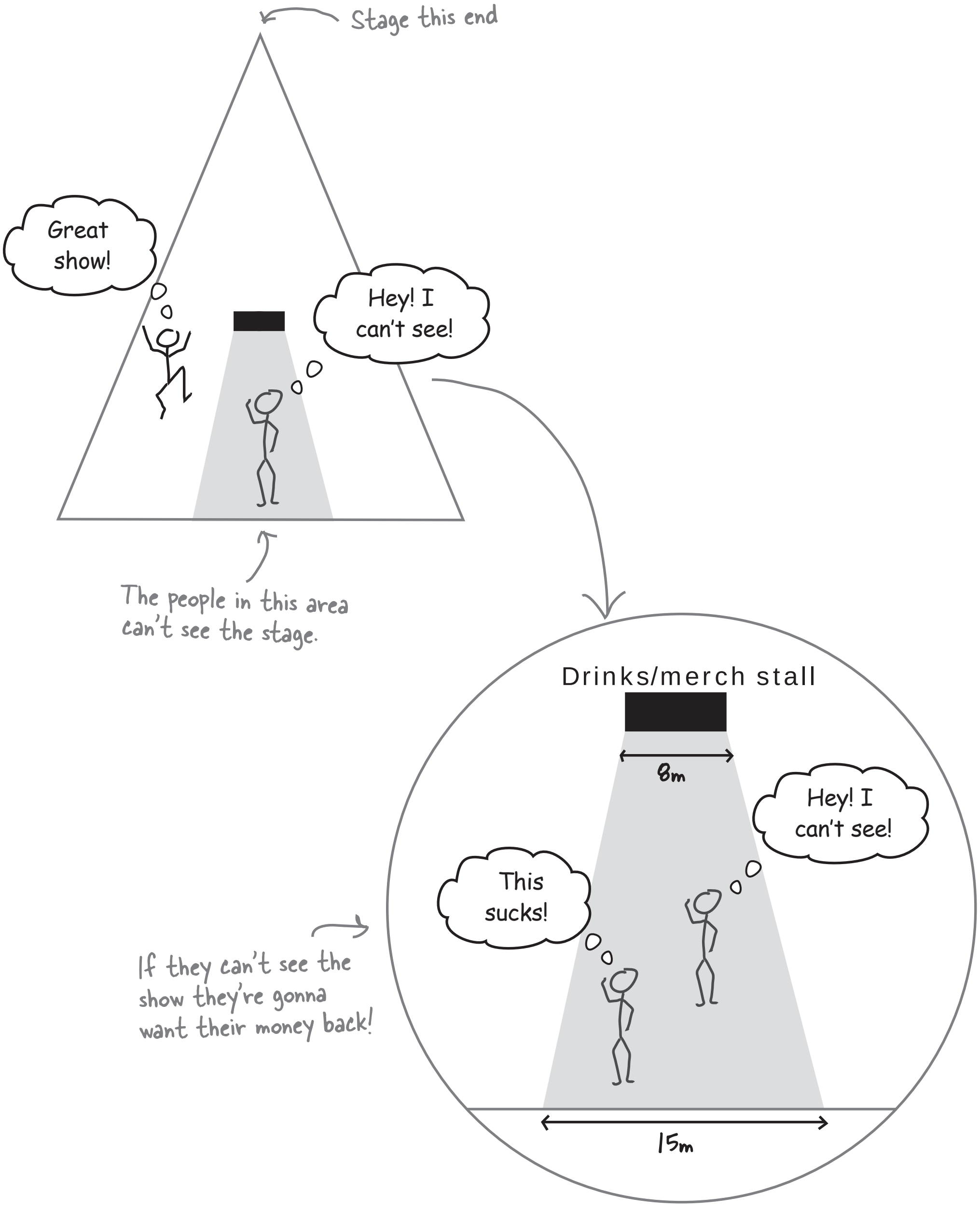
3. You found the centroid of the field—the perfect spot for the drinks and merchandise.

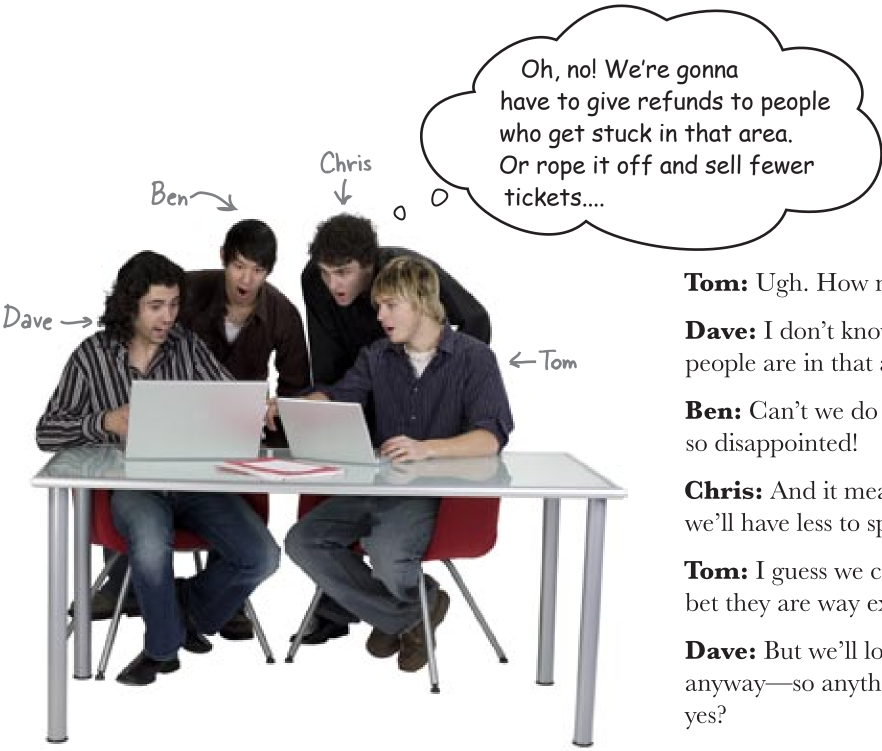




The people behind the drinks stall won't see the stage...

The engineers have started setting up, and now that the drink stand is in place, your buddies realize that it blocks the view for whoever will be standing behind it.





Tom: Ugh. How many refunds?

Dave: I don't know…we'd need to work out how many people are in that area behind the screen.

Ben: Can't we do something else? People are gonna be so disappointed!

Chris: And it means less cash as well—which means we'll have less to split between us all!

Tom: I guess we could hire a giant screen maybe? But I bet they are way expenisve.

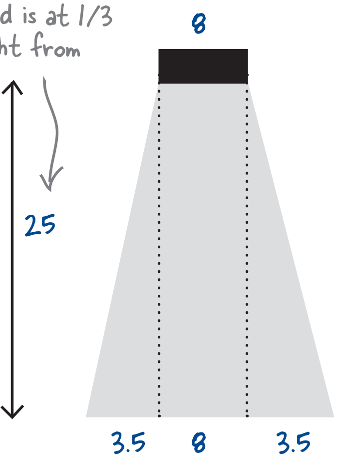
Dave: But we'll lose money if we give refunds anyway—so anything less than that amount is worth it, yes?

Sharpen your pencil

If you had to give refunds to all the people in that area, how much would it cost you? (1 person per 2 square meters and $10 a ticket.)

Sharpen your pencil Solution

The centroid is at 1/3 of the height from the base



If you had to give refunds to all the people in that area, how much would it cost you? (1 person per 2 square meters and $10 a ticket.)

We've got a rectangle and two congruent triangles.

$$\text{Area} = 8 \times 25$$ ← Area of rectangle

$$+ \frac{2 \times 3.5 \times 25}{2}$$ ← Area of triangles, the 2s cancel, which is nice

$$= 8 \times 25 + 3.5 \times 25$$

$$= 200 + 87.5 = 287.5 \text{ sq m}$$

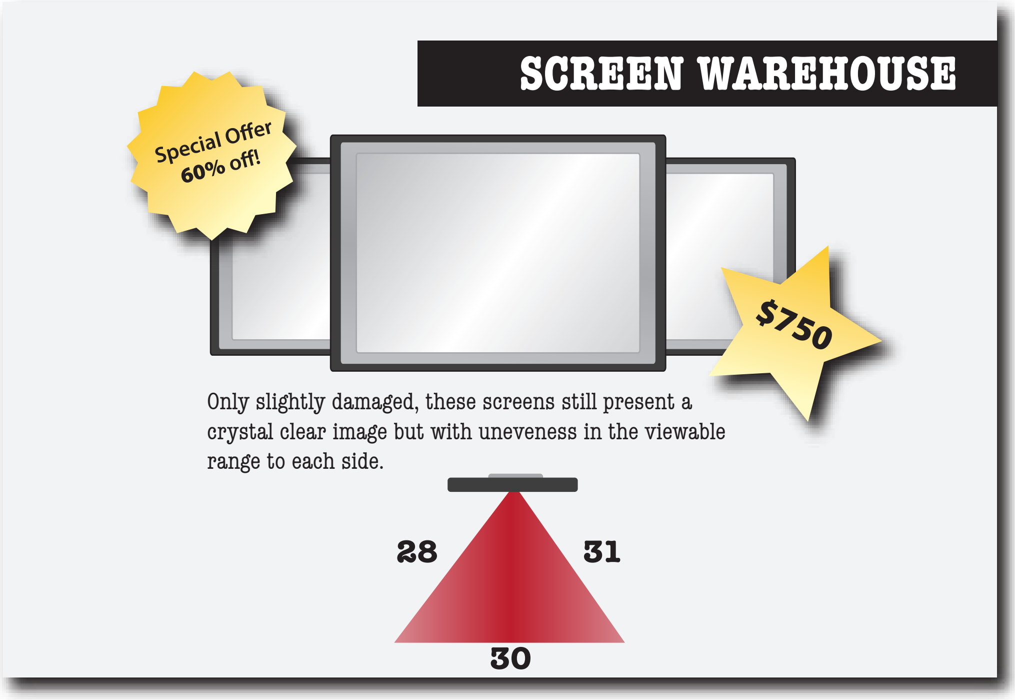
$$= \frac{287.5}{2} = 144 \text{ people} = \$1440$$ ← Round UP to a whole ticket

You need a screen for less than $1,440

The cost of giving refunds to people behind the drinks stall would be $1,440. If you can find a screen to rent for less than that, it would let everybody see the show and you'd still have some profits to split.

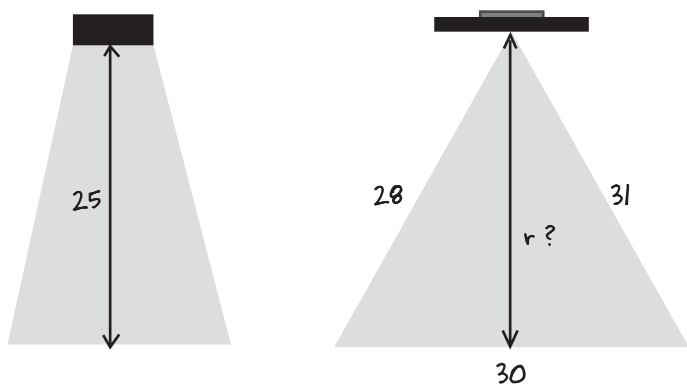


Web ad for the special offer screens



Will the special offer screen still do the job?

The screen is suitable if the viewable range (r) reaches all the way to the back—but you only have the sides given on the specification in the ad.



Sharpen your pencil

What technique from your Geometry Toolbox can help you find the screen's viewable distance, r?

Sharpen your pencil Solution

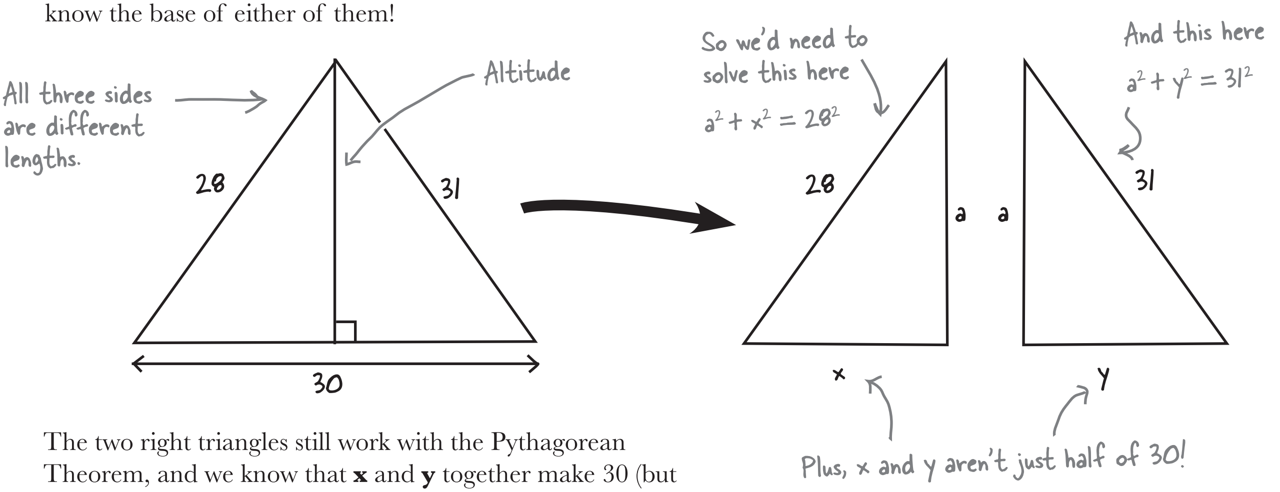
What technique from your Geometry Toolbox can help you find the screen's viewable distance, r?

The Pythagorean Theorem

It's the right answer, but it's also a world of pain...so hold tight for a new tool for your toolbox!

The screen viewing area is a scalene triangle

This means that when you add an altitude, it doesn't bisect the base. Instead of creating two nice, neat ***congruent*** right triangles, you get two ***different*** right triangles. And you don't know the base of either of them!



The two right triangles still work with the Pythagorean Theorem, and we know that **x** and **y** together make 30 (but aren't equal to each other) so we *could* figure this out with a set of three simultaneous algebra equations. If that sounds bad, it kinda is. Don't go there. But in case you're tempted, here's how it starts out:

DANGER
do not cross this line

3 equations

$a^2 + x^2 = 28^2$

$a^2 + y^2 = 31^2$

$x + y = 30$

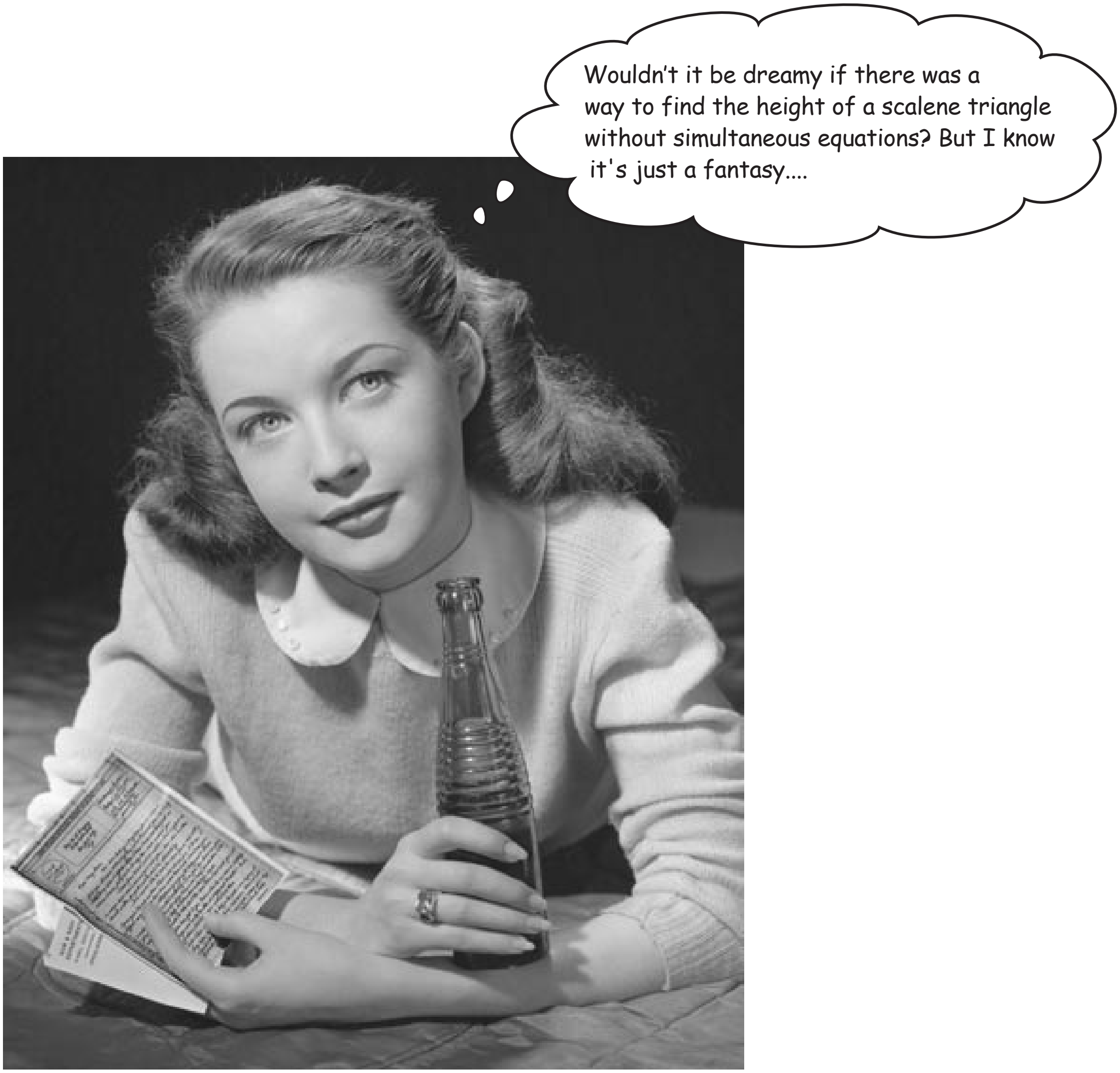
by subtraction

$(a^2 + x^2 = 28^2)$
$-\ (a^2 + y^2 = 31^2)$ → $x^2 - y^2 = 28^2 - 31^2$

$x + y = 30$, so $x = (30 - y)$ → $(30 - y)^2 - y^2 = 28^2 - 31^2$

by substitution

Now...expand this out....


Wouldn't it be dreamy if there was a way to find the height of a scalene triangle without simultaneous equations? But I know it's just a fantasy....

You can find area from sides using Hero's formula

Fortunately there is a formula known as Hero's formula, or sometimes Heron's formula, which lets you find the area of a triangle when you only know the sides. First you find the semi-perimeter—half the perimeter—and then you just pop the numbers into your equation and wham, there's your area. Phew!

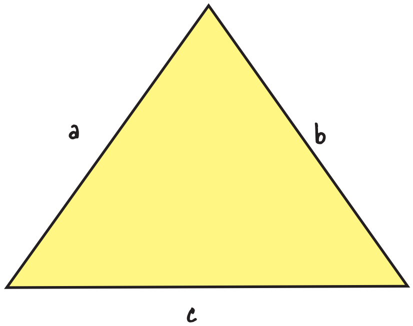
Aptly named, saving you from three variable simultaneous equations!

Hero's formula:

$$\text{Area} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

$$s = \frac{a+b+c}{2}$$

Semiperimeter



1 Find the semiperimeter

The semiperimeter is exactly what it sounds like.

Just add the three side lengths together to get the perimeter, and then divide it by 2.

$$s = \frac{a+b+c}{2}$$

Do this bit first...

...then pop your answer in here, replacing "s."

2 Use the main formula

The main formula has four *s*'s in it. You use the value you got for the semiperimeter for each *s*. Don't forget to get the square root of the whole thing when you're done.

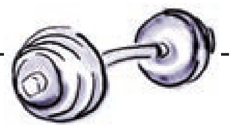
$$\text{Area} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$



Actually, it could be *exactly* what we need!

One of the neat things about geometry is the way ***everything links up***. Like back in Chapter 1, when you proved Benny was innocent because you could find the angle two different ways, and they didn't match up.

Hero's formula doesn't exist in a vacuum—it fits into all the tools you already have in your Geometry Toolbox. And that's why it's a **big** help in finding not just area, but height as well....



BRAIN BARBELL

How could Hero's formula for triangle area also make it easy for you to find the triangle's height?



Brain Barbell Solution

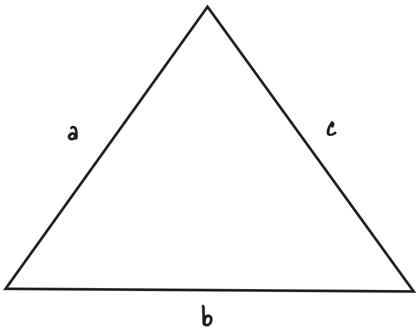
How does Hero's formula for triangle area also make it easy for you to find the triangle's height?

Using the other area formula, area – 1/2 base x height.

So, you can use Hero to find area and then divide by half the base to get height.

Hero's formula and "1/2 base x height" work together

If you know three sides of a scalene triangle, you can use Hero's formula to find the area, and then use the formula you already know to find the height.



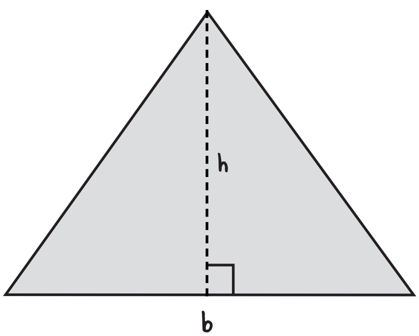
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

Semiperimeter

Take your area value from here...

...and put it into the other area formula to get the height.



$$\text{Area} = 1/2\, b \times h$$

Rearrange

$$h = \frac{2 \times \text{area}}{b}$$

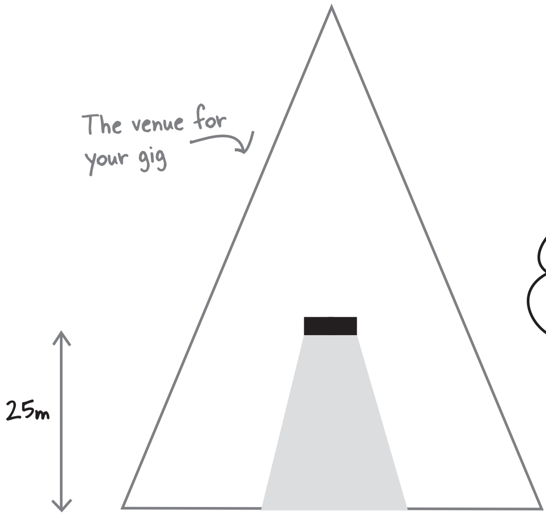
there are no Dumb Questions

Q: So do I actually ever need to do that gnarly three simultaneous equations thing?

A: No. It would work though, so if you ever can't remember Hero's formula, it could get you out of a jam! But yeah—forget it. Sorry we did that to you....

Q: Can I use Hero's formula for isoceles and right triangles, too?

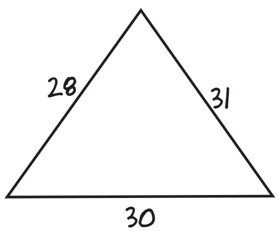
A: For a right triangle, 1/2 base x height is always easier as the two sides on the right angle give you base and height. For an isoceles triangle it's up to you which you find easier.





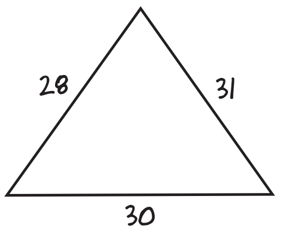
Sharpen your pencil

Will the bargain screen still be visible to the people at the back, 25 meters away?



Sharpen your pencil Solution

Will the bargain screen still be visible to the people at the back, 25 meters away?



Hero's formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$ ← Do this bit first then use it in the area formula.

$$s = (31 + 30 + 28) / 2 = 44.5$$

$$\text{Area} = \sqrt{44.5\,(44.5-31)\,(44.5-30)\,(44.5-28)}$$

$$= \sqrt{44.5 \times 13.5 \times 14.5 \times 16.5}$$

$$= \sqrt{143729} \quad = \quad 379.1 \text{ sq meters}$$

Using conventional, 1/2 base × height, Height = 2 × area/base

$$\text{Height} = 2 \times 379.1 / 30 = 25.27 \text{ m}$$ Perfect !

BULLET POINTS

- Combine tools from your toolbox to get the answer you need.
- There's sometimes more than one way to solve a problem—pick the way that seems like the least amount of work!
- Use the relationship between sides and height to estimate angles.
- Draw sketches or graphs, or use your hands if you're stuck remembering what those side-height-base relationships are.

The rock festival is gonna...rock!

Great venue? Check!

Fencing? Check!

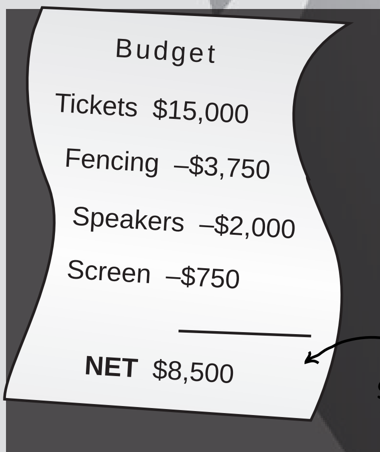
Awesome sound? Check!

Giant video screen? Check!

Sold out? Check!

Way to go! Now—go hang out backstage with the headline act—**Pajama Death**. They're legendary!





Still plenty left for lights, fireworks, backstage parties—excellent!





Pajama Death are headlining. Wicked!



Your Geometry Toolbox

You've got Chapter 4 under your belt and now you've added properties of triangles to your toolbox. For a complete list of tool tips in the book, head to www.headfirstlabs.com/geometry.

Area =

1/2 base × height

The height comes from the altitude of the triangle—it's perpendicular to the base.

Hero's formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

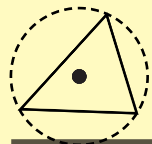
Find the semiperimeter first, then use it to find area.

Perimeter

Add up all the sides of your shape to find the perimeter.

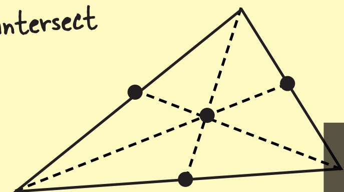
Circumcenter

The center of a circle drawn outside the triangle, through every vertex



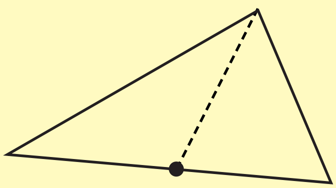
Centroid

The point where all three medians of the triangle intersect



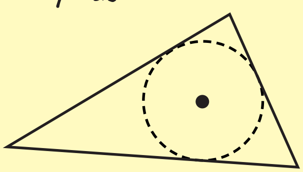
Median

A line from the midpoint of a side to the opposite vertex



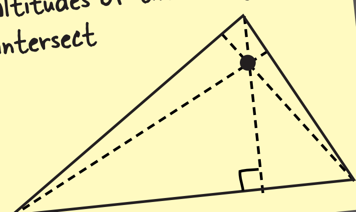
Incenter

The center of a circle drawn inside the triangle, touching every side



Orthocenter

The point where all three altitudes of the triangle intersect



TriangleGrid Solution

Stop the press! Your buddies have secretly lined up a headline act for the festival. Solve the grid puzzle to be the first to know who the mystery band are.

Mystery band

1	PYTHAGOREAN
2	COMPLEMENTARY
3	JULY
4	VERTICALANGLES
5	SIMILAR
6	FACTORTREE
7	ALTITUDE
8	CONGRUENT
9	SUPPLEMENTARY
10	TRIPLES
11	HYPOTENUSE

Clues

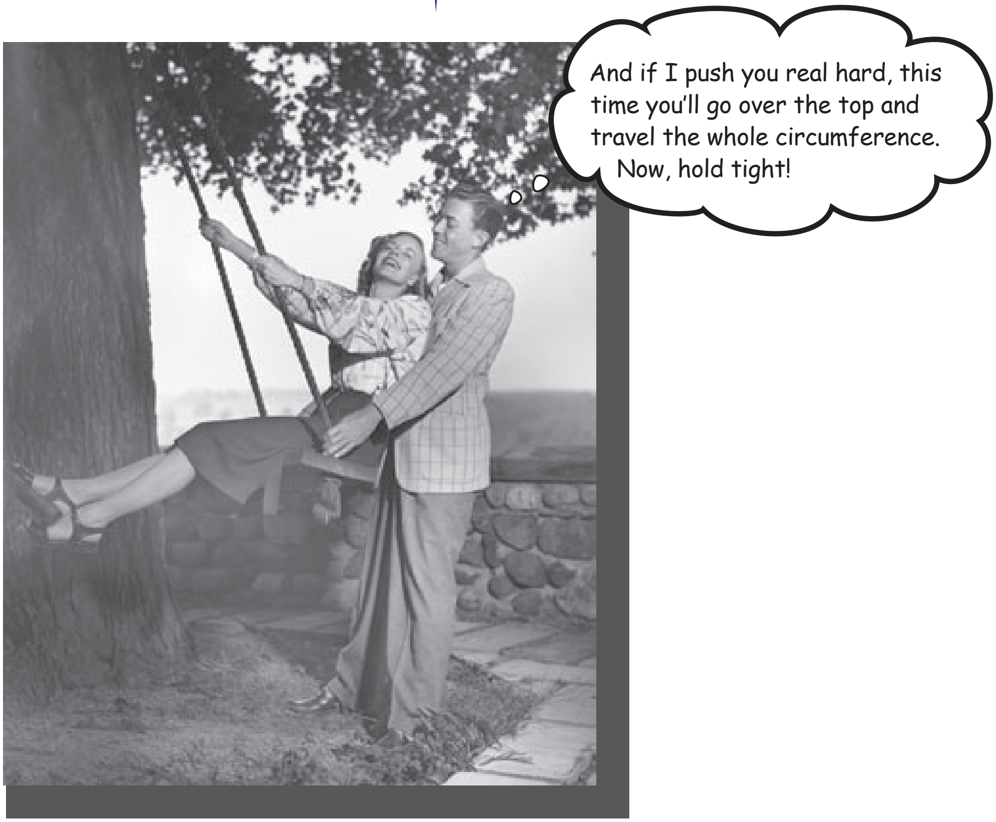
1. You use this theorem to find missing sides of right triangles.
2. Two angles adding up to 90º are known as this.
3. The month before August.
4. When lines cross, they create equal and opposite pairs of these.
5. If shapes have equal angles but are different sizes they are this.
6. To find common factors you can use one of these.
7. The height of a triangle is the length of this.
8. If triangles are similar and the same size then they are this.
9. Two angles adding up to 180º are know as this.
10. 3-4-5 and 5-12-13 are examples of Pythagorean ....
11. The longest side of a right triangle is called this.



Are you a Pajama Death fan? Help the band get their own TV show in Head First Algebra.

5 circles

Going round and round



OK, life doesn't have to be so straight after all!

There's no need to reinvent the wheel, but aren't you glad you're able to use it? From cars to rollercoasters, many of the **most important solutions** to life's problems rely on ***circles*** to get the job done. Free yourself from straight edges and pointy corners—there's no end to the curvy possibilities once you master **circumference**, **arcs**, and **sectors**.

It's not just pizza—it's war!

Everybody knows it—***you want pizza? Go to Mario's!*** But times are changing. The MegaSlice chain is muscling in on Mario's turf, and to make matters worse it looks like they're fighting dirty!

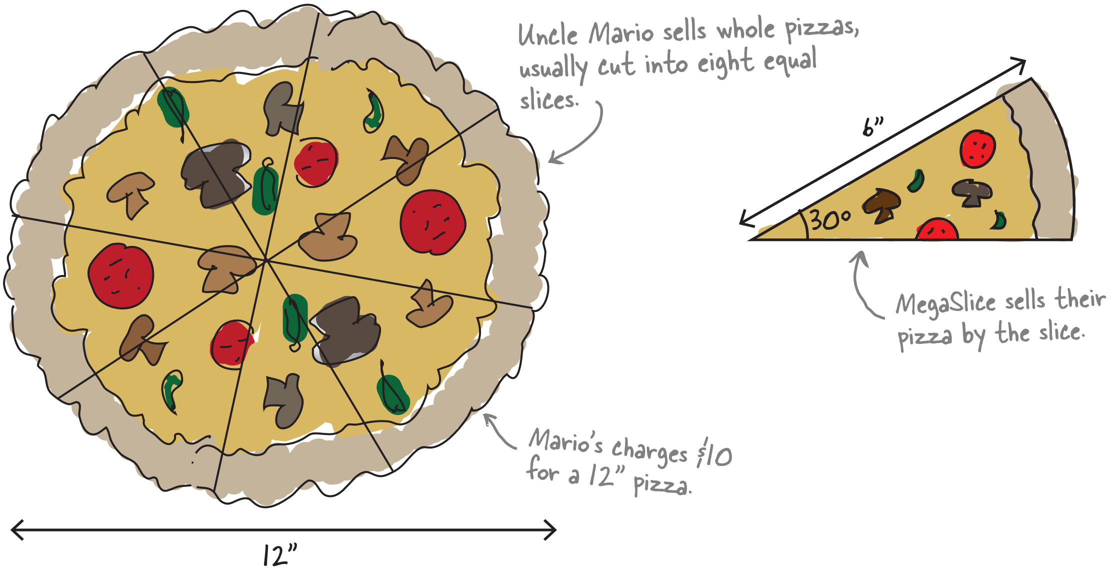


The MegaSlice promo flyer



How does MegaSlice's deal measure up?

MegaSlice claims you get more pizza with their 10-slices-for-$10 deal. You certainly get more slices, but does that mean more pizza?

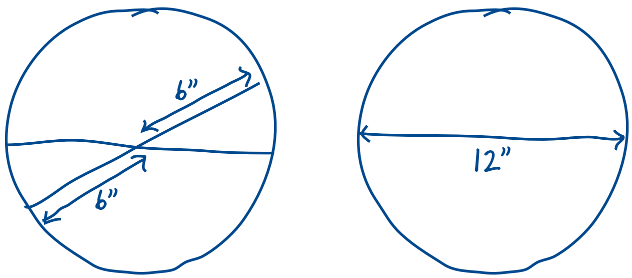


Sharpen your pencil

How big are the pizzas that MegaSlice are cutting up and selling by the slice?

Sharpen your pencil Solution

How big are the pizzas that MegaSlice are cutting up and selling by the slice?

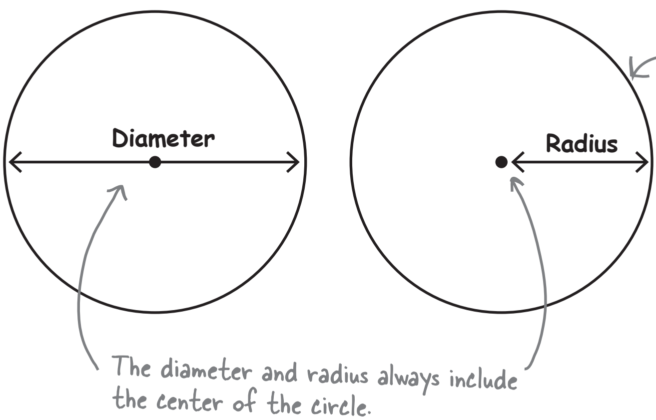


The pizza must be as wide as two of the slice edges—12 inches.

The diameter of a ~~pizza~~ circle is twice its radius

The total width of the pizza—12 inches—is a special circle property we call ***diameter***. The diameter of a circle is the distance from one side to the other, passing through the center. The length of the edge of the pizza slice is called ***radius***, and is the distance from the center to any point on the edge. Diameter is always two times the radius.

It doesn't matter which points on the edge you draw your diameter between, or where you draw the radius to, it's always the same size (just remember to include the center).

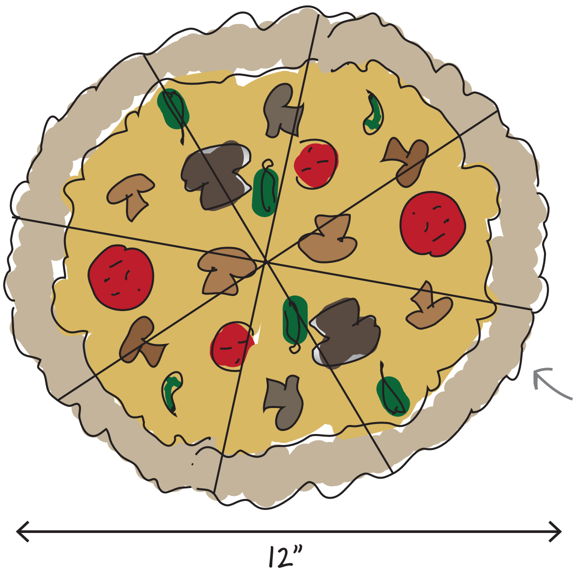


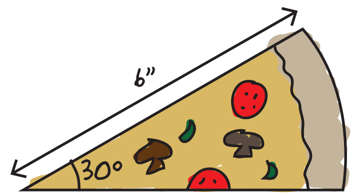
Diameter is always twice the radius:

$D = 2r$

How do slices compare to whole pizzas?

MegaSlice's deal is in slices, and Mario's is for a whole pie. But MegaSlice's 10 slices also come from a 12-inch pizza. So—does that mean that their deal is better, because it's 10 slices and not just 8?





Sharpen your pencil

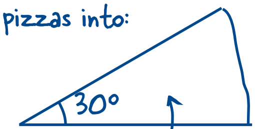
Use the information you have about the MegaSlice slices to work out which is the better $10 pizza deal.

Hint: it's all about the angle....

Sharpen your pencil Solution

Use the information you have about the MegaSlice slices to work out which is the better $10 pizza deal.

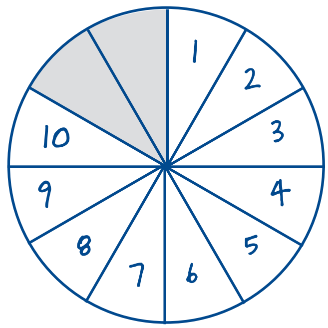
Need to work out how many MegaSlices is MegaSlice cutting each of their pizzas into:



Each slice is 30°

360° in a whole turn

Total slices = $\frac{360°}{30°} = 12$

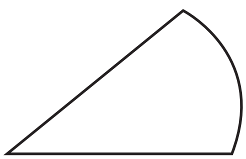


Each MegaSlice pizza is cut into 12 slices—so just 10 slices isn't even the whole pizza!

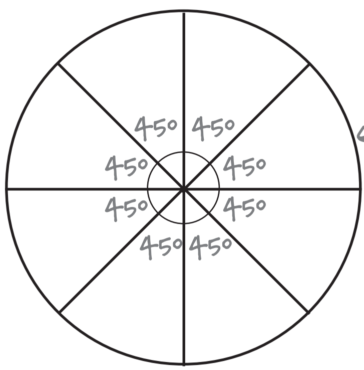
Mario gives you the whole 12" pizza, so Mario's is a better deal.

Sectors of a circle have angles totaling 360°

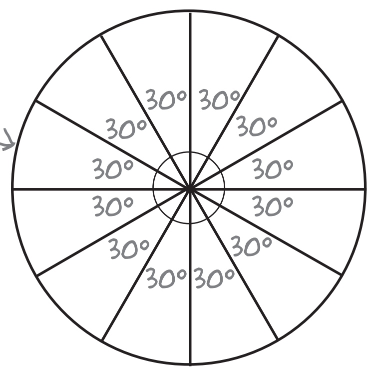
When a circle is divided up using lines that pass through the center of the circle, the resulting "pizza slice" shapes are called ***sectors***. The MegaSlice sectors each have 30° angles, so a full pizza from them would have to be 12 slices (30° × 12 = 360°). But they only give you 10 slices, which means their deal gives you less than a whole 12" pizza!



This is a sector.

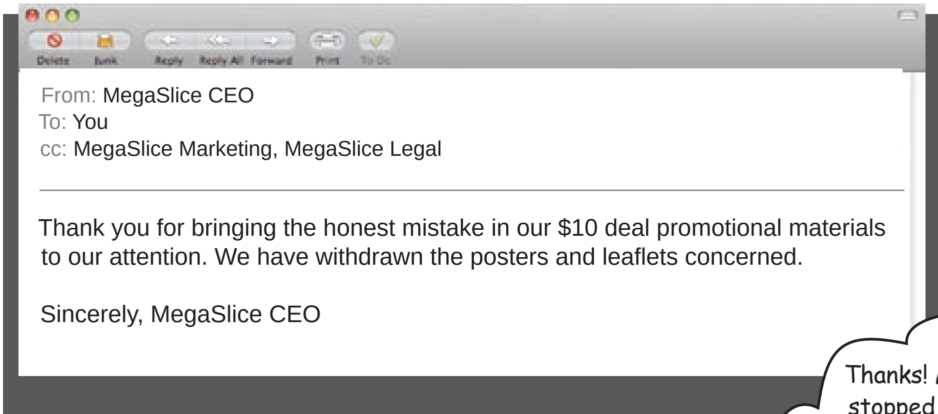


Sector angles always add up to 360°



MegaSlice's $10 deal is a con!

MegaSlice is only offering 10 of the 30º slices in a whole 12" pizza, so while they're offering a greater number of slices than Uncle Mario, it's actually less pizza overall than his deal! Now you've got the hard facts, there's no way MegaSlice can keep claiming to be a better value.


From: MegaSlice CEO
To: You
cc: MegaSlice Marketing, MegaSlice Legal

Thank you for bringing the honest mistake in our $10 deal promotional materials to our attention. We have withdrawn the posters and leaflets concerned.

Sincerely, MegaSlice CEO

MegaSlice is eating humble pie.



Pepperoni crust pizza—but at what price?

Mario's idea is to offer a pepperoni crust on any pizza. He wants to offer the best possible value to his customers so he'll only add the minimum price needed to cover the cost of the extra pepperoni. Sounds simple, right? But Mario has a problem with the sequence of events in the pizza process:

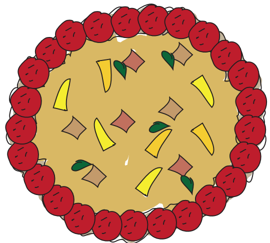
1 The customer rings up with their order.



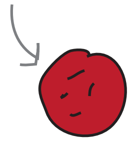
2 Mario says how much it will be.



3 Mario makes the pizza.

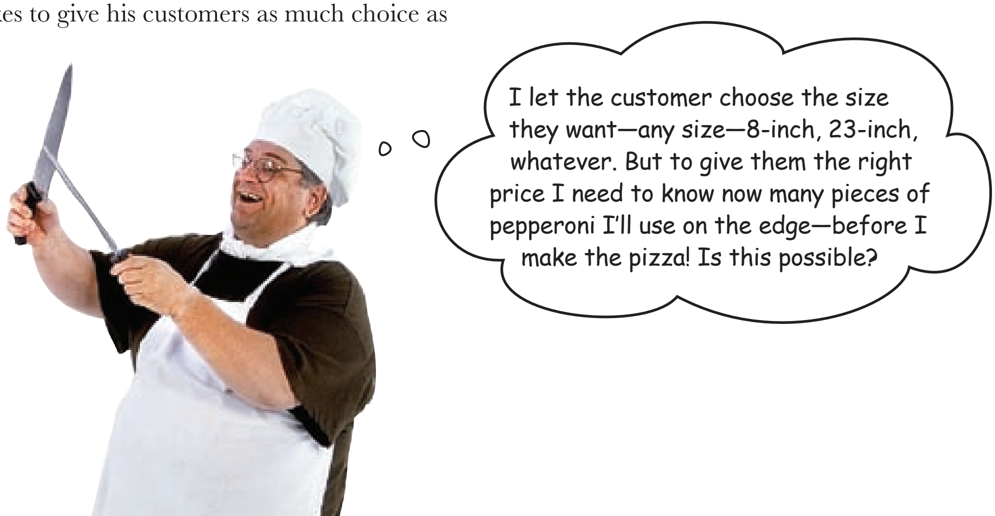


Mario will add 5 cents per piece of pepperoni.



The catch is...Mario's pizzas are custom sized!

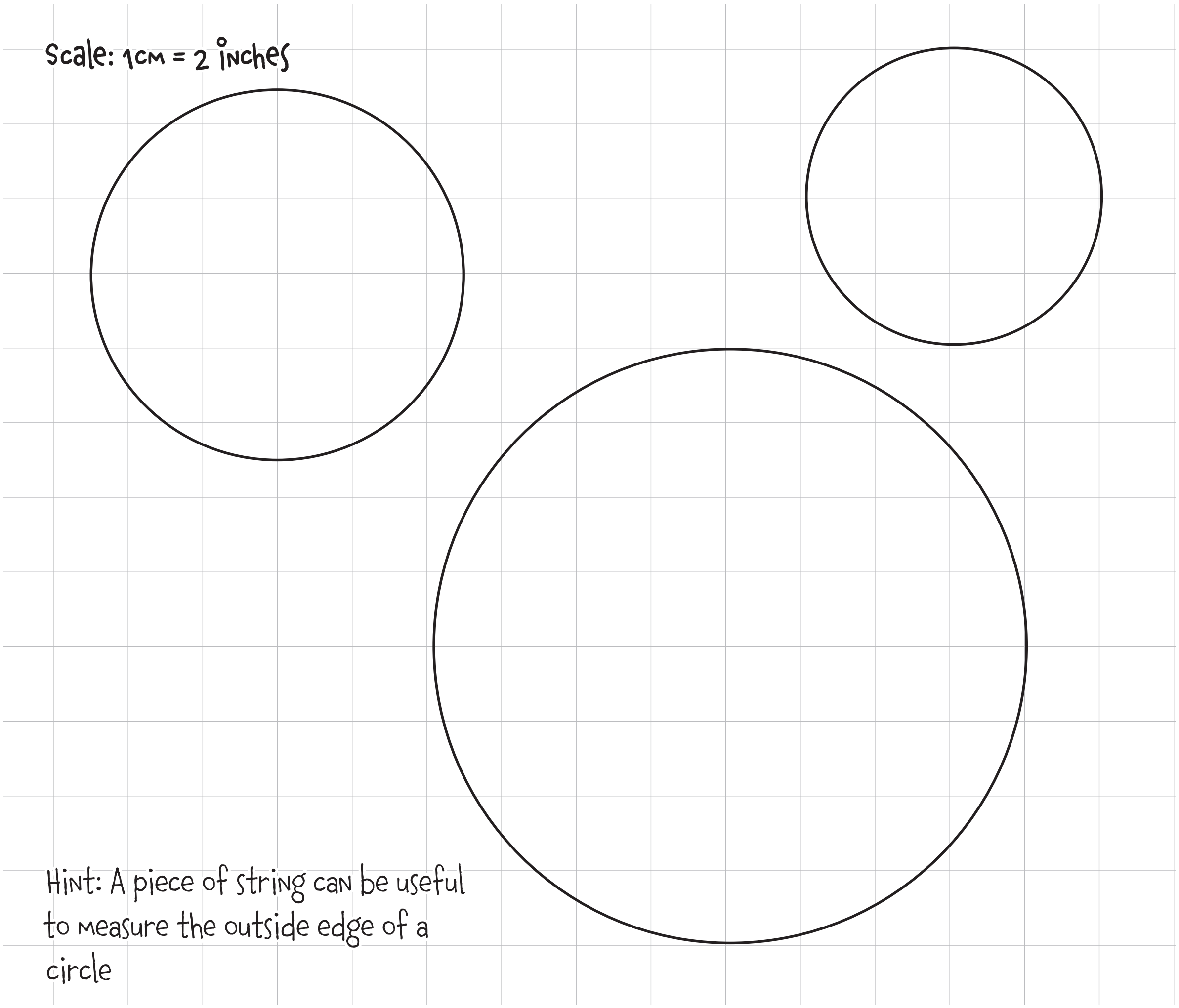
If Mario only offered a couple of pizza sizes you could just make the crust for each size once and then always add the cost of that amount. But Mario likes to give his customers as much choice as possible.



Geometry Detective

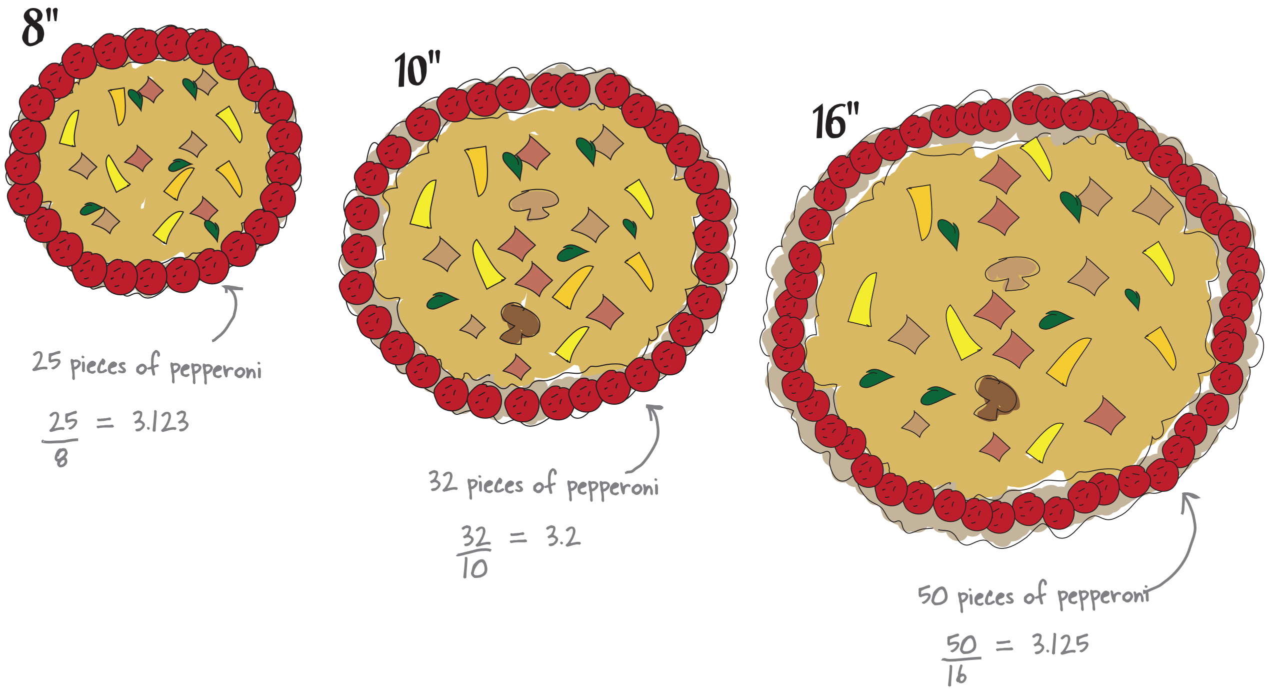
Mario needs a way to work out how many pieces of pepperoni it will take to edge a pizza, based only on the diameter of the pizza. Each piece of pepperoni is 1 inch across.

Your mission is to investigate the relationship between the diameter of each pizza and the "perimeter," or outside edge, of the pizza. How many pieces of pepperoni does Mario need to edge each pizza below? Is there a pattern?



The pepperoni perimeter is 3 (and a bit) times diameter

There's a pattern: no matter what size the pizza, the pepperoni perimeter is a little more than three times the diameter.

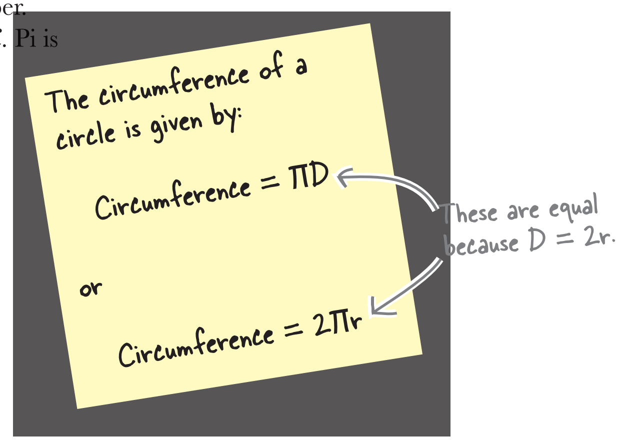


A circle's circumference is diameter x Pi

In geometry terms, a circle's perimeter is called the ***circumference***, and no matter how big or small that circle is, you can find the circumference by multiplying the diameter by the same number. Roughly it's just barely over three, and we call this number ***Pi***. Pi is usually written using this symbol:

$$\pi$$

Pi





"Pi" is actually a letter from the Greek alphabet.

It's used to stand in for the number that you always get when you divide a circle's circumference by its diameter.



You can probably find a Pi button on your calculator. It might give you 3.14159, or it might give you something with even more decimal places.

No calculator? Substitute the fraction 22/7 for a close approximation to Pi.

BRAIN POWER

Why do you think "math people" find Pi, or π, more useful than 3.14159265358979323846264338327950...?

there are no Dumb Questions

Q: When you showed the formula for circumference, you put πD but also 2πr—does it matter which I use?

A: D is diameter, and r is radius, so the two formulas always give the same answer since D = 2r. It doesn't matter which you use—it just depends on whether you've got the diameter or the radius available.

Q: So, in like two pages you've shown Pi as "just barely over three," 3.14159, a way-long decimal, and even 22/7... which is it?

A: Pi is what is known as an *irrational* number. That means that even if you used a billion decimal places you couldn't write it down completely. Depending on what you're using Pi for you might need to use a more or less precise version of it. On an exam, you'll usually be told what to use, otherwise just take whatever your calculator gives you when you press the π button, and remember to round your answer to fit with the question.

OK, so Pi is really nothing more than a placeholder for the ratio between a circle's circumference and its diameter? I guess it's quicker than writing or saying 3.14159265358979 32384626433832795...?

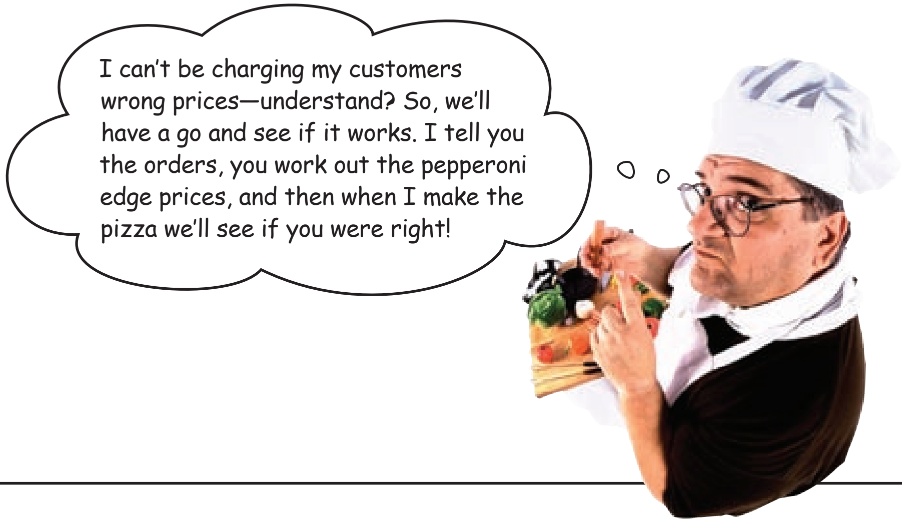


Exactly.

Math geeks talk about Pi as if it's kind of magic—and it's certainly pretty useful because it works for every circle, every time—but really Pi is just a quick way of saying "that big long number that you get when you divide circumference by diameter."

Mario wants to put your pepperoni crust pricing formula to the test

So, is this "Pi" thing just theoretical, or does it actually work in the real world? It's time to put your formula to the test.





Exercise

Mario wants to charge 5 cents per piece of pepperoni. How much does he need to add to the price of the following orders if they all want the new pepperoni crust? (Use your calculator and round up or down to the nearest whole piece of pepperoni.)

1: A 14" TexMex with extra jalapenos

2: A 20" "The Works" (hold the mushrooms)

3: Two 6" kids' Hawaiians, one with extra pineapple

Exercise Solution

Mario wants to charge 5 cents per piece of pepperoni. How much does he need to add to the price of the following orders if they all want the new pepperoni crust? (Use your calculator and round up or down to the nearest whole piece of pepperoni)

1: A 14" TexMex with extra jalapenos

$14 \times \pi = 43.98 = 44$ pieces of pepperoni

44×5 cents = \$2.20

2: A 20" "The Works" (hold the mushrooms)

$20 \times \pi = 62.83 = 63$ pieces of pepperoni

63×5 cents = \$3.15

If you forgot to round your pieces of pepperoni before you worked out the price you'd have slightly different answers here—like \$3.14. Remember to round at the right time!

3: Two 6" kids' Hawaiians, one with extra pineapple

$2 \times 6 \times \pi = 37.7 = 38$ pieces of pepperoni

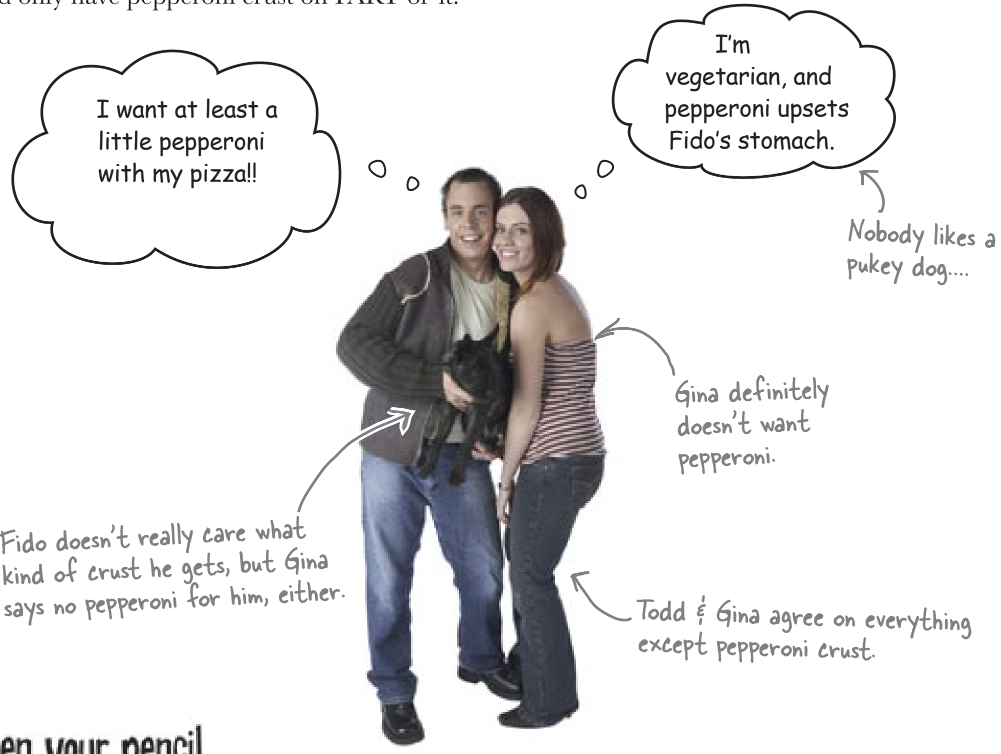
This order is for 2 pizzas.

38×5 cents = \$1.90



The customers are always ~~right~~ fussy

Pizza is for sharing, right? But some of Mario's customers want to share a pizza and only have pepperoni crust on PART of it.



Sharpen your pencil

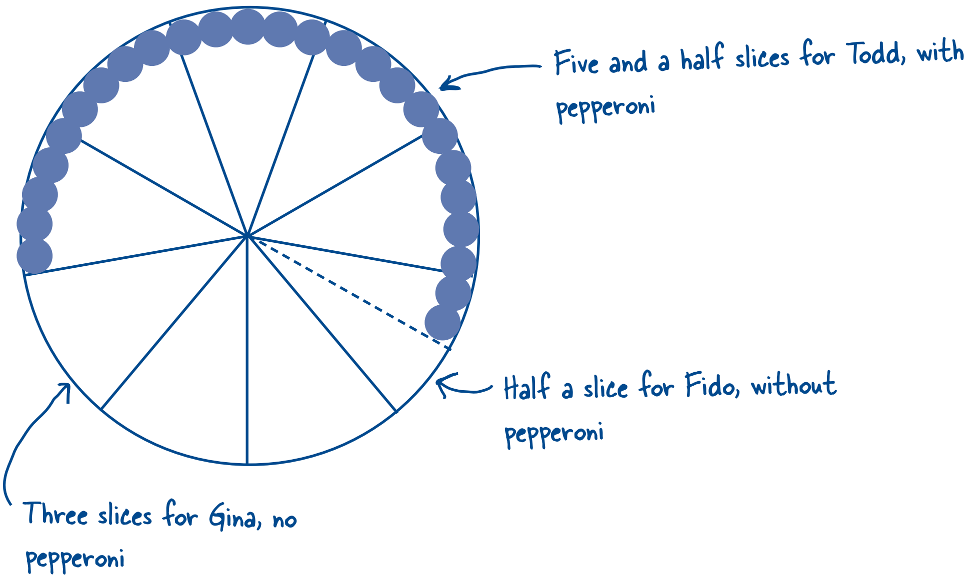
Todd and Gina always order an 18-inch Super Veggie deep-dish pizza. They ask for it cut into nine equal slices. Gina eats three, Todd has the other six, but he always gives his last half-slice to Fido.

How much extra should Mario charge for their complicated pepperoni crust requirements?

Sharpen your pencil Solution

Todd and Gina always order an 18-inch Super Veggie deep-dish pizza. They ask for it cut into nine equal slices. Gina eats three, Todd has the other six, but he always gives his last half-slice to Fido.

How much extra should Mario charge for their complicated pepperoni crust requirements?



Total pepperoni required if we were going to edge the whole pizza with it: → $18 \times \pi = 56.55$ pieces ← You might have rounded here first—your answer should be the same in this question.

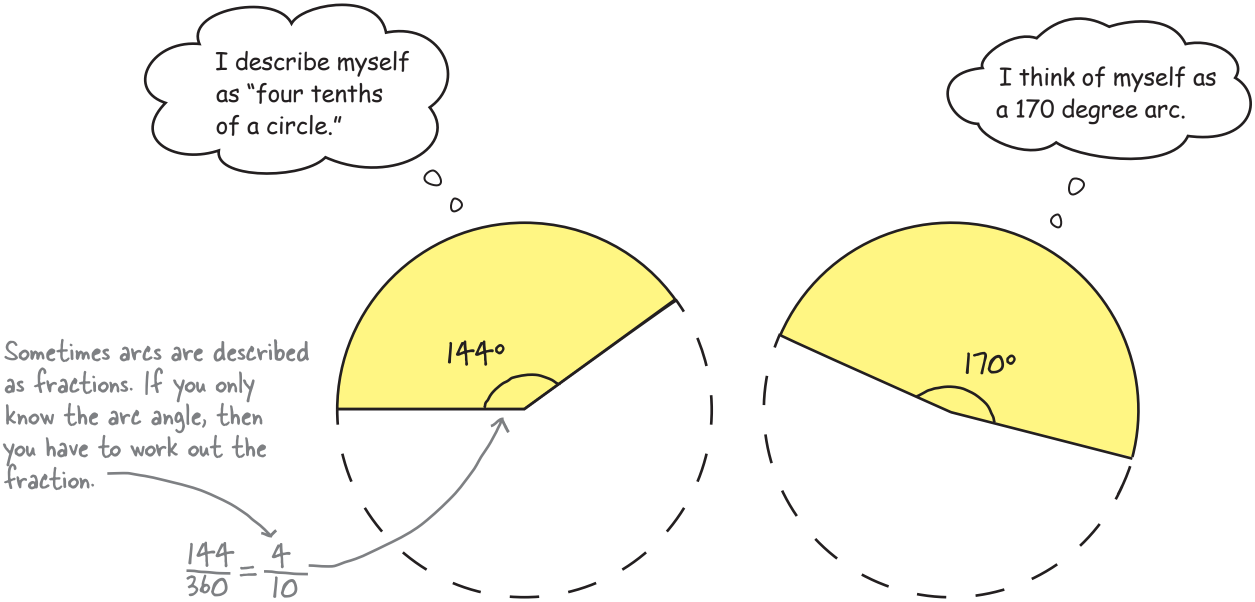
Pepperoni required for the 5.5 slices Todd eats: → $56.55 \times \frac{5.5}{9} = 34.55$ = 35 to nearest whole piece ← If you figured out separately that this was 61% (or .61) and used that instead, that works, too.

Price for pepperoni crust: → $35 \times 5¢ = \$1.75$

An arc is a section of the circumference

Covering part of the pizza crust with pepperoni creates an ***arc***. That's geometry jargon for a section of the circumference of a circle—whether it goes almost all the way around a circle, or just a tiny part of it.

Arcs can be described by the proportion of the circumference they cover—a semicircle covers half—or by the angle of the corresponding sector (the arc is the outside curved part of the sector).



Arc length is circumference x sector angle / 360°

To find the length of an arc, first you need to find the circumference of the circle the arc is part of. That means that you need either the diameter or radius of the circle. Then you can use the sector angle to find out the length of the part of the circumference that your arc covers.

$$\text{Arc Length} = 2\pi r \times \frac{\text{Sector Angle}}{360°}$$

This calculation helps you find the fraction of the circle that the sector represents. If you already know it's 2/5 or 1/12, then just use the fraction instead!

Mario's business is booming!

Mario's customers love being able to choose how much of their pizza has pepperoni crust. Even when they can't agree on pepperoni, they can agree that Mario's is the perfect pizza to suit them.





BULLET POINTS

- Circumference $= \pi D = 2\pi r$
- Diameter $= 2r$
- Arc Length $= 2\pi r \times \dfrac{\text{Sector Angle}}{360°}$

But MegaSlice is at it again...

Just when Mario thought MegaSlice had learned its lesson, MegaSlice is back with a TV commercial claiming their restaurants give you more for your money. And on the surface it sounds like a pretty good deal.





Sharpen your pencil

What property of the pizzas do you need to compare to be able to work out whether MegaSlice's claims are true?

Sharpen your pencil Solution

What property of the pizzas do you need to compare to be able to work out whether MegaSlice's claims are true?

Area

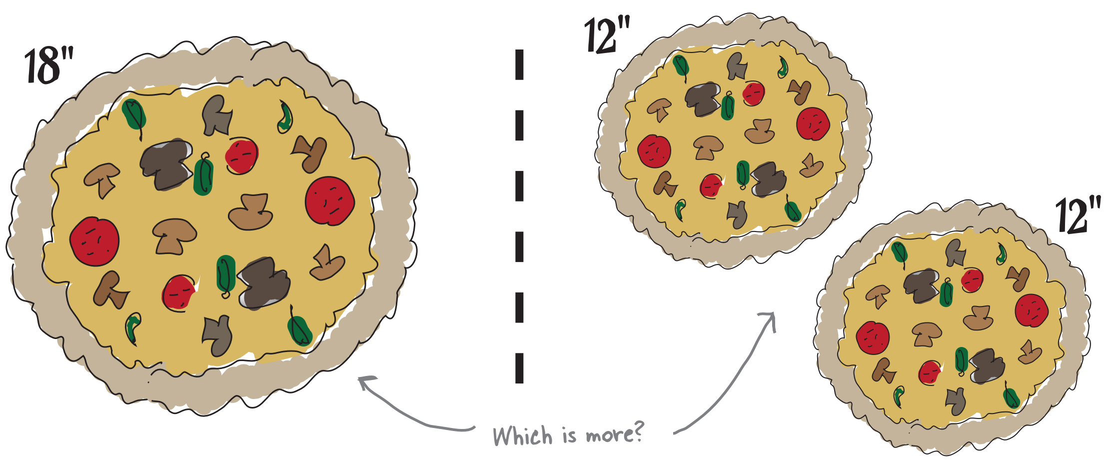
If you put "Volume" here, you're right, but as all Mario's pizzas have the same thickness, area will also cover it. More on that in Chapter 7!

We need to find the area of the two pizza deals

MegaSlice offers two 12" pizzas for the same price as one of Mario's 18" pizzas.

So, which deal is more AREA of pizza, because that's the property you actually get to eat.

OK, you could kind of nibble the circumference if you wanted....





Well, I'm not sure we even need to work this one out. Two times twelve is more than eighteen, so MegaSlice's deal is more pizza.

Frank: Not so fast. I'm not sure it's that simple.

Joe: What's hard about it? 12 + 12 = 24. 24 is way more than 18. Game over.

Jim: But that's not area—that's diameter!

Joe: Yeah—and?

Jim: Well, I'm not sure that you can just add up diameters and then compare them instead of comparing area.

Joe: Why not? Anyway, we don't even know how to find the area of a circle. Hey—is this making you guys hungry?

Frank: Shut up! This is serious. We need to find a way to compare the actual pizza area, and not just the diameters.

Jim: Agreed, but where do we start?

Frank: Well, I guess we should just start from what we do know and go from there.

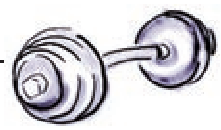
BRAIN BARBELL

Which technique from your Geometry Toolbox could help you work out the pizza areas approximately?

Triangle area = 1/2 base x height

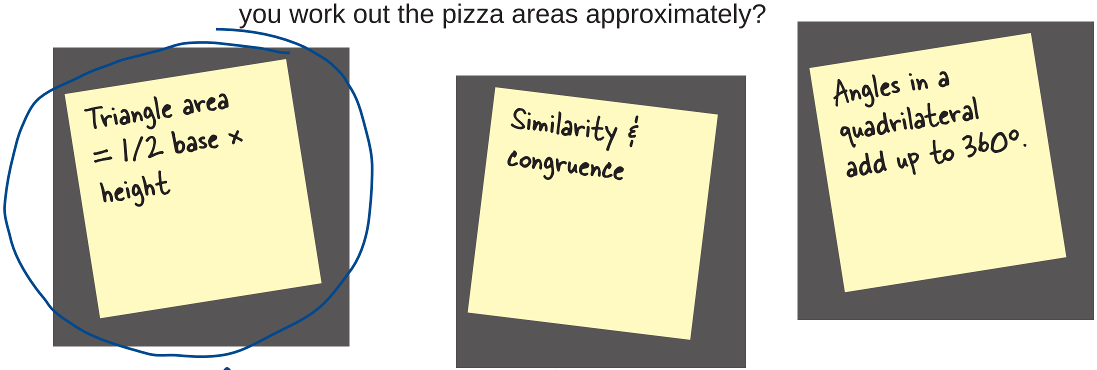
Similarity and congruence

Angles in a quadrilateral add up to 360°.



Brain Barbell Solution

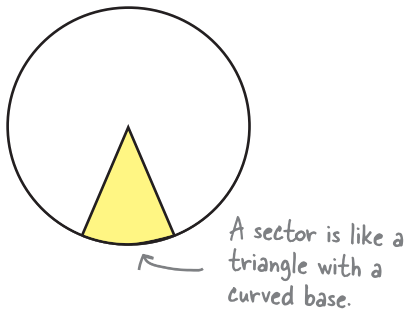
Which technique from your Geometry Toolbox could help you work out the pizza areas approximately?

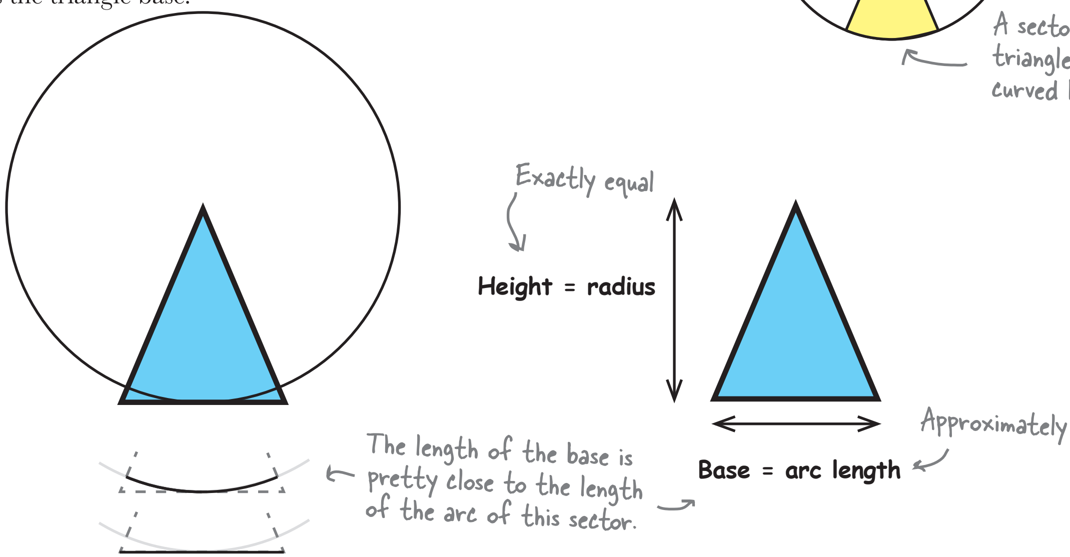


If you pretended that a slice (sector) was a triangle, you could work out the area of that slice and then add up all the slices.

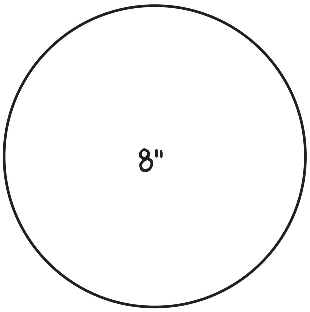
Each sector (slice) is a triangle (kind of)

A sector of a circle isn't actually a triangle with three straight edges, but it's pretty close. You can find the approximate area of that sector by using the radius of the circle as the triangle height, and the arc length as the triangle base.





Geometry Detective



This pizza has a diameter of 8 inches.

Using radius to stand in for height, and arc length to stand in for base, use the triangle area formula to investigate how you can approximate the circle area.…

1. By chopping the pizza base into 6 equal slices

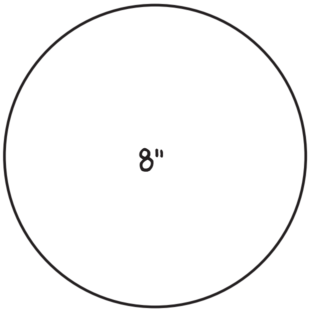
2. By chopping the pizza base into 10 equal slices

3. By chopping the pizza base into 30 equal slices

What do you notice as you chop it into more and more slices?

Geometry Detective Solution

This pizza has a diameter of 8 inches.



Using radius to stand in for height, and arc length to stand in for base, use the triangle area formula to investigate how you can approximate the circle area....

Here's the triangle area formula from your Geometry Toolbox.

Triangle area = 1/2 base × height

Use this to find the triangle (or sector) area, and then multiply by the number of slices we're using to make the circle.

$$\text{Slices} \times \text{radius} \times \frac{\text{circumference}}{\text{slices}} \times \frac{1}{2} = \text{area}$$

1. By chopping the pizza base into 6 equal slices

$$6 \times 4 \times \frac{2\pi \times 4}{6} \times \frac{1}{2} = 50.2654 \text{ square inches}$$

2. By chopping the pizza base into 10 equal slices

$$10 \times 4 \times \frac{2\pi \times 4}{10} \times \frac{1}{2} = 50.2654 \text{ square inches}$$

3. By chopping the pizza base into 30 equal slices

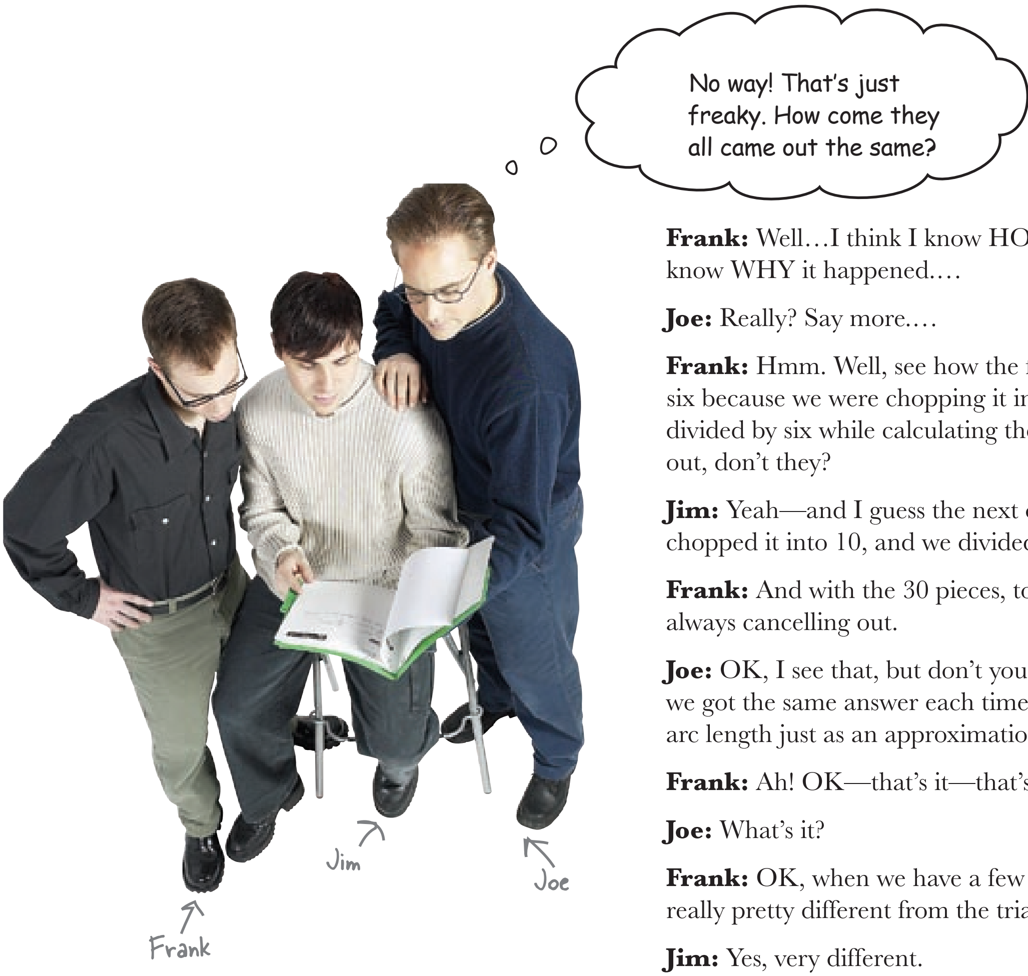
$$30 \times 4 \times \frac{2\pi \times 4}{30} \times \frac{1}{2} = 50.2654 \text{ square inches}$$

We didn't tell you how many decimal places to use—but if your answers are coming out equal with just a couple of decimal places it's always worth checking whether they're still equal with more decimal places, too.

What do you notice as you chop it into more and more slices??

It doesn't seem to matter how many slices you use to get the area approximately...the answer is always the same?!?

You might have put something different—as long as you noticed that the answer was always the same number, you're on the right path.



Frank: Well…I think I know HOW it happened, I just don't know WHY it happened.…

Joe: Really? Say more.…

Frank: Hmm. Well, see how the first time we multiplied by six because we were chopping it into six slices, and then we divided by six while calculating the arc length? They cancel out, don't they?

Jim: Yeah—and I guess the next one does as well—when we chopped it into 10, and we divided and multiplied by 10.

Frank: And with the 30 pieces, too. The number of slices is always cancelling out.

Joe: OK, I see that, but don't you think it's kind of weird that we got the same answer each time, even though we were using arc length just as an approximation for the triangle base?

Frank: Ah! OK—that's it—that's exactly it!

Joe: What's it?

Frank: OK, when we have a few big slices the arc length is really pretty different from the triangle base.

Jim: Yes, very different.

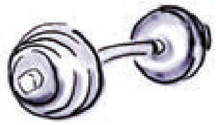
Frank: But, the more slices we chop the circle into, the nearer the arc length is to the triangle base length. So, eventually—if we chop the circle into like a million pieces—it's not an approximation anymore, it's a perfect fit!

Jim: Yes. Cool! In fact, I think we've just found the formula for the area of a circle.

BRAIN BARBELL

Based on the pattern you found in your investigation, what do YOU think the formula for the area of a circle might be?

Try starting with a word equation, and then work it up in algebra if you're feeling confident.



Brain Barbell Solution

Based on the pattern you used in your investigation, what do YOU think the formula for the area of a circle might be?

The pattern for finding area based on triangle area was:

$$\text{area} = \text{slices} \times \text{radius} \times \frac{\text{circumference}}{\text{slices}} \times \frac{1}{2}$$

These cancel

$$\text{area} = \text{radius} \times \text{circumference} \times \frac{1}{2}$$

You might have stopped here, which is great work, or you might have gone a step further….

$$\text{Circumference} = \pi D = 2\pi r$$

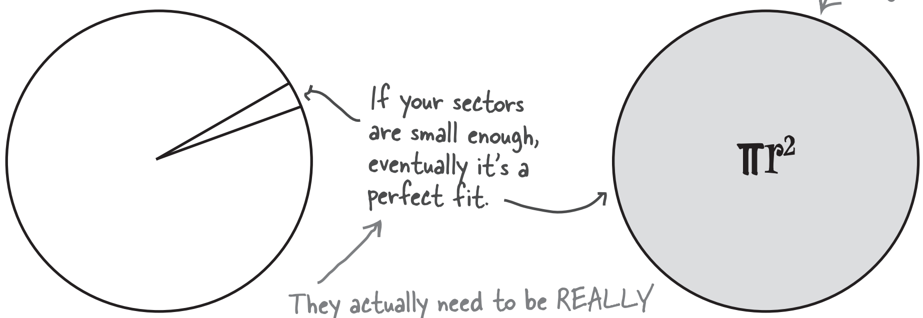
$$\text{area} = \text{radius} \times \text{circumference} \times \frac{1}{2}$$

$$\text{area} = \text{radius} \times \cancel{2}\pi r \times \frac{1}{\cancel{2}}$$

$$\text{area} = r \times \pi r = \pi r^2$$

Area of a circle = πr^2

It's true—if you crammed enough narrow triangles into a circle, eventually you'd get a perfect fit. You'd need an infinite number of triangles, but it turns out the number of triangles cancels neatly, so wham—you just found the formula for the area of a circle!



Sharpen your pencil

Use the circle area formula to work out the area of the pizza in each deal. Is MegaSlice really a better value?

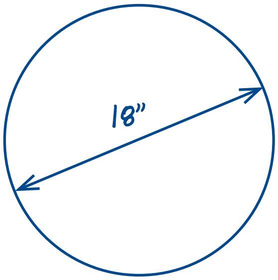
If your calculator doesn't have a Pi button then you can use 22/7 to get an approximate value for Pi.

Sharpen your pencil Solution

Use the circle area formula to work out the area of the pizza in each deal. Is MegaSlice really a better value?

The circle area formula → $Area = \pi r^2$

If your calculator doesn't have a Pi button and you used 22/7 instead, your numbers will be slightly different but the final answer will be the same.

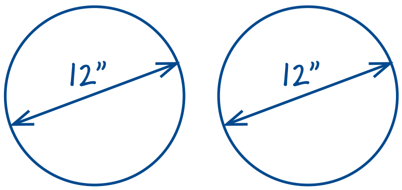


Mario's 18" pizza: $radius = \frac{18}{2} = 9$ inches

Pizza size is diameter: divide it in half to get radius.

$Area = \pi \times 9^2 = \pi \times 81 = 254.5$ sq inches

Just press that "Pi" button on your calculator.



MegaSlice 12" pizza: $radius = \frac{12}{2} = 6$ inches

$Area = 2 \times \pi \times 6^2 = 2 \times \pi \times 36 = 226.2$ sq inches

2 pizzas

Mario's is MORE!

there are no Dumb Questions

Q: Eek—2πr and πr^2 look pretty similar don't you think? Is there a way to keep them straight so I don't mix them up?

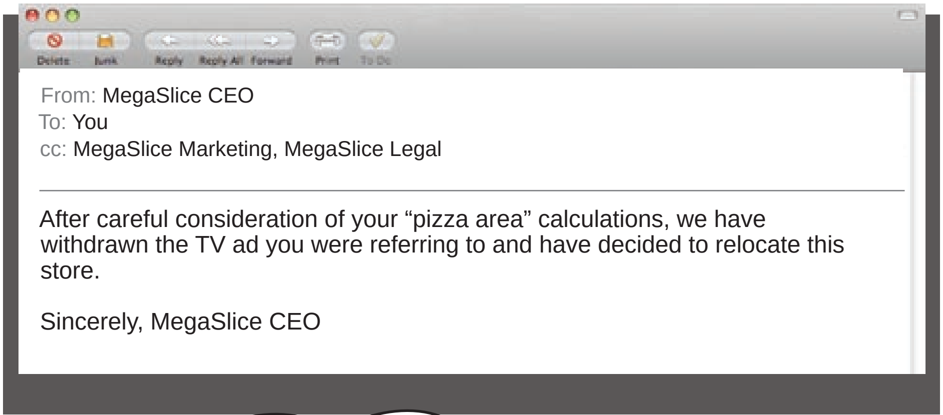
A: There are a couple of tricks that might help you. First of all, if you can remember that circumference is also found from πD then you can compare that and find which one matches (2πr). Also, if you can remember that you learned about circumference before area, the 2 comes first on the circumference formula, and comes after in the area formula.

Q: What if I have to find just the area of a slice of pizza, and not the whole circle?

A: Sector area is easy to find—just like you used its angle to find the length of an arc, you can use the sector angle to find sector area. Find the area of the circle then divide it by 360 (to find the area of a 1 degree sector) and multiply by the angle of your sector.

Mario's pizza is here to stay

That was some pretty impressive work—finding the formula for the area of a circle is no small deal, and for Mario it's a huge win in the pizza wars.



From: MegaSlice CEO
To: You
cc: MegaSlice Marketing, MegaSlice Legal

After careful consideration of your "pizza area" calculations, we have withdrawn the TV ad you were referring to and have decided to relocate this store.

Sincerely, MegaSlice CEO



As part of a balanced diet of course....



Your Geometry Toolbox

You've got Chapter 5 under your belt and now you've added properties of triangles to your toolbox. For a complete list of tool tips in the book, head over to www.headfirstlabs.com/geometry.

Circumference $= \pi D = 2\pi r$

Area $= \pi r^2$

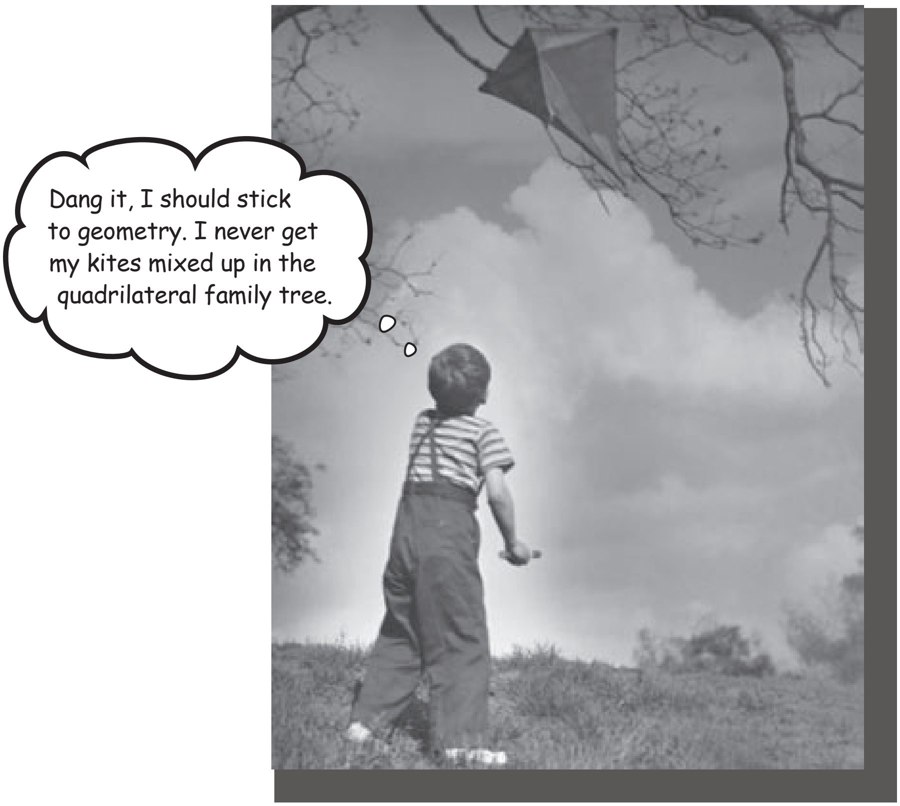
Radius $= \dfrac{\text{diameter}}{2}$

Sector area $= \pi r^2 \times \dfrac{\text{Sector angle}}{360}$

Arc length $= 2\pi r \times \dfrac{\text{Sector angle}}{360}$

6 quadrilaterals

It's hip to be square



Maybe three isn't the (only) magic number.

The world isn't just made up of triangles and circles. Wherever you look, you'll see **quadrilaterals**, shapes with four straight sides. Knowing your way 'round the quad family can save you a lot of time and effort. Whether it's **area**, **perimeter**, or **angles** you're after, there are ***shortcuts galore*** that you can **use to your advantage**. Keep reading, and we'll give you the lowdown.

Edward's Lawn Service needs your help

Edward runs his own lawn mowing and edging service, and over the summer months, demand is high. He's offered you a job to help him cope with all the extra business.

Edward charges clients based on lawn area. That way, clients with a small area of lawn get charged a lower amount than clients with a large lawn.

Edward's Lawn Service

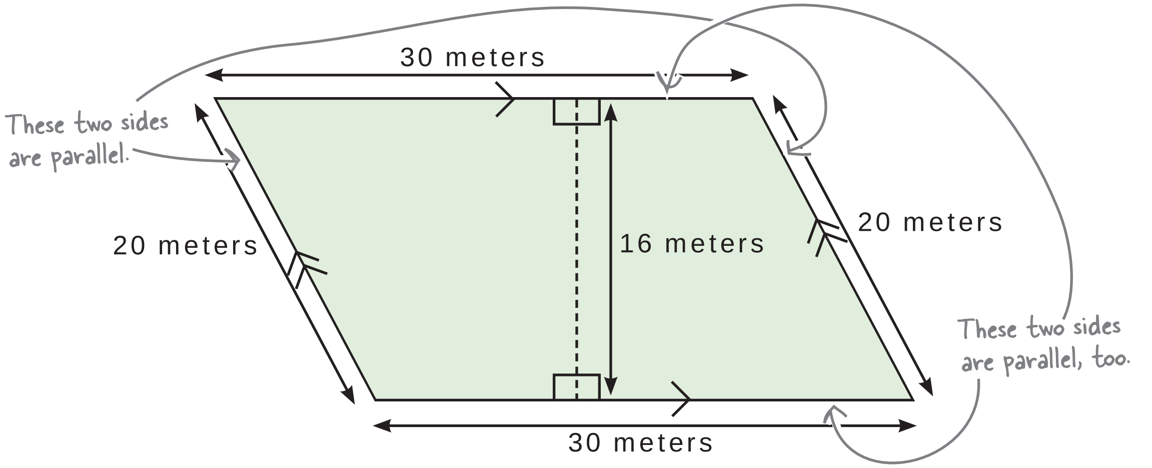
Cost per square meter - $0.10

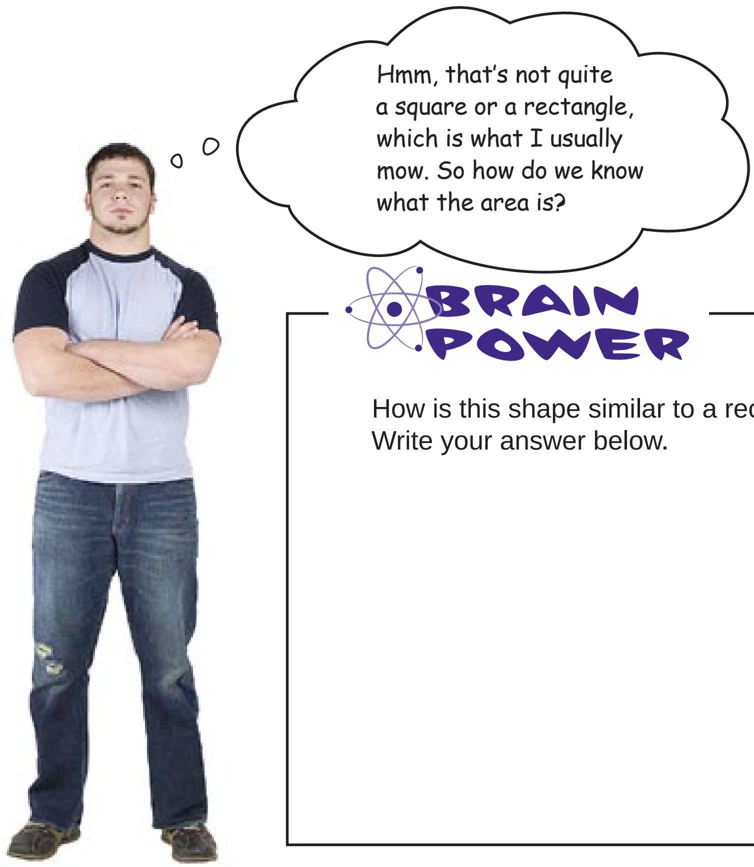
(Payable weekly. Cost includes one lawn cut per week, with all lawn edges trimmed and neatened.)



Your first lawn

The first client Edward hands over to you has a not-so-square lawn. How much should you charge for mowing it?





BRAIN POWER

How is this shape similar to a rectangle? How is it different? Write your answer below.

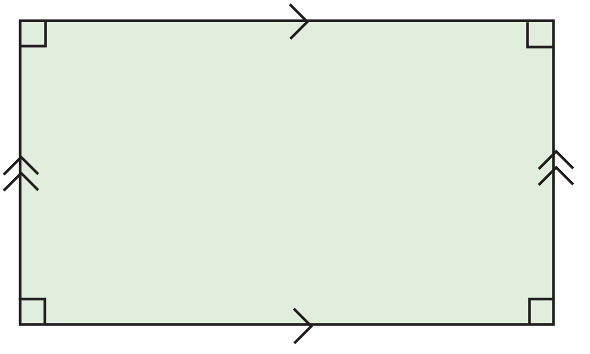
The lawn is a parallelogram

It kinda looks like a rectangle, but squished down so the sides are slanted. A ***parallelogram*** is a four-sided shape whose opposite sides are parallel to each other.

A rectangle:

Rectangles have two pairs of parallel sides. The parallel sides are also congruent.

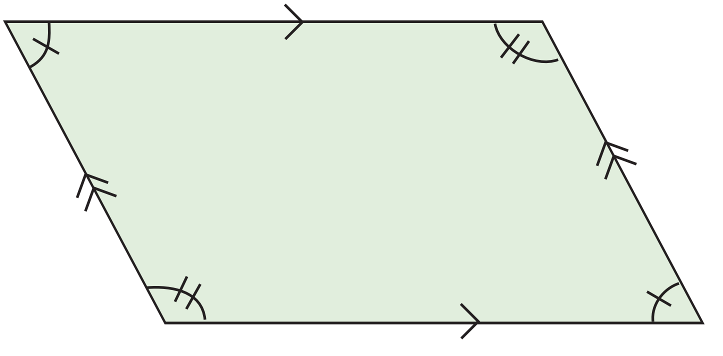
Each of the four corners forms a right angle.



A parallelogram:

Parallelograms have two pairs of parallel sides, too, and like a rectangle, the parallel sides are also congruent.

Unlike a rectangle, the corners of a parallelogram aren't right angles. The most we can say is that opposite corner angles are congruent.



So what should you charge?

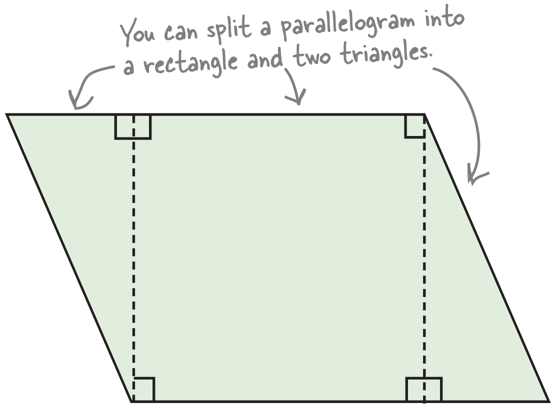
We need to calculate the charge for mowing a lawn in the shape of a parallelogram, and to do this, we need to work out the area. You know how to do that for a rectangle, but would that work for a parallelogram, too?

A parallelogram is a four-sided shape whose opposite sides are parallel.

Let's split the parallelogram

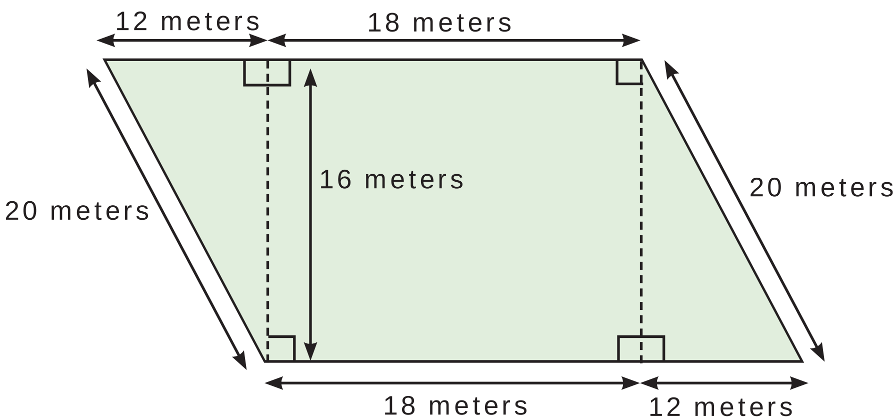
Think back to your work in previous chapters. When you needed to find the total area of a shape you were unfamiliar with, you split your shape up into other simpler shapes you were more familiar with.

If you look carefully, you'll see that the parallelogram is actually made up of a central rectangle, with a right angle triangle on either end. This means that to find the area of the parallelogram, we can find the areas of these shapes, and then add them up together.



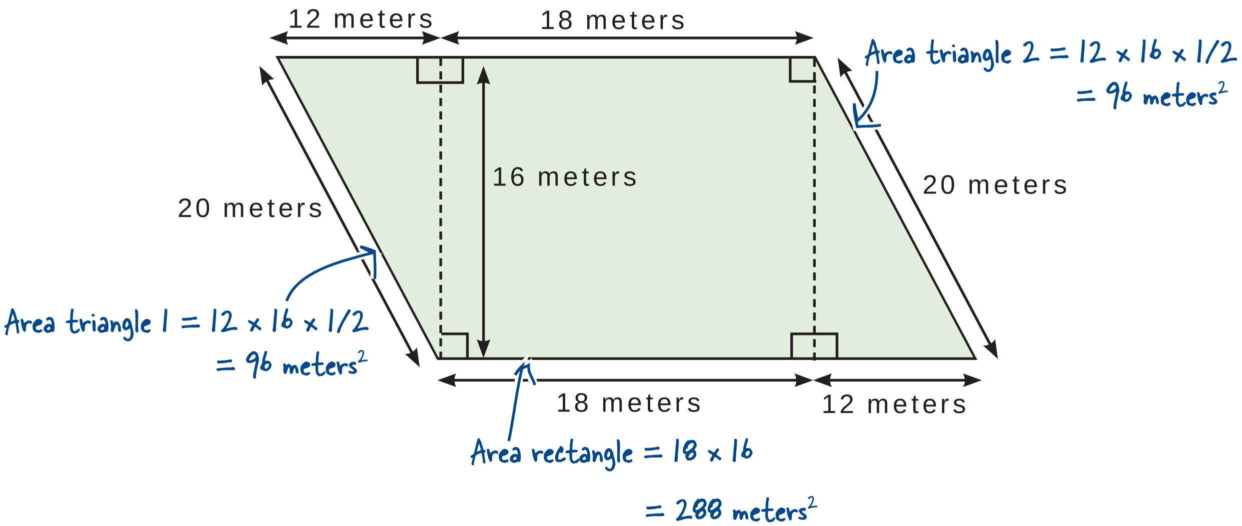
Sharpen your pencil

What's the total area of the lawn that Edward has asked you to mow? How much should you charge? We've added on some extra measurements to get you started.



Sharpen your pencil Solution

What's the total area of the lawn that Edward has asked you to mow? How much should you charge?



Area of parallelogram = Area triangle 1 + Area rectangle + Area triangle 2

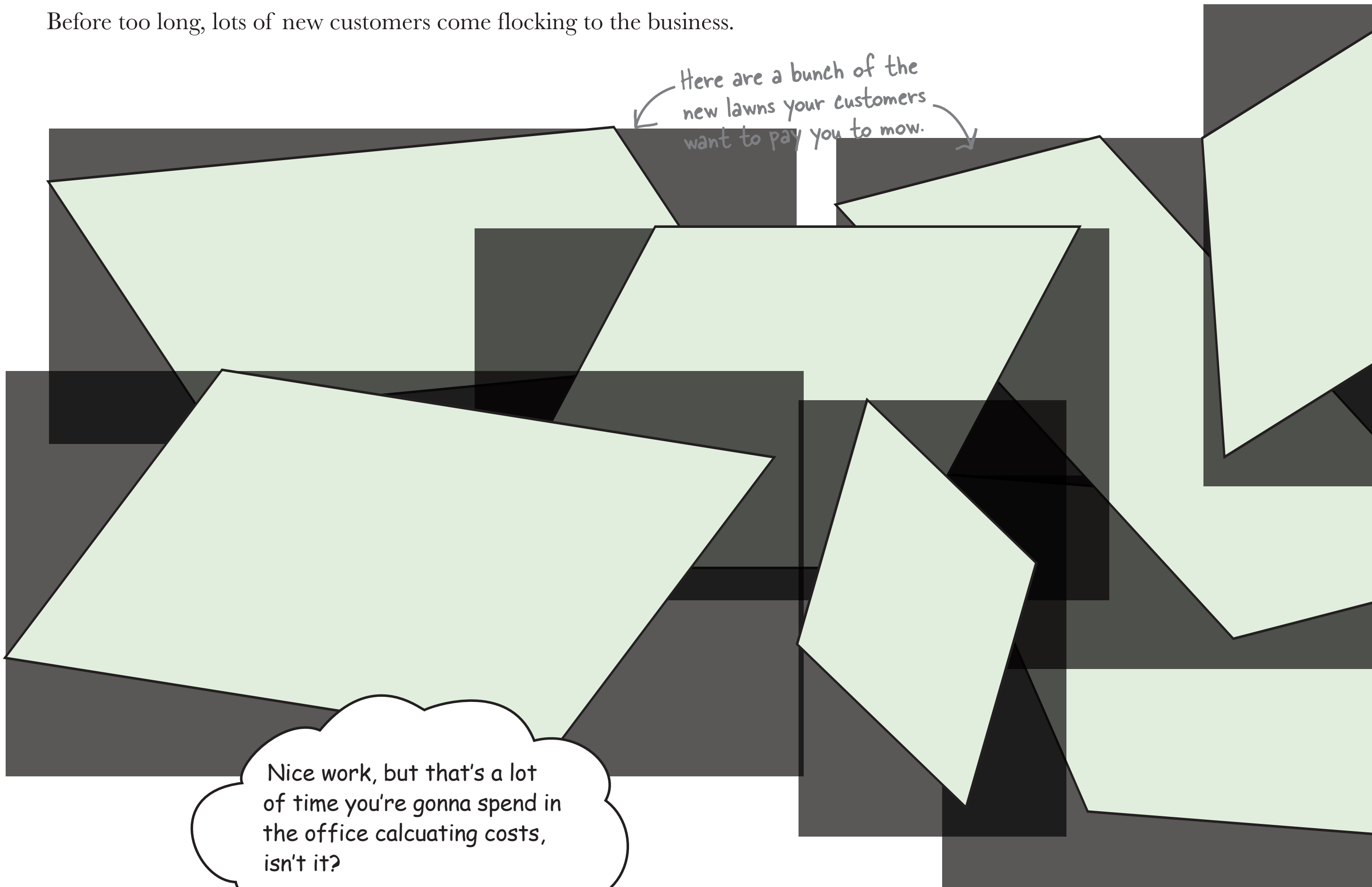
= 96 + 288 + 96

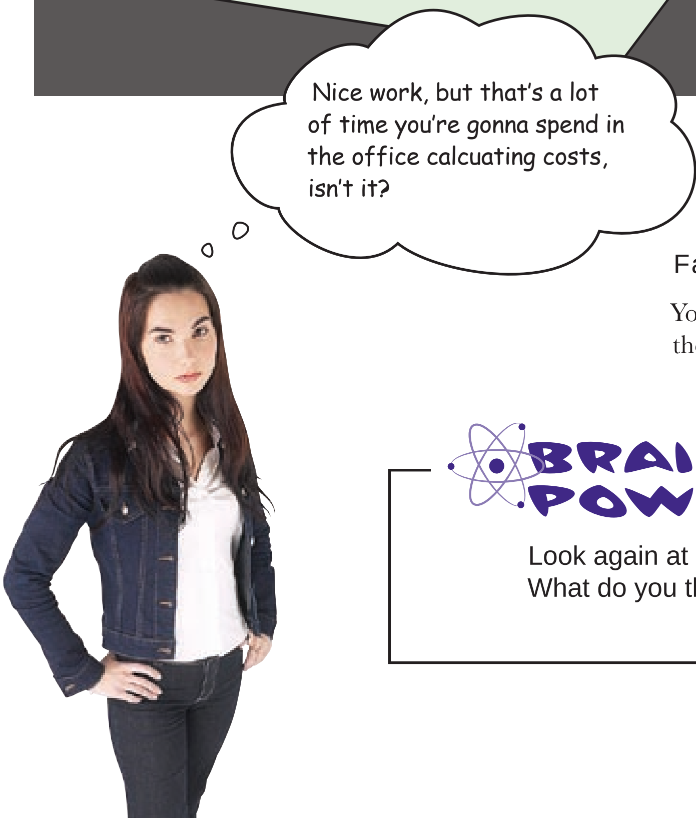
= 480 meters²

Charge = \$0.10 x 480 = \$48.00

Business is booming!

Before too long, lots of new customers come flocking to the business.



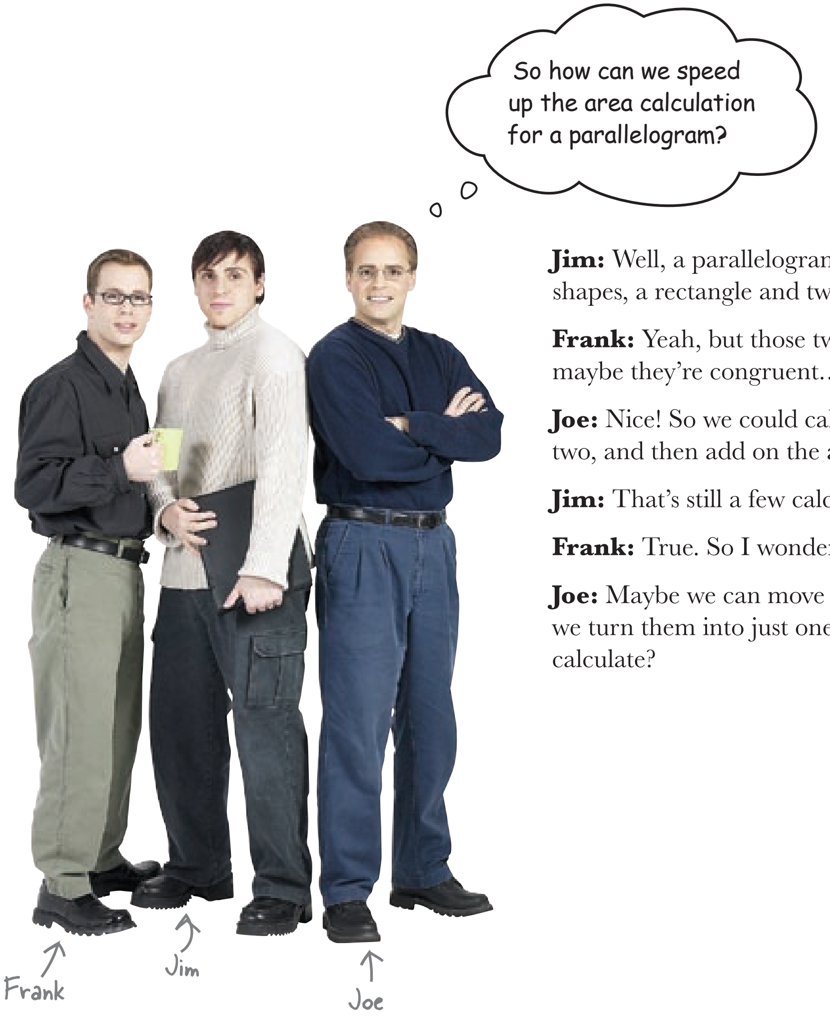


Fair enough.

You can't bill customers if you don't have time to actually mow their lawns. There is probably a way to speed things up a bit.…

BRAIN POWER

Look again at the rectangle and triangles that make up a parallelogram. What do you think you can do to speed up your calculations?



Jim: Well, a parallelogram is basically made up of just three shapes, a rectangle and two triangles.

Frank: Yeah, but those two triangles look the same, right? So maybe they're congruent....

Joe: Nice! So we could calculate one triangle area, multiply by two, and then add on the area of the rectangle.

Jim: That's still a few calculations per lawn, duh!

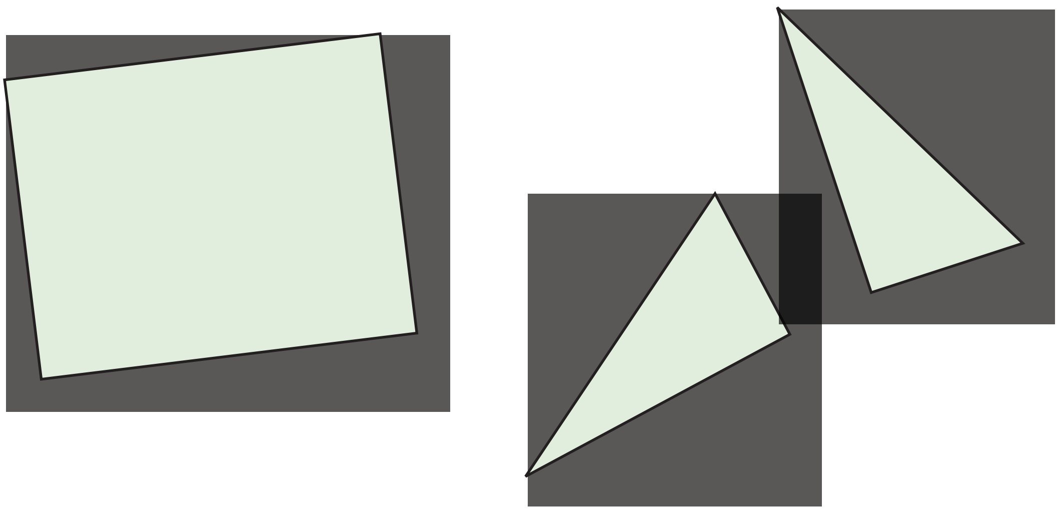
Frank: True. So I wonder if we could take it any further?

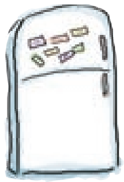
Joe: Maybe we can move these shapes around a bit. Could we turn them into just one shape whose area we know how to calculate?



Geometry Magnets

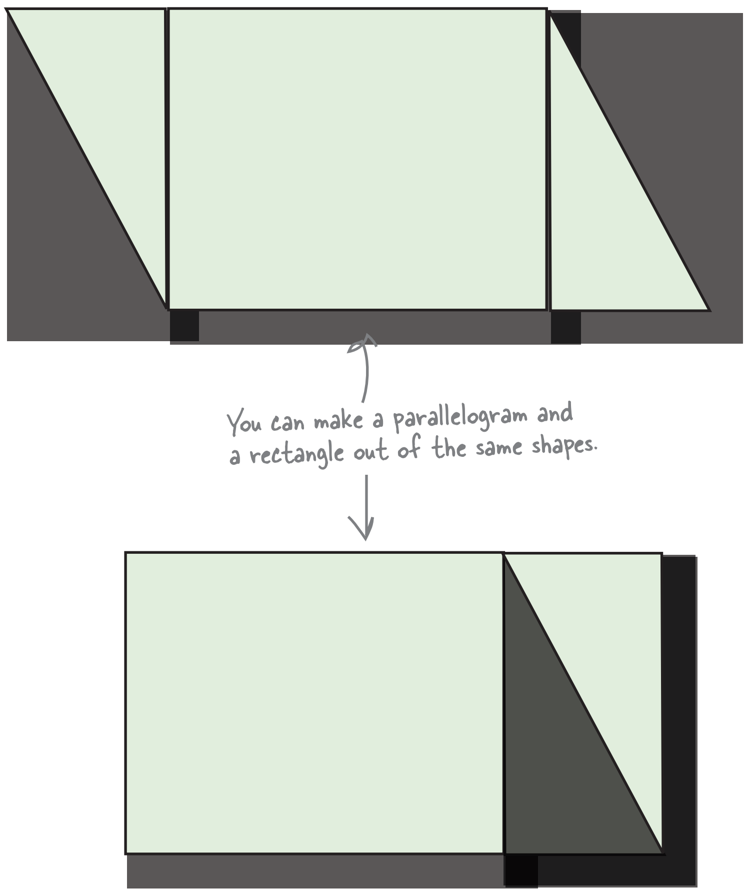
Let's see if we can figure out a quicker way of calculating the area of a parallelogram. First, arrange the shapes below to form a parallelogram. Then, see if you can rearrange them to form a rectangle. What does this tell you about a more general formula for finding the area of *any* parallelogram?





Geometry Magnets Solution

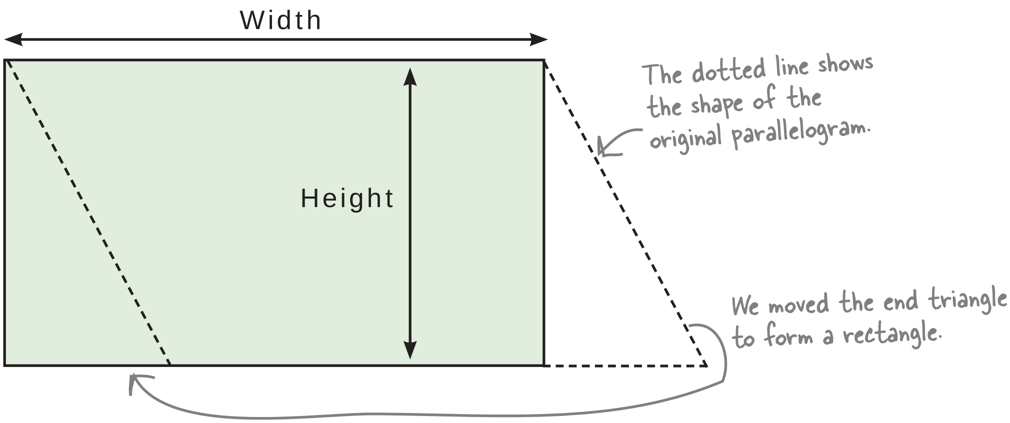
Let's see if we can figure out a quicker way of calculating the area of a parallelogram. First, arrange the shapes below to form a parallelogram. Then, see if you can rearrange them to form a rectangle. What does this tell you about a more general formula for finding the area of *any* parallelogram?



The areas of both the parallelogram and the rectangle must be the same because they're made up of the same shapes. So you can find the area of a parallelogram by finding the area of a rectangle with the same base length and height.

If you don't like what you're given, change it

True, geometry has lots of rules, but moving things around doesn't mean you're breaking them. By moving the shapes you created in the parallelogram around, you were able to form a rectangle. And you know that it has the same height and same base width as the original parallelogram. So now…you know how to find the area of a parallelogram!

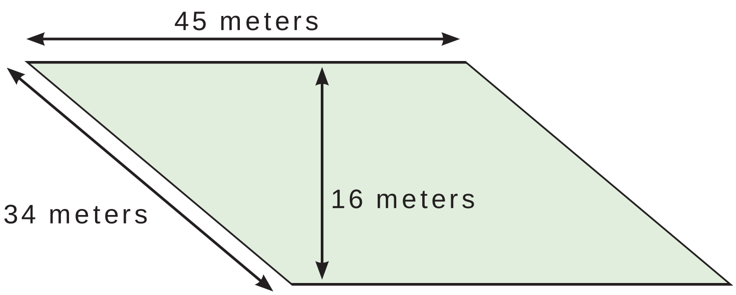


Parallelogram Area = Base Width × Height

Look familiar? It's the same formula you'd use to calculate the area of a rectangle.

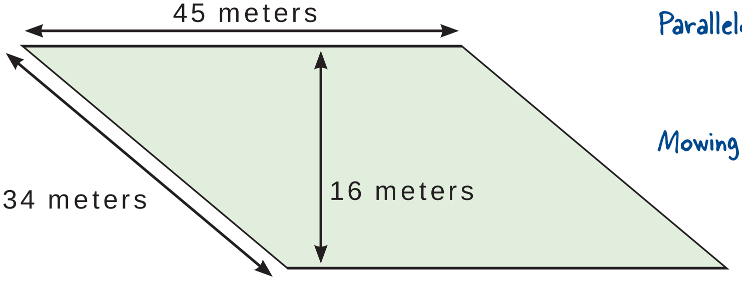
Sharpen your pencil

Here's a new parallelogram lawn you've been asked to mow. What should you charge for mowing it?



Sharpen your pencil Solution

Here's a new parallelogram lawn you've been asked to mow. What should you charge for mowing it?



Parallelogram area = 45 x 16

= 720 $meters^2$

Mowing charge = $0.10 x 720

= $72.00

there are no Dumb Questions

Q: What if I only have one pair of opposite sides? Do I still have a parallelogram?

A: No. In order for a shape to be a parallelogram, both pairs of opposite sides must be parallel. If you only have one pair of opposite parallel sides, your shape isn't a parallelogram.

Q: Can I still use the same formula for finding the area for that shape?

A: This formula is specific to parallelograms. But keep reading, we're going to cover more than just parallelograms and we might have just the formula you need.

Q: What if I only know the length of the sides of a parallelogram and not the height? Can I still work out the area?

A: You can't, because you need to know what the height is. The reason for this is that the degree to which the parallelogram "slants" can make a big difference to the overall area. As an example, if the sides of a parallelogram are tilted so that they're almost horizontal, you'll have a much smaller area than if the sides are almost vertical.

Q: But what if I have extra information such as the internal angles of the parallelogram? Can I work out the area then?

A: Yes, it is possible to find out the area, but it takes a bit more work, not to mention some trigonometry!

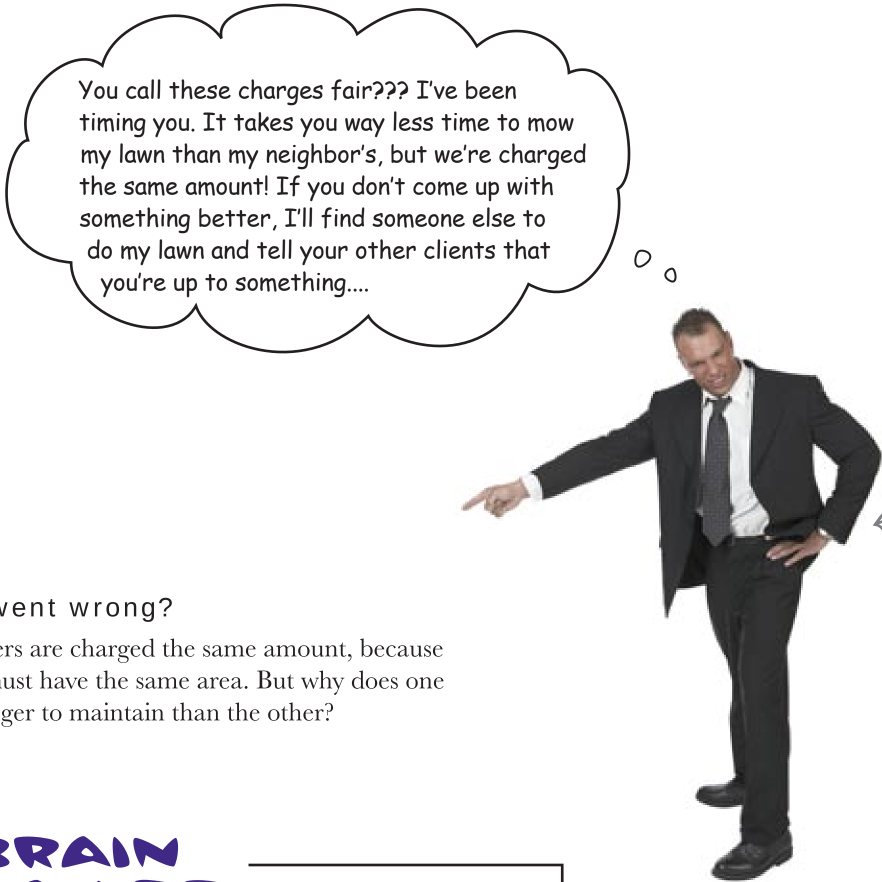
Q: Are opposite sides always the same length?

A: Yes they are, because opposite sides of a parallelogram are always congruent (same angle, same length). If opposite sides are different lengths, you don't have a parallelogram.

Q: What about opposite angles? They look the same to me.

A: Yes, for parallelograms opposite angles are always congruent. Similarly, consecutive angles are supplementary. You can use the angle skills you developed earlier in the book to work this out.

But people are upset with Ed's prices...



Your next customer. Can anyone say too much caffeine?

So what went wrong?

Both customers are charged the same amount, because their lawns must have the same area. But why does one lawn take longer to maintain than the other?

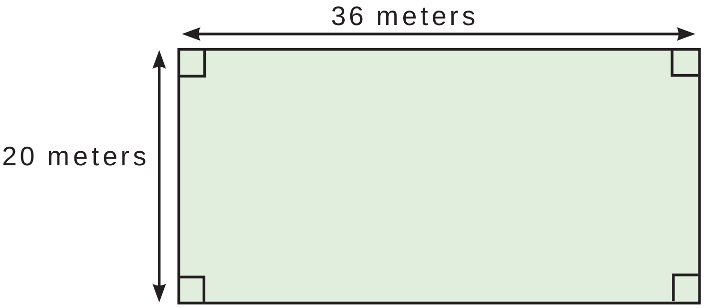
BRAIN POWER

What other factors are involved? Why could it take different lengths of time to maintain the lawns if the lawn areas are the same?

Let's compare the two lawns

They should be the same, but let's see if we can track down what went wrong by comparing the areas of the two lawns. The angry customer has a rectangular lawn, and his neighbor has a lawn in the shape of a parallelogram.

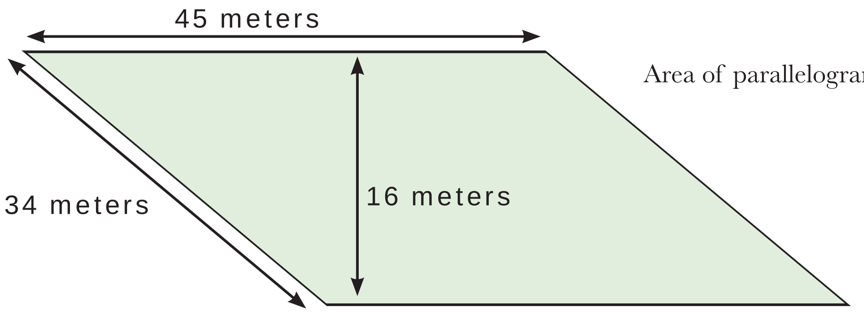
Lawn 1–a rectangle



Area of rectangle = 36×20

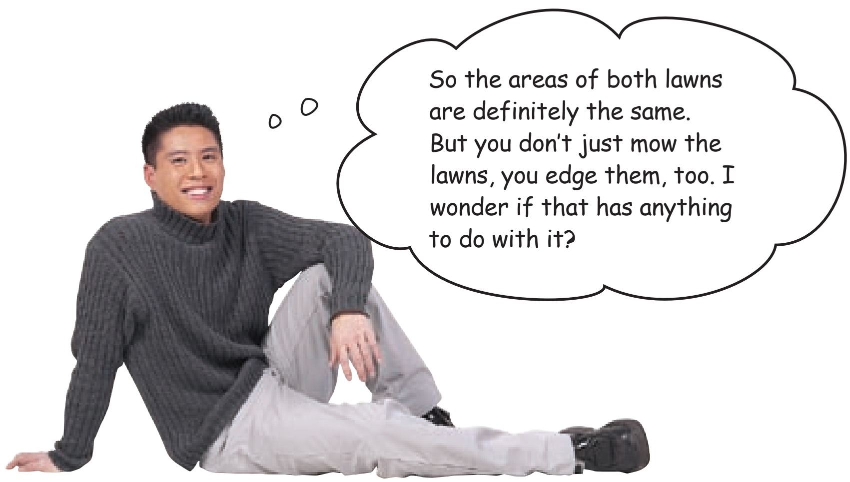
$= 720 \text{ meters}^2$

Lawn 2–a parallelogram



Area of parallelogram = 45×16

$= 720 \text{ meters}^2$



The lawns need edging, too

Of course! When you mow lawns for Edward, you have to use an edger to trim where the grass ends, around the outside of each lawn.

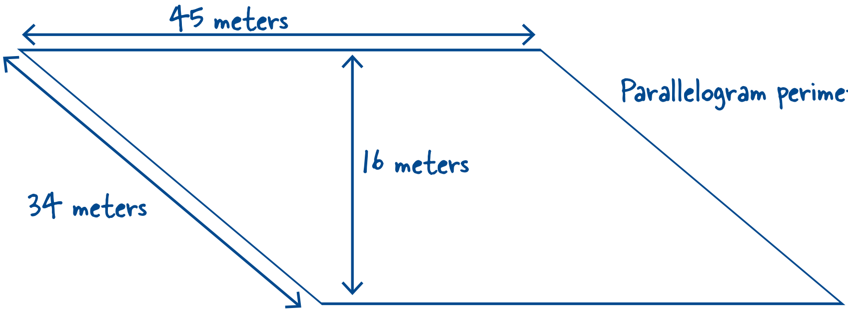
The edge of the lawn is the lawn's ***perimeter***. ← Remember this from Chapter 4?

Sharpen your pencil

Calculate the perimeters of the two lawns. How does this account for the difference in time spent on each lawn? What do you think Edward should do?

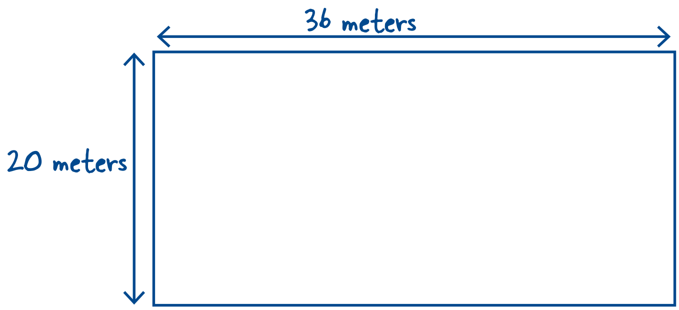
Sharpen your pencil Solution

Calculate the perimeters of the two lawns. How does this account for the difference in time spent on each lawn? What do you think Edward should do?



Parallelogram perimeter = 45 + 34 + 45 + 34

= 158 meters



Rectangle perimeter = 36 + 20 + 36 + 20

= 112 meters

It takes longer to maintain the parallelogram lawn because the perimeter is so much greater and therefore takes longer to edge. Edward's charges need to change so that lawn perimeter is considered, too.

Same shape, different perimeters

Your friends ran into a similar problem back in Chapter 4 when they were picking out a venue for the rock festival—the biggest field didn't necessarily have the most perimeter. For our lawn calculations, even if we know the area of a particular lawn, we can't make any assumptions about its perimeter or how long it will take to edge the lawn. And currently, Edward's charges are only based on lawn area.

Same Area ≠ Same Perimeter

there are no Dumb Questions

Q: So it's not just triangles that have perimeter?

A: No, not at all. The perimeter of a shape is basically the edge around the outside of a shape, whether it's a triangle, rectangle, parallelogram, or some other shape.

Q: Are there any shortcuts we can take in calculating the perimeter of a particular shape?

A: It all depends on the shape.

You calculate the perimeter of a shape by adding the lengths of all its sides together. You can take a few shortcuts if some of these sides are the same length.

As an example, a square has four sides that are all the same length, so the perimeter of a square is just 4 times the length of one side.

Q: What about for a parallelogram?

A: A parallelogram has two pairs of congruent sides, so there are two unique side lengths. You can find the perimeter of a parallelogram by adding these two unique side lengths together, and then multiplying by 2.

Q: What if two shapes have the same area and the same perimeter? Does this mean they're the same shape?

A: Not automatically, no, you need more information. Many different shapes can have the same area and the same perimeter.

Q: More information? Like what?

A: Well, a good starting point is if you know what sort of shapes you're dealing with. This will tell you how many sides your shapes have, and how many of these are congruent.

As an example, if you're told you have two squares and they both have the same area, the squares must be congruent. This is because you calculate the area of a square by calculating (side length)^2. The only way the squares can have the same area is if their sides are the same length.

In general, if you know what type of shape you're dealing with, you can use the properties of the shape to help you deduce further facts.

BULLET POINTS

- A parallelogram is a four-sided shape whose opposite sides are parallel to each other.
- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Consecutive angles of a parallelogram are supplementary.
- To calculate parallelogram area, multiply the width by the height:
- To calculate parallelogram perimeter, add together the length of each side.

Edward changed his rates...

Edward was psyched that you figured out what was going on, and he's changed his pricing to include both the area *and* perimeter of the lawn.



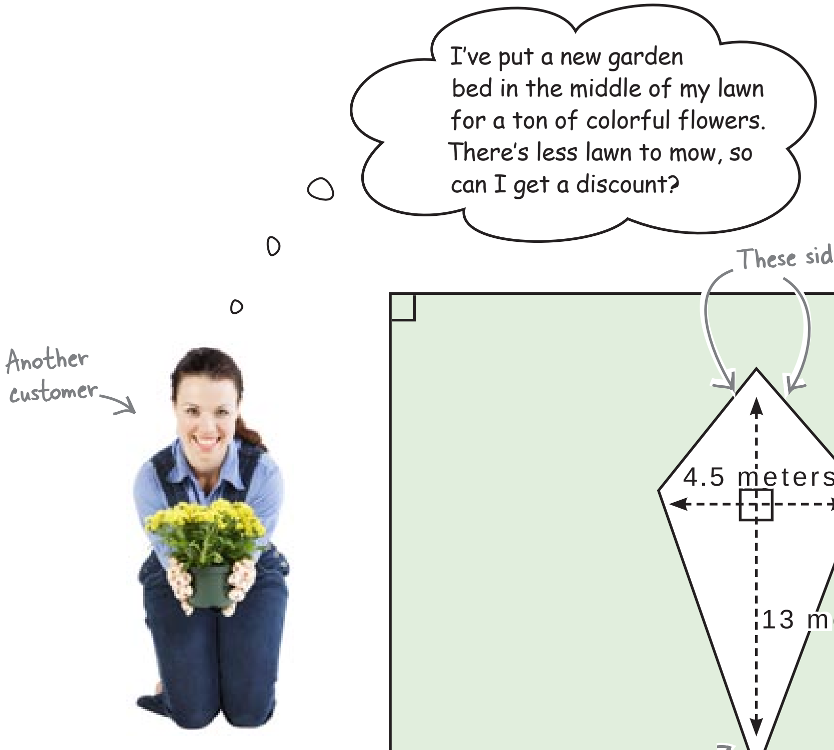
Edward's Lawn Service

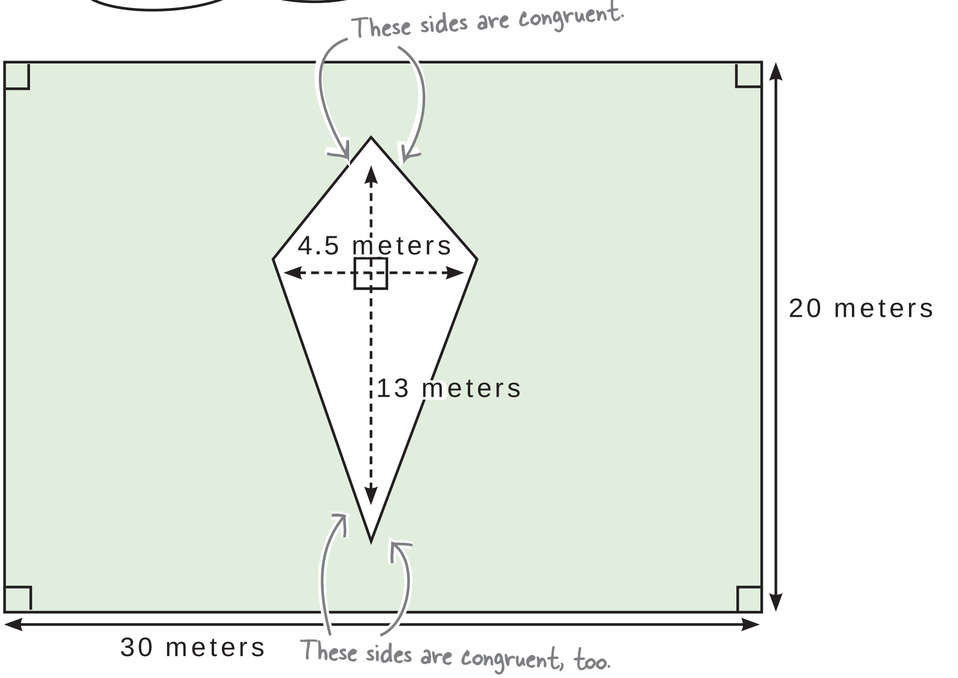
Lawn cutting cost - $0.05 per square meter

Lawn edging cost - $0.10 per meter

(Payable weekly)

...and the customers keep flooding in





Sharpen your pencil

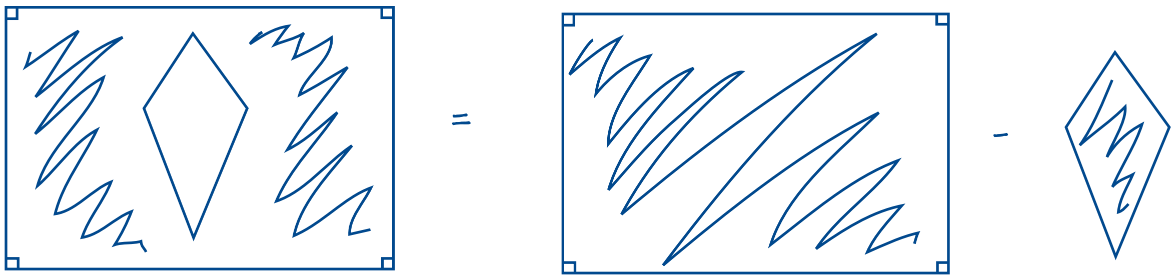
Check out the sketch of the lawn. How would you calculate just the area of the lawn, without the flower bed? Describe what you need to do in words and sketches, and assume you have all the measurements you need.

Sharpen your pencil Solution

Check out the sketch of the lawn. How would you calculate just the area of the lawn, without the flower bed? Describe what you need to do in words and sketches, and assume you have all the measurements you need.

Area of lawn = Area of the main lawn rectangle - Area of the flower bed

OR

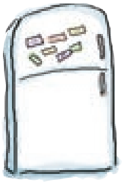


So to find the total area of the lawn we have to mow, we need to find the area of the lawn as though the flower bed wasn't there, and then subtract the area of the flower bed.



True, it isn't a rectangle or a parallelogram.

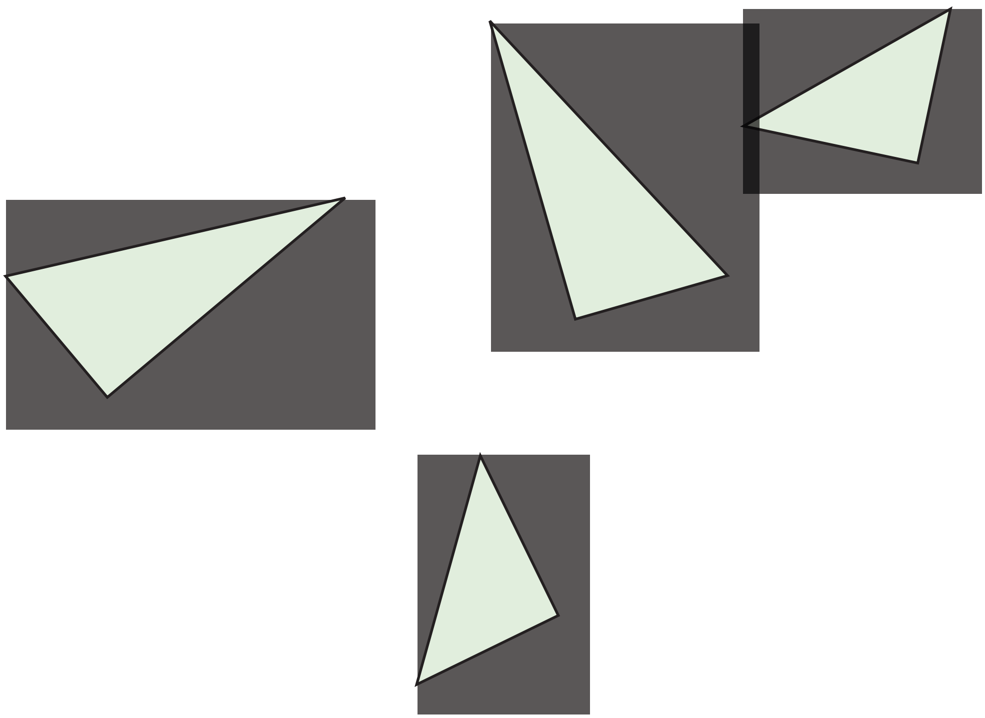
The opposite sides of the flower bed aren't parallel. Could you use a technique from earlier in this chapter to try to figure out the area of that shape?

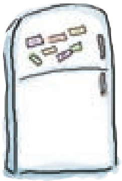


Geometry Magnets

Arrange the triangles below to form the same shape as the flower bed. Then see if you can rearrange them to form a rectangle. What's the base of this rectangle? What's the height? What does this tell you about how you might find the area of the flower bed?

Hint: you'll need to flip some of the shapes over to get the rectangle.

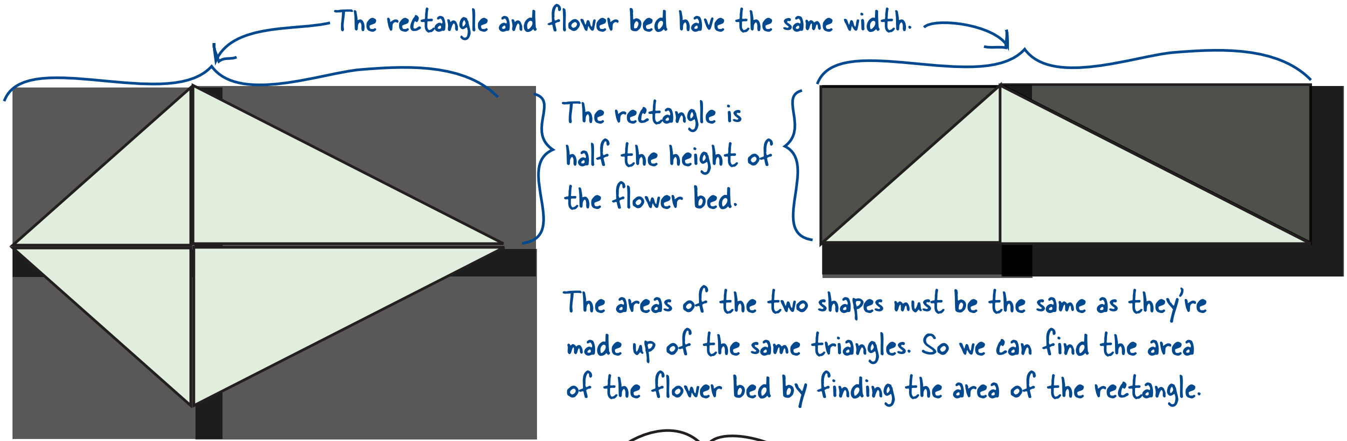


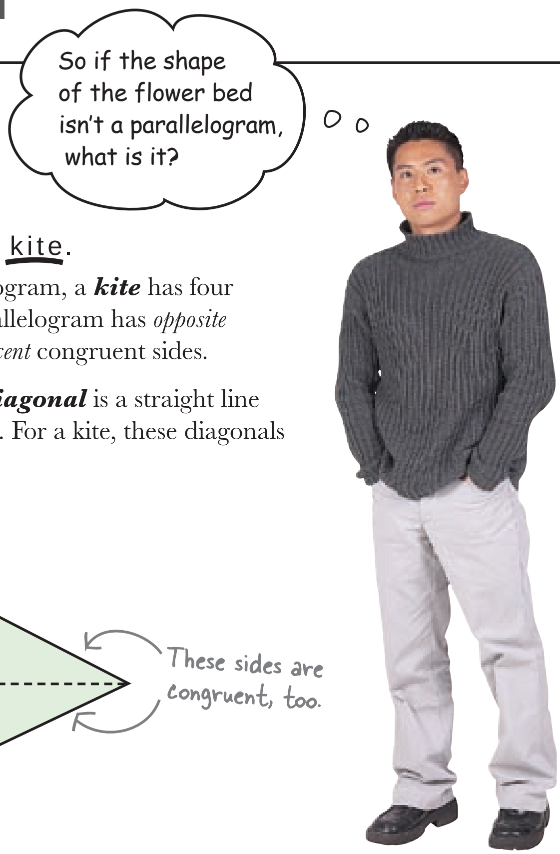


Geometry Magnets Solution

Rearrange the triangles below to form the same shape as the flower bed. Then see if you can rearrange them to form a rectangle. What's the base of this rectangle? What's the height? What does this tell you about how you might find the area of the flower bed?

Hint: you'll need to flip some of the shapes over to get the rectangle.

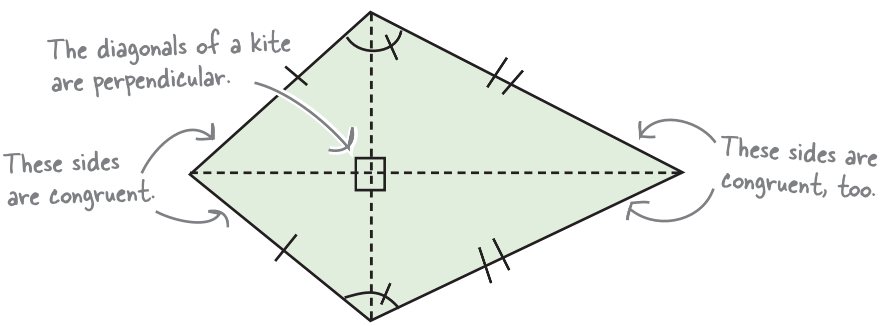




Just what it looks like: it's called a kite.

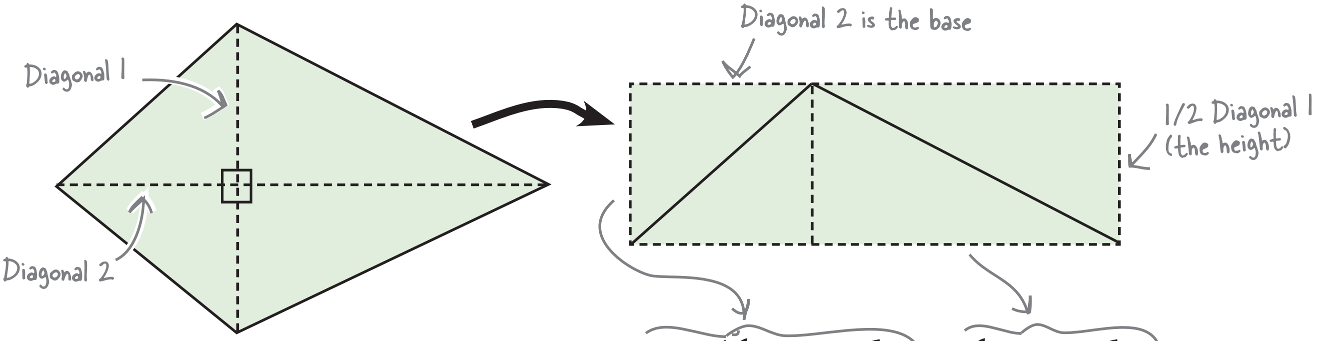
No crazy geometry jargon here! Like a parallelogram, a ***kite*** has four sides. There's one big difference though. A parallelogram has *opposite* congruent sides, but a kite has two pairs of *adjacent* congruent sides.

The diagonals of a kite are important, too. A ***diagonal*** is a straight line that connects one corner to the corner opposite. For a kite, these diagonals are always perpendicular to each other.



Use diagonals to find the area of the kite

Just like the parallelogram, there's a shortcut we can take to find the area of a kite. All we need to do is multiply the lengths of the two diagonals together, and divide by 2:

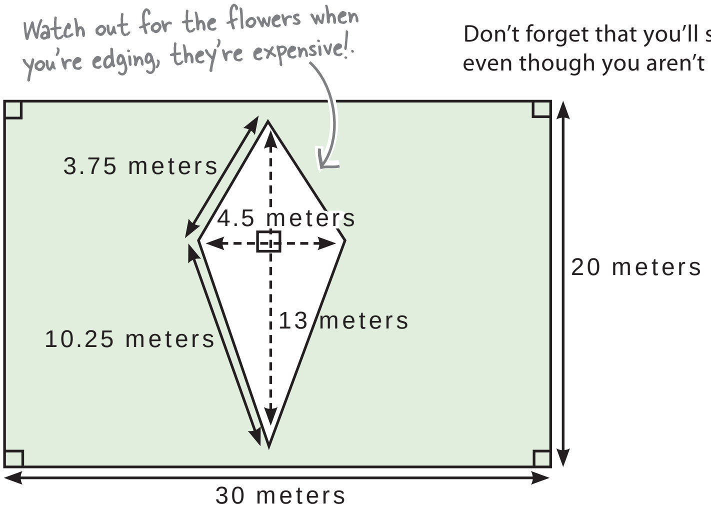


Kite Area = 1/2 (diagonal 1 × diagonal 2)

Sharpen your pencil

How much should you charge for maintaining this lawn? We've added in some extra measurements to help you.

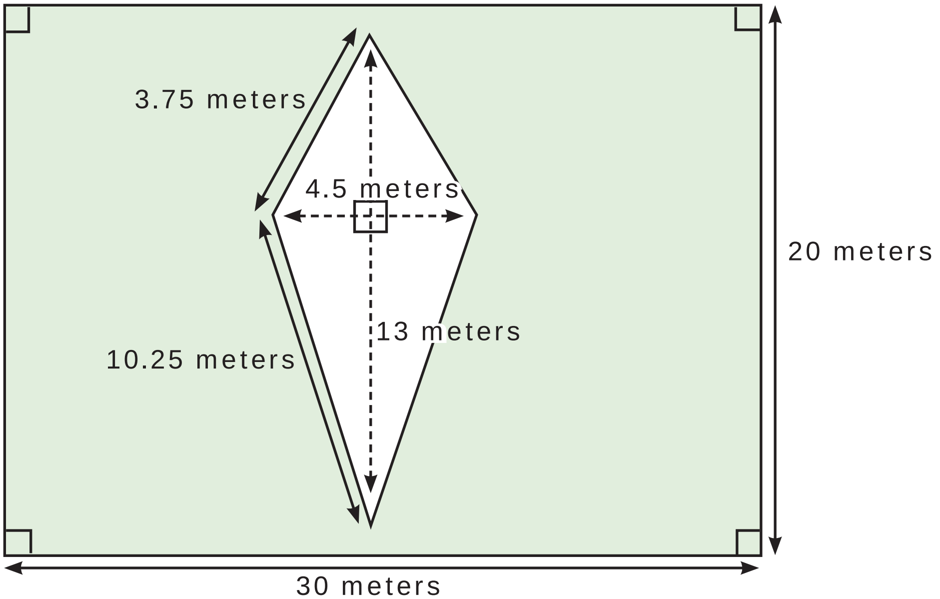
Don't forget that you'll still need to edge round the flower bed, even though you aren't mowing it.



Sharpen your pencil Solution

How much should you charge for maintaining the lawn? We've added in some extra measurements to help you.

Don't forget that you'll still need to edge round the flower bed, even though you aren't mowing it.



Area of rectangle $= 20 \times 30$

$= 600$ meters^2

Perimeter of rectangle $= (2 \times 20) + (2 \times 30)$

$= 40 + 60$

$= 100$ meters

Area of kite $= 1/2\ (4.5 \times 13)$

$= 1/2\ (58.5)$

$= 29.25$ meters^2

Perimeter of kite $= (2 \times 3.75) + (2 \times 10.25)$

$= 7.5 + 20.5$

$= 28$ meters

Total area to maintain $= 600 - 29.25$

$= 570.75$ meters^2

Total perimeter to maintain $= 100 + 28$

$= 128$ meters

Total charge $= \$0.05 \times 570.75 + \$0.10 \times 128$

$= \$28.54 + \$12.80$

$= \$41.34$

Round to 2 decimal places.

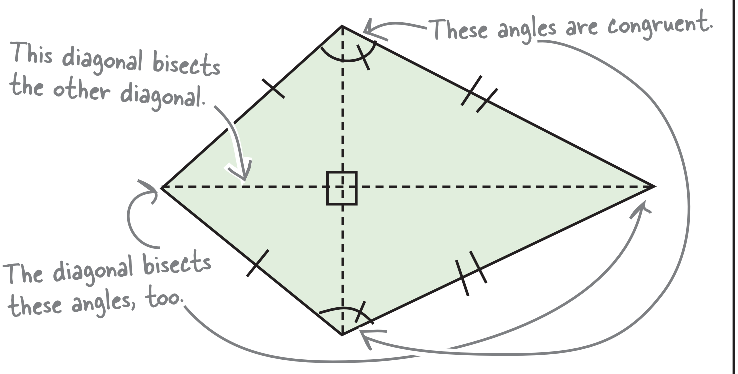
Kites Up Close



Diagonals

The diagonals of a kite are perpendicular, but that's not all there is to say about them.

For starters, one diagonal bisects the other, meaning it chops the other diagonal in half. It also bisects the pair of opposite angles, and if you look at the remaining pair of angles, they're congruent, too. So there's a lot you can know about a kite without having to do any calculations!



Area and perimeter

As you discovered earlier, you find the area of a kite by multiplying together the lengths of the two diagonals and dividing by two. To find the perimeter, remember that there are two pairs of congruent sides so you only have to add the two different sides together and multiply by two.

there are no Dumb Questions

Q: So the diagonals of a kite are perpendicular. What about the diagonals of a parallelogram, are they perpendicular, too?

A: In general, parallelograms don't have perpendicular diagonals.

The diagonals of a parallelogram are still important though. If a shape is a parallelogram, then its diagonals bisect each other. Try adding diagonals to the parallelograms earlier in the chapter and you'll see what we mean.

Q: Could I have calculated the area of the kite by splitting it into simpler shapes like before?

A: You could, but it would have taken you much longer to calculate. All you really need to do is multiply the two diagonals together and divide the result by 2.

Q: The kites we've looked at in this chapter look symmetrical. Is that a coincidence?

A: No, not at all. Every kite is symmetrical along one diagonal.

Q: Can a shape be both a parallelogram and a kite?

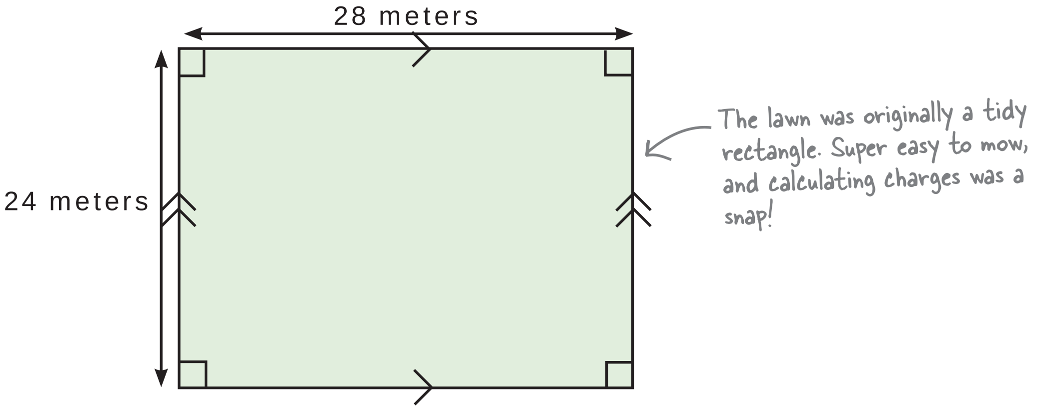
A: Yes it can. A shape is a parallelogram and a kite if it fits the description of both. In other words, it must have two pairs of separate adjacent congruent sides, and also the opposing sides must be parallel. This means that all four sides must be congruent.

An example of a shape that is both a parallelogram and a kite is a square. All four sides are congruent, and opposite sides are parallel. We'll get to that in a little bit....

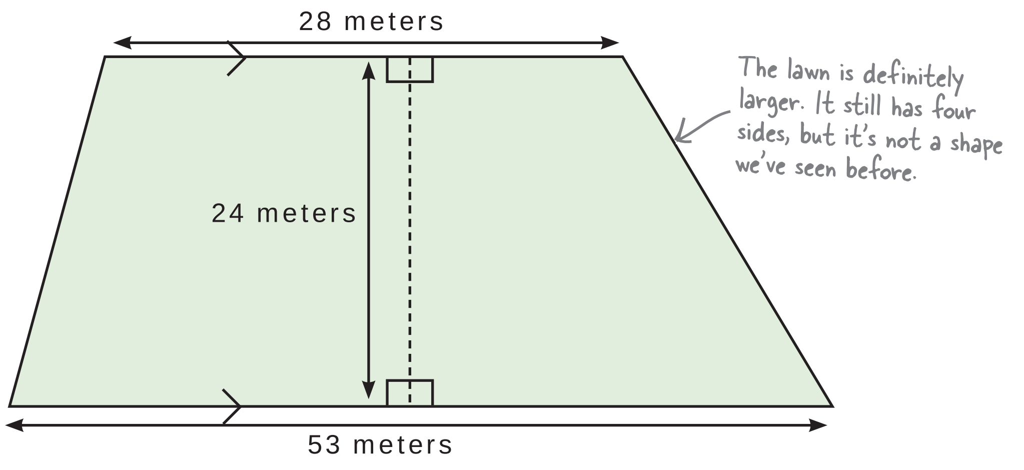
Landowners, unite

Just when all the lawns seemed under control, Edward ran into a little snag. He turned up at a customer's house to find that he has been buying up adjacent land. The neat rectangular lawn that Edward had been mowing for months has been transformed into…something else.

Before:



After:



It's going to take longer to mow and edge this new lawn, and Edward needs a hand working out the charges. He took some rough measurements and gave them to you over the phone, but he didn't get all the sides. So what do we do with a lawn like this?

Sharpen your pencil

Check out the shape of the new lawn. How do you think we could go about finding the area of a shape like this?

You've **GOT** to be kidding! This is way too easy, we've done all this before. All we need to do is split the shape into a rectangle and two triangles, and find the total area. Duh!

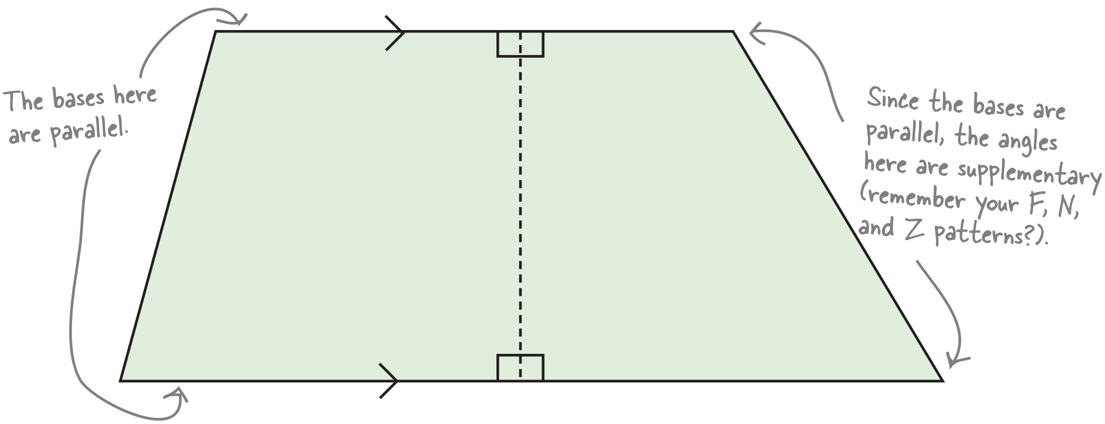


Really?

We can't do exactly the same thing we did before because we don't have all the measurements we'd need. Let's take a closer look.

There are some familiar things about this shape

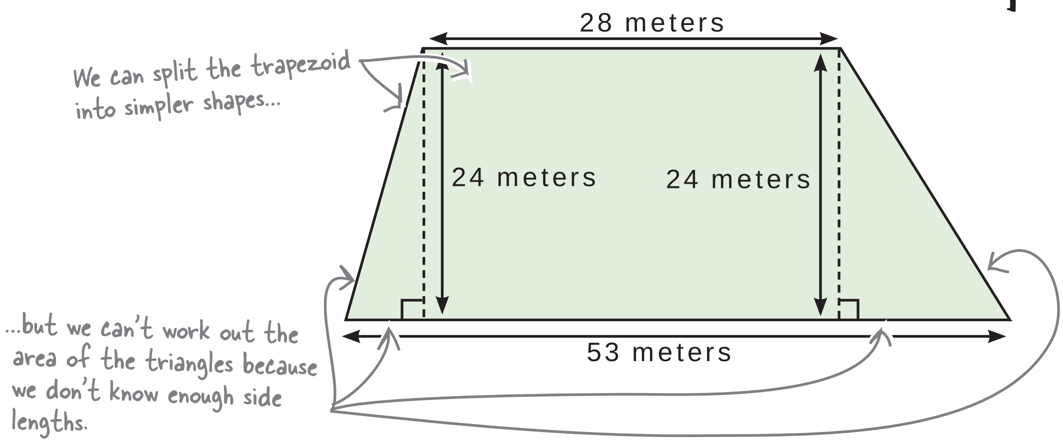
It's a four-sided shape, but it only has one pair of parallel sides. These parallel sides are called ***bases***. We call this shape a ***trapezoid***.



But what's the area?

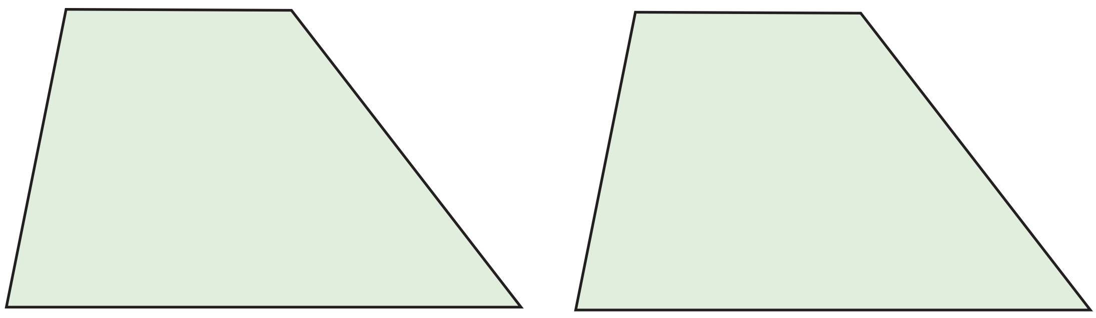
In theory, we could split the trapezoid into a rectangle and two right triangles, and then add together the area of each shape. The trouble is, we don't have enough information to do this. We know that they're both right triangles, but we don't know what the other two angles are, and we only know the length of one of the sides. So what can we do instead?

A trapezoid has exactly one pair of opposite parallel sides.



Sharpen your pencil

Instead of splitting the shape up, let's add to it and see if that helps us figure out how to find the area of a trapezoid. Here are two congruent trapezoids. Draw them together so that they form a single parallelogram.



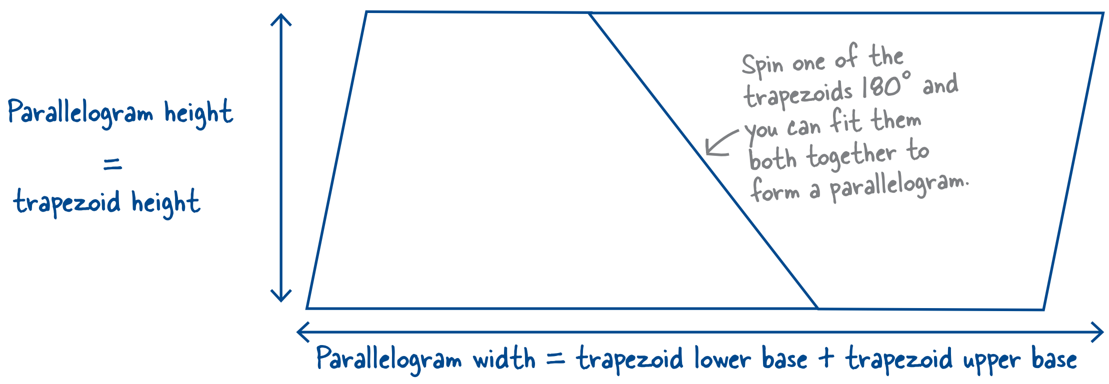
What's the width of the parallelogram?

What's the height of the parallelogram?

What ideas does this give you about how to calculate the area of a trapezoid?

Sharpen your pencil Solution

Here are two congruent trapezoids. Draw them together so that they form a single parallelogram.



What's the width of the parallelogram?

The trapezoid upper and lower bases added together.

What's the height of the parallelogram?

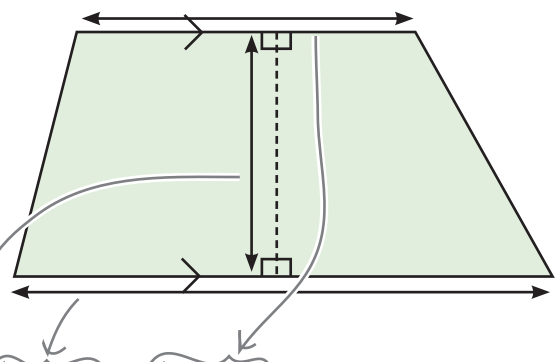
The trapezoid height

What ideas do you have about how to calculate the area of a trapezoid?

The parallelogram is made up of two trapezoids. This means that the area of each trapezoid must be half that of the parallelogram, and we know that the area of a parallelogram is equal to height x width.

Calculate trapezoid area using base length and height

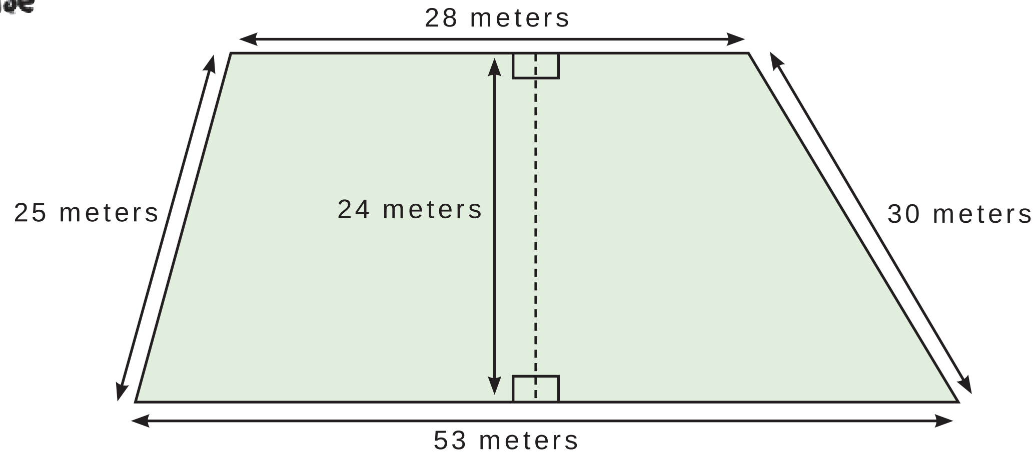
So what we've discovered is that, like with the other shapes you've encountered so far, there's a shortcut we can take if we want to find the area of a trapezoid. All we need to do is add together the upper and lower base lengths, multiply by the height, and then divide by two.



$$\text{Trapezoid Area} = \frac{\text{height}\ (\text{base } 1 + \text{base } 2)}{2}$$

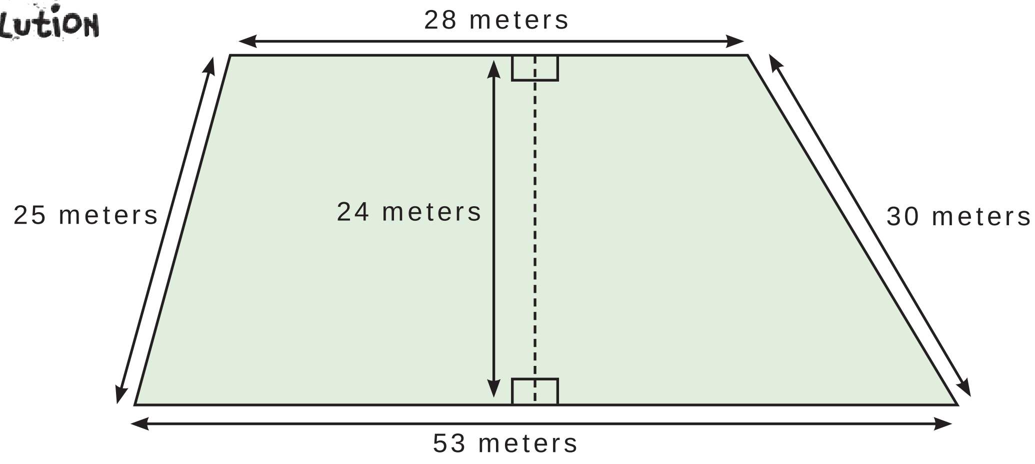


What's the charge for maintaining the trapezoid lawn? Thankfully, Ed went back and collected some extra measurements to help you out with the perimeter.



Exercise Solution

What's the charge for maintaining the trapezoid lawn? Thankfully, Ed went back and collected some extra measurements to help you out with the perimeter.



Trapezoid area = 1/2 × 24 × (28 + 53)

= 12 × 81

= 972 meters^2

Trapezoid perimeter = 28 + 30 + 53 + 25

= 136 meters

Maintenance charge = \$0.05 × 972 + \$0.10 × 136

= \$48.60 + \$13.60

= \$62.20



The Trapezoid Exposed

This week's interview:
What's So Special About Trapezoids?

Head First: Hey, Trapezoid, nice to have you on the show tonight.

Trapezoid: It's a real pleasure. I don't seem to get out as much as some of the other shapes.

Head First: What do you mean?

Trapezoid: Well, everyone knows about Square and Rectangle, although they're kind of boring if you ask me. Kite gets good press because it looks like… well…a kite. But me? Everyone forgets about me.

Head First: I notice that you didn't mention Parallelogram. Don't you guys have something in common?

Trapezoid: Some would say we're family, and I guess you could say we're kinda like cousins. I have only one pair of parallel sides, and Parallelogram has two. But don't you think that's just being greedy? Parallelogram's just a lopsided, funny-looking Rectangle if you ask me, and I don't limit myself like that.

Head First: But aren't you just like Triangle but with one of the points chopped off?

Trapezoid: That's one way you could talk about me, yes, but the key thing is that I have one pair of parallel sides, and Triangle doesn't. Um, I wouldn't talk to Triangle about that, he's still a bit sensitive.

Head First: But don't you have one nice thing in common with Triangle?

Trapezoid: You did your research, didn't you? We've all heard of an Isosceles Triangle, right? Well, Trapezoids can be Isosceles, too.

Head First: What's special about that?

Trapezoid: It's still a Trapezoid, but with congruent legs. And by legs I mean the sides that aren't bases. Isosceles Trapezoid is a bit more regular than me, there's more symmetry about him. He's thinking about getting into modeling, from what I hear. At the very least, it must make shaving much easier.

Head First: What about diagonals?

Trapezoid: What about them? Sure, I have them, but I can't say much about them. Now Isosceles Trapezoid, he has congruent diagonals, and he has some congruent angles, too. Some guys have all the luck.

Head First: Well, that's all we have time for, Trapezoid, thanks for stopping by.

BULLET POINTS

- A trapezoid is a four-sided shape with one pair of parallel sides called bases.
- Since the bases are parallel, this means that you have two pairs of supplementary angles.
- An isosceles trapezoid is a trapezoid with congruent "legs." The lower base angles are congruent, the upper base angles are congruent, and the diagonals are congruent, too.

The quadrilateral family tree

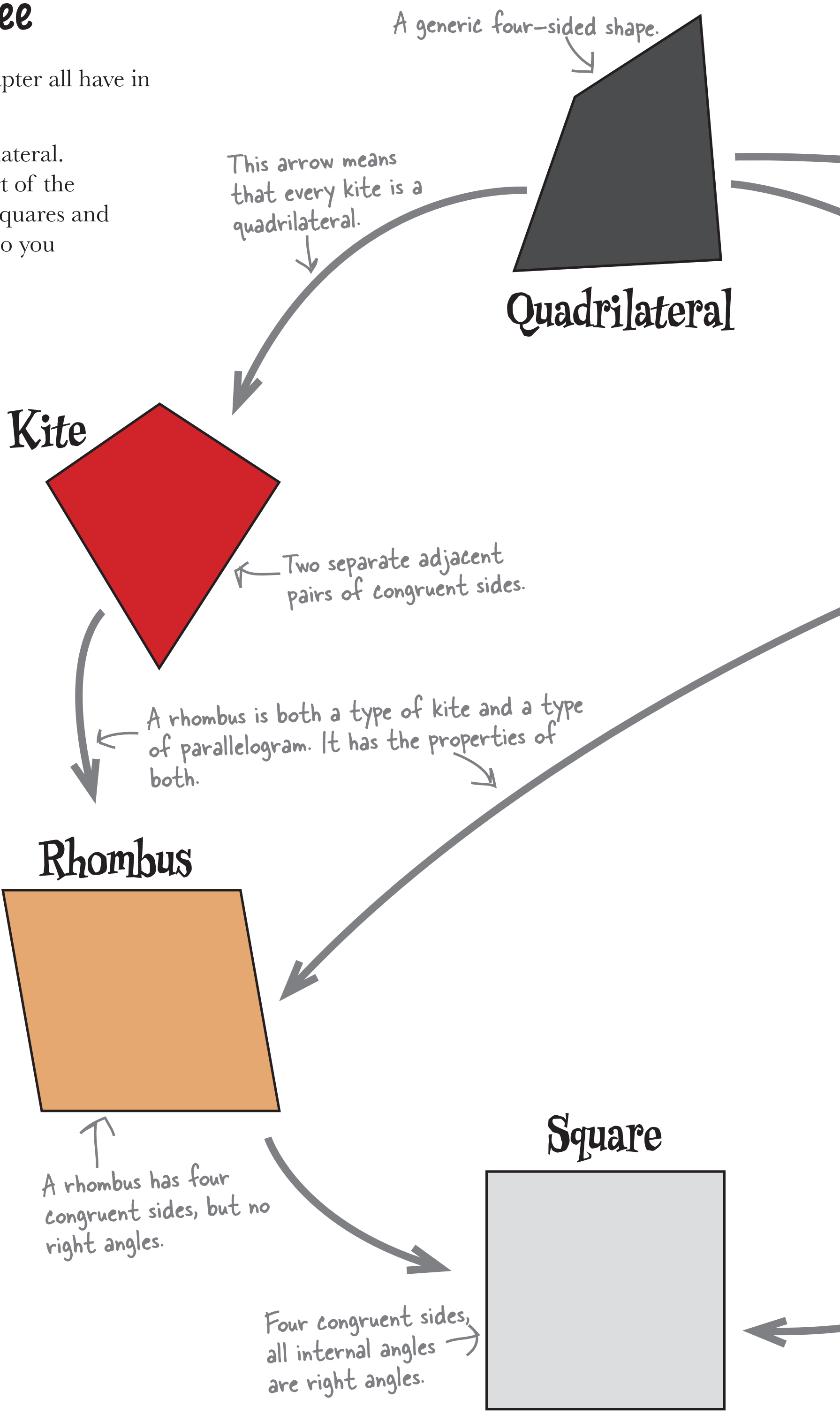
There's one key thing that the shapes in this chapter all have in common—they all have four sides.

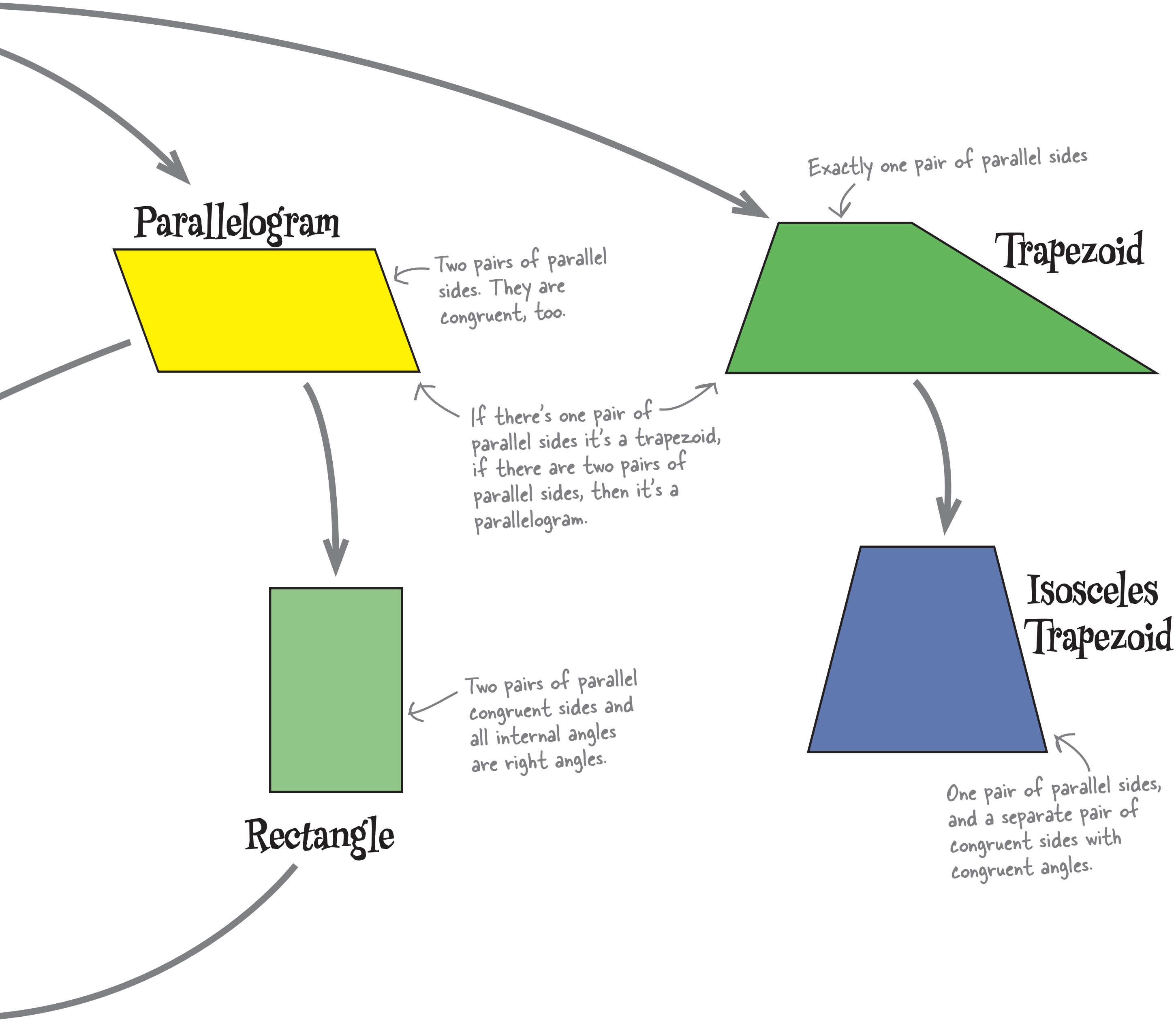
Any shape that has four straight sides is a quadrilateral. Parallelograms, kites, and trapezoids are all part of the quadrilateral family, along with shapes such as squares and rectangles. Here's the quadrilateral family tree so you can see how they're all related.

The arrows show you the hierarchy

The thick arrows on the family tree show you the relationships between the shapes. Any shape at the end of an arrow is a more specialized form of the shape that the arrow is coming from. So a kite is a type of quadrilateral since it shares the same properties as a quadrilateral—it has four sides, with two pairs of separate adjacent sides. Similarly, a square is type of rhombus, which means it is also a type of kite.

The family tree also shows you which shapes are *not* directly related. That means a kite is not a type of parallelogram and a rectangle is not a type of trapezoid.




Parallelogram
Two pairs of parallel sides. They are congruent, too.
Trapezoid
Exactly one pair of parallel sides
If there's one pair of parallel sides it's a trapezoid, if there are two pairs of parallel sides, then it's a parallelogram.
Rectangle
Two pairs of parallel congruent sides and all internal angles are right angles.
Isosceles Trapezoid
One pair of parallel sides, and a separate pair of congruent sides with congruent angles.

there are no Dumb Questions

Q: So how does knowing which different quadrilaterals relate to each other help me?

A: Some shapes can be classified as more than one type of quadrilateral, and if you know which groups each shape belongs to, you can use the properties of each group to help you calculate things like area and perimeter.

Q: But surely a shape can only be in one group at a time?

A: No, it's really a hierarchy. As an example, a square isn't just a square, it's also a kite as it has two pairs of adjacent sides.

Q: But all four sides are the same length.

A: Yes, but because it has two pairs of adjacent sides that are equal, that makes it a kite, too. Another way of looking at it is that a kite with four right angle corners where the sides are all the same length is called a square.

Q: But how does this really help me?

A: If a shape belongs to a group then you can find the area and perimeter using the formulas for that group. As an example, suppose you have a square and all you know about it is the length of the diagonals. How would you go about finding the area? One approach would be to calculate the sides of the square using the length of the diagonal. The area would then be the square of the side length. A simpler approach, however, would be to remember that a square is also a kite, and use the kite formula to find the area. In other words, multiply the diagonals together and divide by 2.

Q: Oh, I get it. If I can remember how all the quadrilaterals relate to one another, I can save time calculating things like area.

A: Absolutely!

Q: So what's a rhombus?

A: A rhombus is quadrilateral where all four sides are the same length. It's a bit like a skewed square.

Q: Oh, I see. So does that mean that a square is a rhombus, too?

A: Well done! Yes, that's totally right. But remember, the reverse isnt necessarily true. Just because a square is a rhombus, it doesn't mean that every rhombus is also a square.

Q: So is a trapezoid a type of parallelogram?

A: A parallelogram has to have two pairs of parallel sides, and a trapezoid has to have exactly one pair of parallel sides. This means that a trapezoid is not a parallelogram, as trapezoids don't have two pairs of parallel sides.

Q: The family tree shows that a parallelogram is not a type of trapezoid. In another book, I saw a diagram where the two are shown as related. Why's that?

A: That's an interesting question.

We've defined a trapezoid as having *exactly* one pair of parallel sides. Since a parallelogram has *two* pairs of parallel sides, it doesn't meet the requirements for being in the trapezoid club. It has too many pairs of parallel sides.

Sometimes other people define a trapezoid as having *at least* one pair of parallel sides. If you define a trapezoid in this way, then a parallelogram is a type of trapezoid, as it has more than one pair of parallel sides.

Q: I've heard of a trapezium, too. What's one of those?

A: It all depends where in the world you are.

In the U.S., a trapezium is a quadrilateral with no parallel sides.

Outside the U.S., a trapezium is the name given to a shape with one pair of parallel sides—what people in the U.S. call a trapezoid.

You've entered the big league

Thanks to your skill with quadrilaterals, Ed's lawn mowing business has really blown up.

We're raking the money in now, and it's all thanks to you. We've even been approached by the local fancy golf course for a multi-year contract. Want to be my business partner?



That means a much bigger cut of the profits for you!





Your Geometry Toolbox

You've got Chapter 6 under your belt and now you've added quadrilaterals to your toolbox. For a complete list of tool tips in the book, head over to www.headfirstlabs.com/geometry.

Quadrilateral

A flat shape with four straight sides.

Parallelogram

Has two pairs of parallel sides that are also congruent.

Opposite corner angles are congruent.

Area = height x base width.

Rectangle

Has two pairs of parallel congruent sides.

Every corner is a right angle.

Area = width x height.

Trapezoid

Has one pair of parallel sides called bases.

Area = 1/2 x height x (base 1 + base 2)

Rhombus

Has four congruent sides.

Is both a parallelogram and a kite—so you can use the area calculation of either shape.

Kite

Has two pairs of separate adjacent congruent sides.

Diagonals are perpendicular.

Area = 1/2 x diagonal 1 x diagonal 2

7 regular polygons

It's all shaping up



Want to have it your way?

Life's full of compromises, but you don't have to be restricted to triangles, squares, and circles. **Regular polygons** give you the **flexibility** to demand exactly the shape you need. But don't think that means learning a list of new formulas: *you can treat 6-, 16-, and 60-sided shapes the same*. So, whether it's for your own creative project, some required homework that's due tomorrow, or the demands of an important client, you'll have the tools to **deliver exactly what you want**.

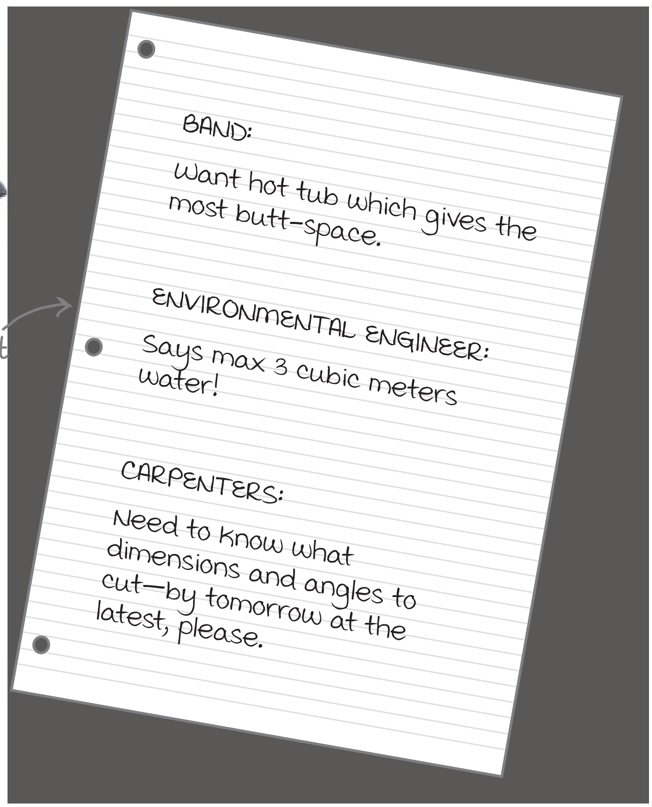
We need to choose a hot tub



Choosing a hot tub? Now, that sounds like a job that should be super easy. But of course there's a catch: everybody's got an opinion on what this hot tub needs to be like…

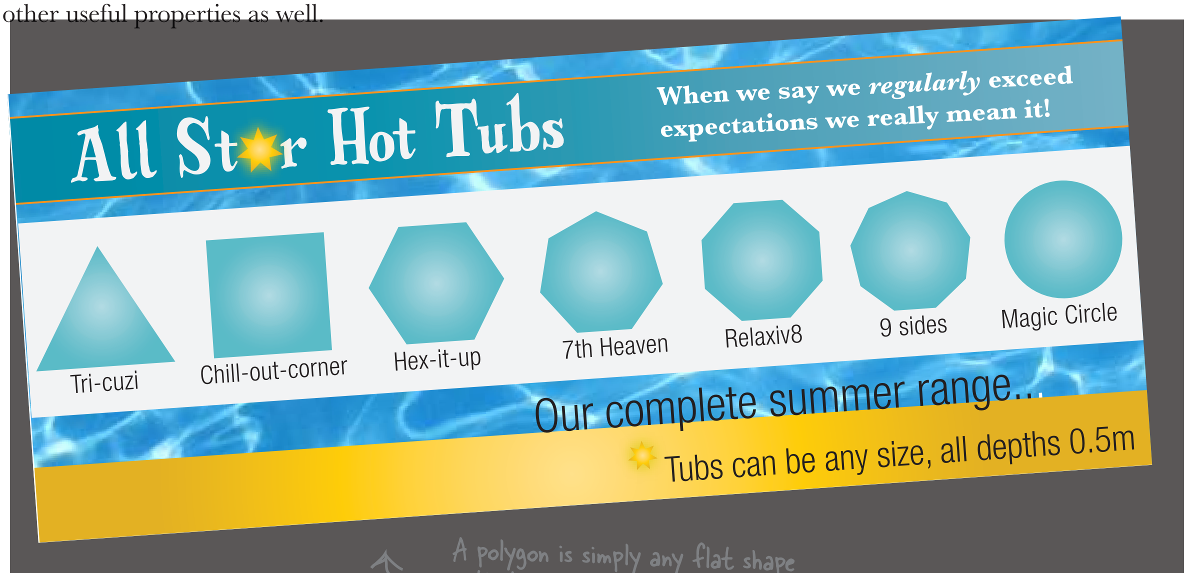
…and they need it all sorted out by tomorrow.

A bunch of different people each have requirements about this hot tub.

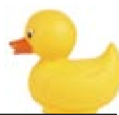


All the hot tubs are regular polygons

The local hot tub suppliers have said they'll rush through an order for any hot tub in their range—all of which they describe as ***regular polygons***. The word ***regular*** is another way of saying equilateral—all the sides are the same length—but it means the polygons have some other useful properties as well.



A polygon is simply any flat shape with three or more sides. You've already worked with a bunch of them, like triangles and quads.



Sharpen your pencil

What properties of the hot tubs do you need to work out to satisfy:

1. The bands? ....

2. The environment engineer? ....

3. The carpenters? ....

....

The carpenters need to make an appropriately sized hole in the floor for the tub to go in.

How does it help you that the tubs are all ***regular*** polygons?

....

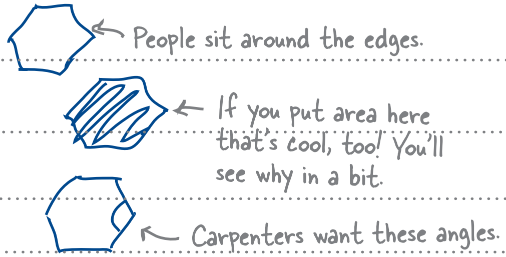
....

Sharpen your pencil Solution

What properties of the hot tubs do you need to work out to satisfy:

1. The bands? *Perimeter*
2. The environment engineer? *Volume*
3. The carpenters? *Internal angles*

   Side lengths



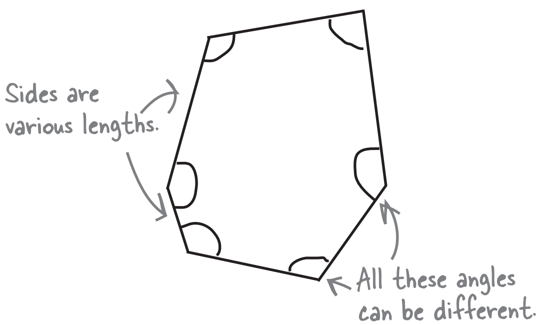
How does it help you that the tubs are all ***regular*** polygons?

Since all the sides on a regular polygon are the same length, then all the angles are equal too, so we've only got to find one side and one angle—all the others will match.

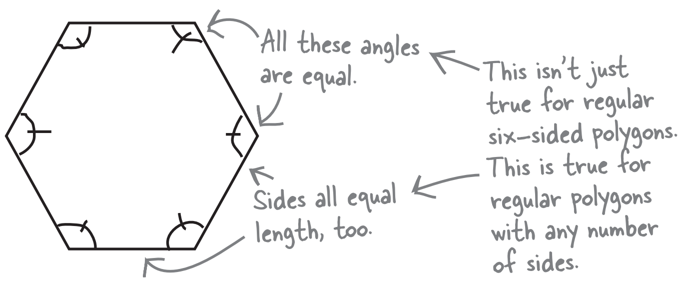
Regular polygons have equal sides and angles

Any six-sided polygon could have six different side lengths and six different angles. But fortunately for us, all the hot tubs are *regular* polygons. A six-sided regular polygon has six sides of equal length and six equal angles.

Six-sided polygon:

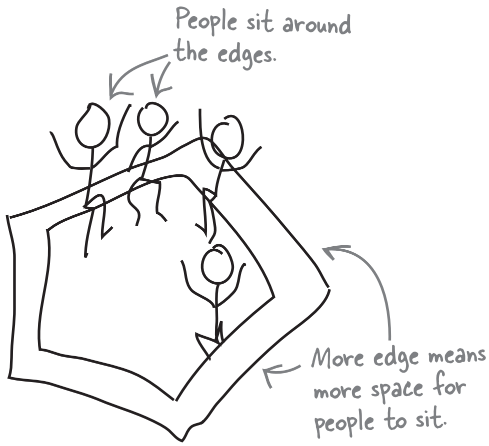


REGULAR six-sided polygon:



Butt-space is all about perimeter

The number of people who can sit in the tub at the same time depends on the perimeter of the tub. More perimeter = more butt-space = more people.



So we need the hot tub with the biggest perimeter? But we've also got to limit the water...so it's not as simple as just "bigger is better," is it?



That's true—in fact this hot tub problem is a lot deeper than it first appears.

First, you've got to compare perimeters of polygons which are all different shapes, and second you've got to work to a limited amount of water…3 cubic meters.

BRAIN POWER

Is there a pattern you can use to work out the perimeters of your regular polygon shaped hot tubs? What would you need to know first?

Is 3 cubic meters of water a lot or a little?

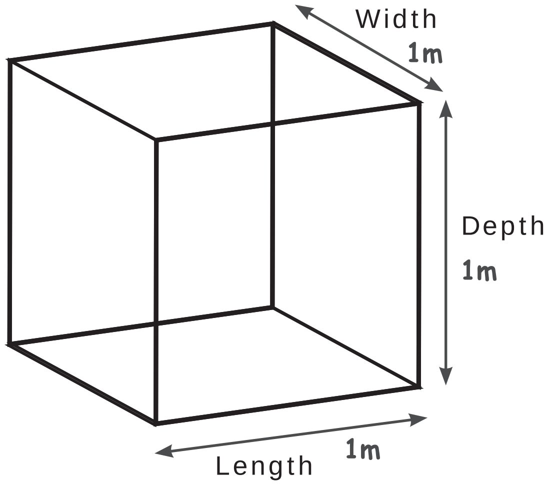
Typical engineer. Instead of giving you the amount of water in terms of a measure that might actually make some sense, like cups or spoonfuls or gallons, he's talking about ***cubic meters***. So, how much is 3 cubic meters of water? A bathful? A pondful? A swimming poolful?

Let's start by think about what just 1 cubic meter actually is:

Volume of a cube = length x width x depth

Cubic meter = 1m x 1m x 1m

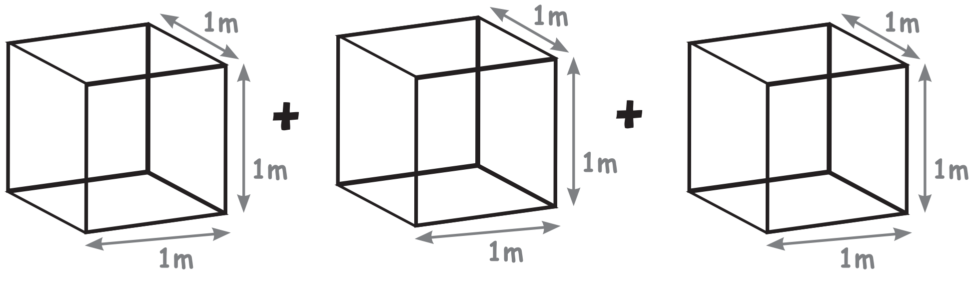
= $1m^3$



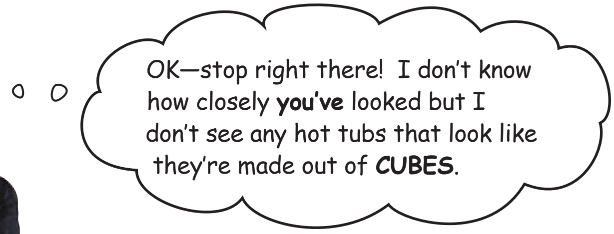
A cube-shaped container with length, width, and depth all of one meter holds exactly one cubic meter of water.

So 3 cubic meters of water is the volume of water you could store in three of those one meter cubes:

3 cubic meters =





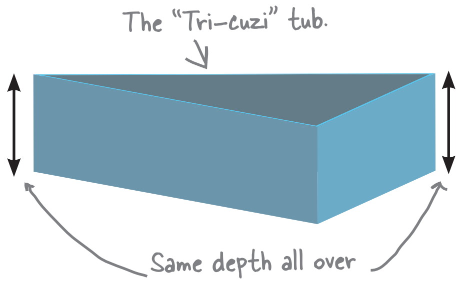


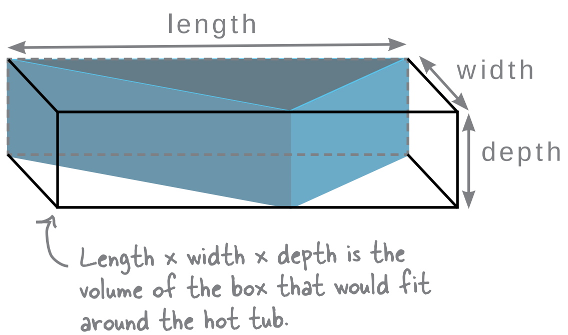
That's true.

In fact, thinking about cube shapes can only take us so far, because, aside from the square-shaped one, the hot tub range isn't made out of cubes.

How do the hot tubs compare to a cube?

Like a cube, all the hot tubs have a uniform depth. They also have straight sides at a right angle to the base. But for most of the shapes of hot tub, calculating the volume using ***length* × *width* × *depth*** would give you the volume of a box that you could fit the hot tub inside, not the water that the hot tub would contain. It might be handy, but it's not what we're looking for!





Volume of a hot tub = ? × depth

What tool from your Geometry Toolbox could we use to calculate the volume, instead of "length × width"?

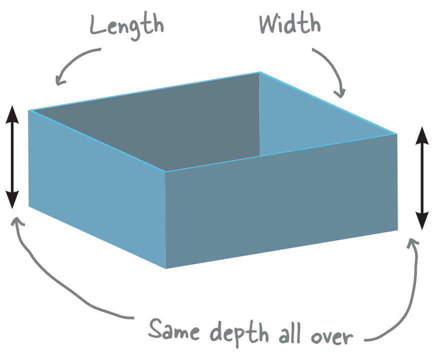
Hot tub volume is area x depth

Calculating the volume of some 3D shapes can get pretty gnarly, but for the hot tub range it's simple. The tubs have straight sides, so the volume can be found from area × depth.

These "straight-sided" shapes are known as prisms—more about those in Head First 3D Geometry.

[Thanks! Marketing xx]

AREA = length x width



there are no Dumb Questions

Q: This book is called *Head First 2D Geometry*, right? So how come we're talking about 3D in this chapter?

A: Volume is definitely a 3D topic, and we cover it in much more detail in *Head First 3D Geometry*, but it's not too bad to dip your toes in the water is it? Also, we're about to turn this problem back into a 2D one on the next page.

Q: You're going to turn a 3D problem into a 2D problem? How does that work?

A: The third dimension in the hot tubs problem is depth. Once we don't have to work with the depth anymore it's just a 2D problem we're left with. Hold that thought to the bottom of the next page.

Q: What if the tub was deeper at one end than the other? Or had curved sides?

A: The *area* × *depth* formula only applies to 3D shapes with straight sides, all of the same depth, which are perpendicular to the base. If the hot tub was deeper at one end we'd need a different way of working out its volume.

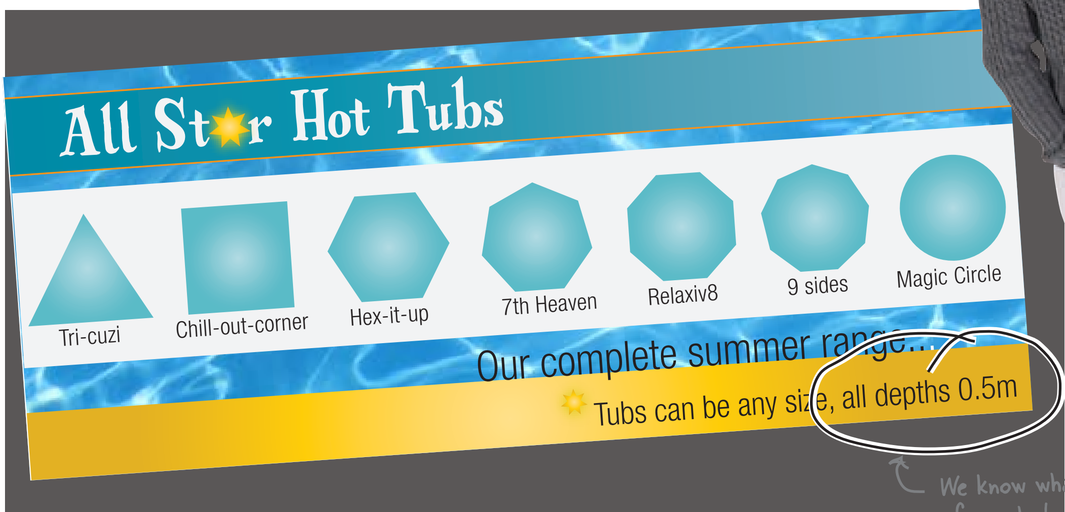
Ok—so all the hot tubs have the same depth. Does that help simplify the problem?



Yes. All the depths are the same, so only the area of the hot tubs varies.

The All Star Hot Tubs range is a fixed depth: 0.5 meters.

You can have your tub wider and longer, but you can't have it deeper.





BRAIN BARBELL

The maximum volume allowed is 3 cubic meters, and all the tubs are 0.5 meters deep. What's the maximum area our hot tub can have?

Brain Barbell Solution

The maximum volume allowed is 3 cubic meters. Handily, all the tubs are 0.5 meters deep. What's the maximum area our hot tub can have?

Volume = Area × Depth → $3 = \text{Area} \times 0.5$

$$\frac{3}{0.5} = \text{Area}$$

Hope you remembered the units! → 6 square meters = Area ← And now you've got a 2D problem...phew!

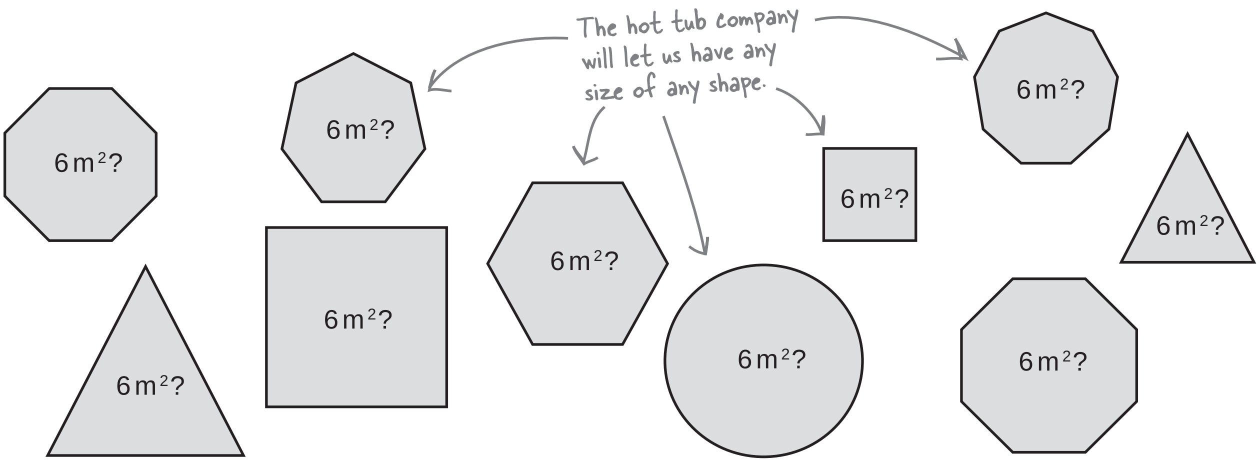
The hot tub's area must be 6m²

Technically, anything less than or equal to six would be OK with the engineers, but we want max butt-space, so we want to push it to the limit.

Whichever shape of tub we choose, the area of the hot tub must be no more than 6 square meters. That way we know for sure that it'll only need 3 cubic meters of water to fill it, and the environmental engineers will be happy.

As will we! Saving water is kind of important.

The shape and size of the tub is completely flexible—it's up to us to work out what size and shape of tub will give us 6 square meters of area and also meet the band's requirement: maximum butt-space!



Which hot tub shape gives the most butt-space?



Jill: No—even for triangles that didn't work—remember?

Joe: Oh, yeah. Well, I guess we just have to divide up the work then.

Frank: What work?

Joe: Well, we'll have to do a bunch of calculations for each shape—like try with the sides at 1 meter, 2 meters, 3 meters, and so on. Working out the area and perimeter for each. Find the ones closest to 6 square meters of area and then compare them and maybe refine the calculations....

Jill: Whoa. No way. That's a ton of work! We'll never get it done for tomorrow.

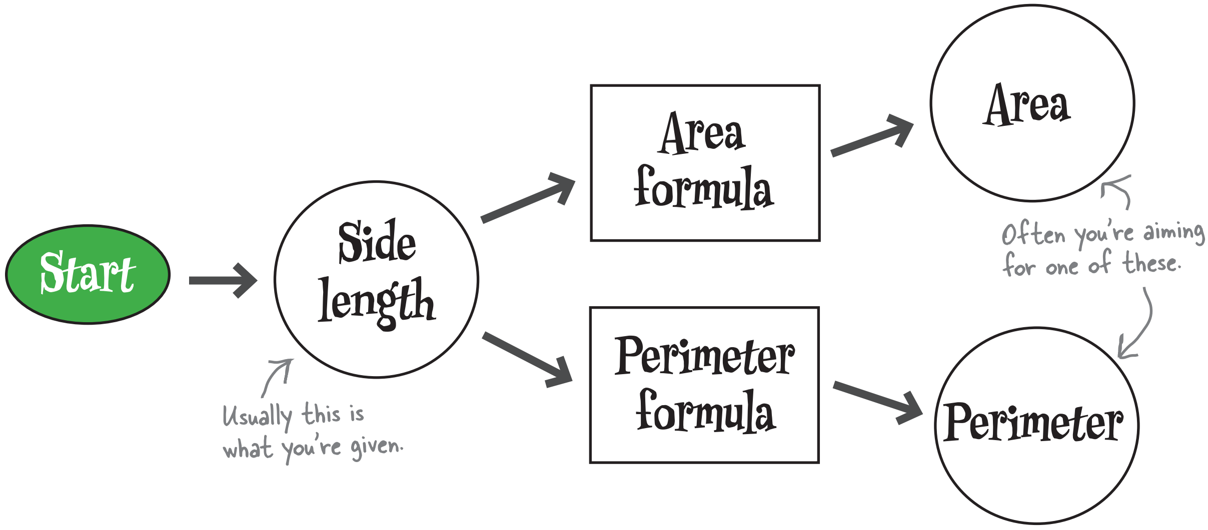
Frank: I don't think we need to do it that way anyway. Do you think we could use the formulas we know for area, and sort of rewind them to find the side lengths?

Joe: Rewind them? Rewind a formula? What?

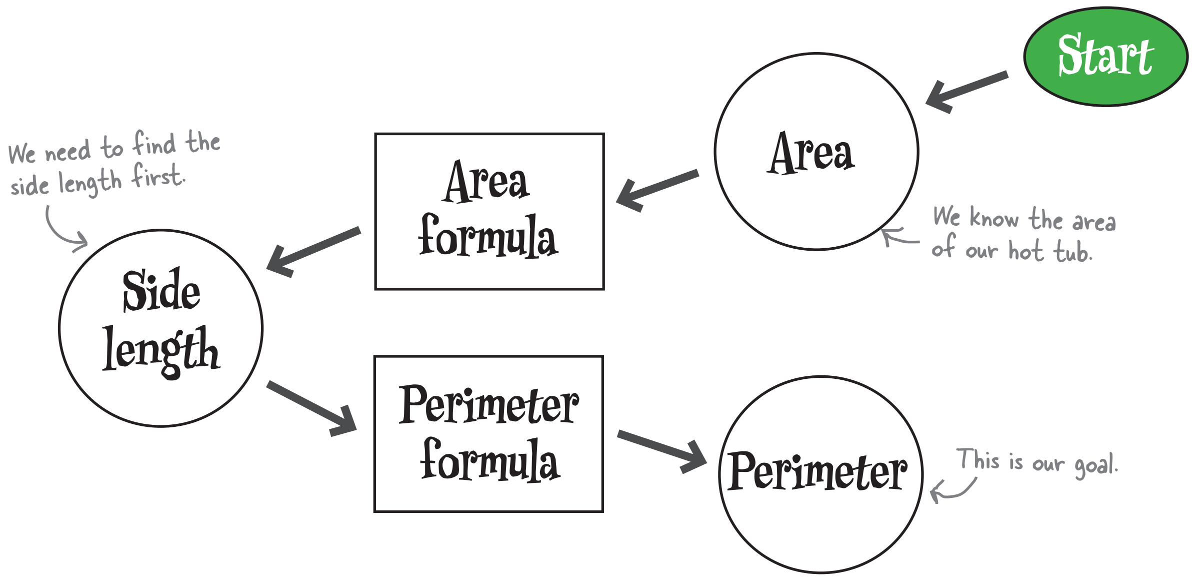
What do YOU think Frank means? Would it work?

Work backward from area to find butt-space

Normally, you're given the lengths of the sides of a shape and you push them through a couple of formulas to get area and perimeter:



But in this case, you know your hot tub *area*, and you need to find the length of the sides that would give that area, so you can use them to find the perimeter (butt-space). To do this you need to work backward.

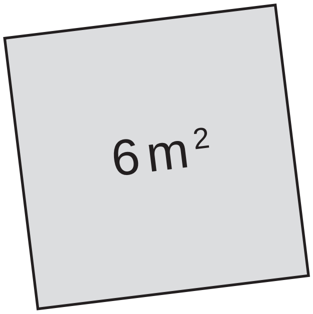


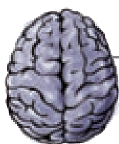


Use any formula you know for area of a square to find the side lengths of the square hot tub with 6 square meters area.

Then use the side length to find the perimeter.

If each butt requires 0.5 meters of perimeter, how many butts fit in this square hot tub?





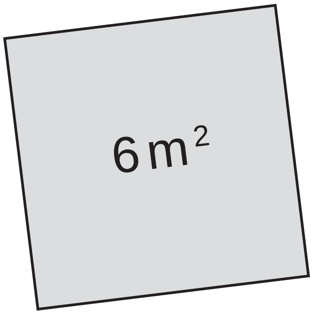
Reviewing isn't cheating! Quads area formulas: page 272.

Exercise Solution

Use any formula you know for area of a square to find the side lengths of the square hot tub with 6 square meters area.

Then use the side length to find the perimeter.

If each butt requires 0.5 meters of perimeter, how many butts fit in this square hot tub?



Area of a square = side length × side length

$6 = s^2$

If you had width × height that works too—but in a square the width and height are the same so it's even easier!

$\sqrt{6} = s$

2.45 meters = s ← *Length of each side.*

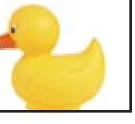
Perimeter of a square = 4s

Perimeter = 4 × 2.45 = 9.8 meters

Units help you keep track of what each number means.

Each butt takes 0.5 meters

total butts = $\frac{9.8}{0.5}$ = 19.6 butts



there are no Dumb Questions

Q: Half a meter per butt? Really?

A: OK, so we all know that butt-size is one of life's interesting variables. Obviously not all butts are the same size, but in this problem that's not too important. Even if you had a bunch of people with more generously proportioned butts, you'd still fit more into the biggest perimeter, so it's still a useful comparison.

Q: 19 butts or 19.6 butts? I've never met anyone with 0.6 of a butt.

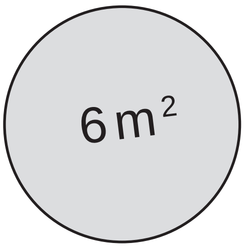
A: That's true, you'd really round down to 19. Only whole butts can get in the tub (unless you count a really skinny one as a 0.6). That 0.6 gives a little extra room to the 19 folk in there though, so it's worth keeping those decimals in case we end up choosing between two tubs which both have butt counts between 19 and 20.

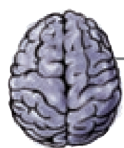
Is 19.6 butts a lot or a little?

One number on its own isn't enough to choose a hot tub—you want the MOST possible butt-space for your 6 square meters of area. You need to tackle the other hot tubs shapes to find out if they're better or worse than the square tub.

Sharpen your pencil

Work the formula for circle area backward to get the butt count for the circle shaped hot tub.

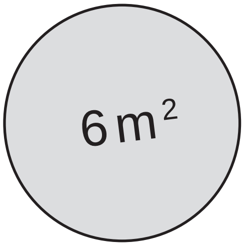




Reviewing isn't cheating! Circle area formulas: page 234.

Sharpen your pencil Solution

Work the formula for circle area backward to get the butt count for the circle shaped hot tub.



Circumference in the case of a circle

$$\text{perimeter} = 2\pi r$$

$$\text{Butts} = \frac{\text{perimeter}}{0.5}$$

$$\text{circle area} = \pi r^2$$

$$6 = \pi r^2$$

$$\frac{6}{\pi} = \frac{6}{3.1415} = r^2$$

$$1.9m^2 = r^2$$

$$1.38m = r$$ ← radius

$$P = 2 \times 3.1415 \times 1.38$$

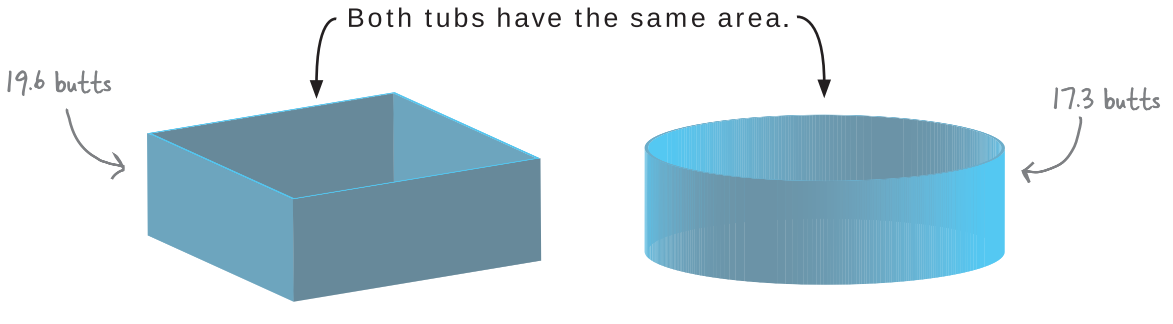
$$P = 8.67m$$ ← That's m for meters, not a new variable!

$$\text{Butts} = \frac{8.67}{0.5} = 17.3 \text{ butts}$$

Or 17.4 butts if you didn't round as you worked—either is OK.

The square tub beats the circle tub

So, when it comes to providing butt-space for a 3 cubic meter volume of water, the square shaped hot tub is better than the circle shaped one.

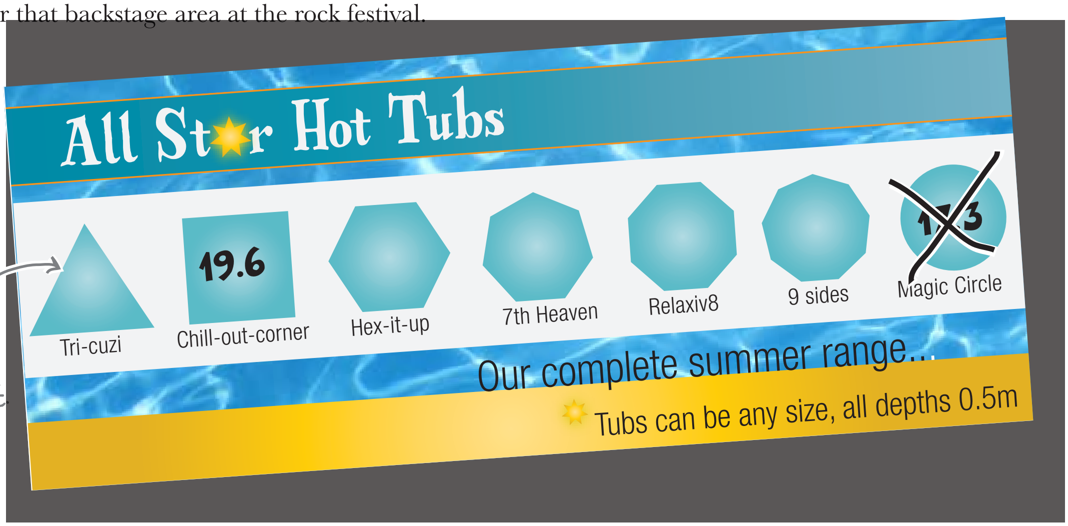


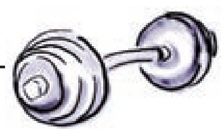
Both tubs are the same depth.

Two tubs down, five to go

If there were only two tub designs your work would be pretty much done, but there are another five shapes of hot tub to compare in order to find the perfect hot tub for that backstage area at the rock festival.

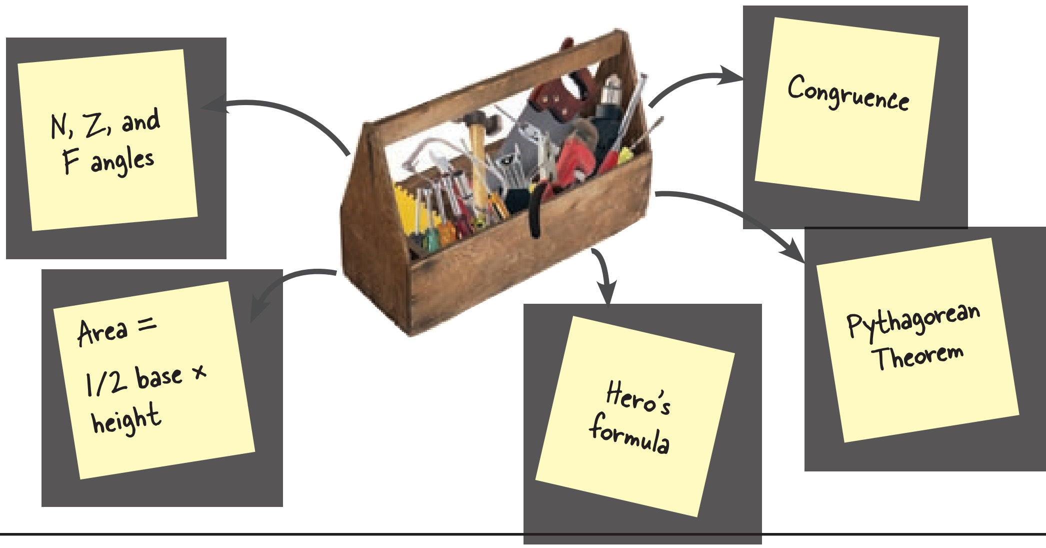
Let's take the triangle hot tub next.

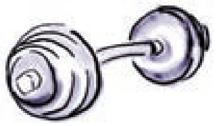




BRAIN BARBELL

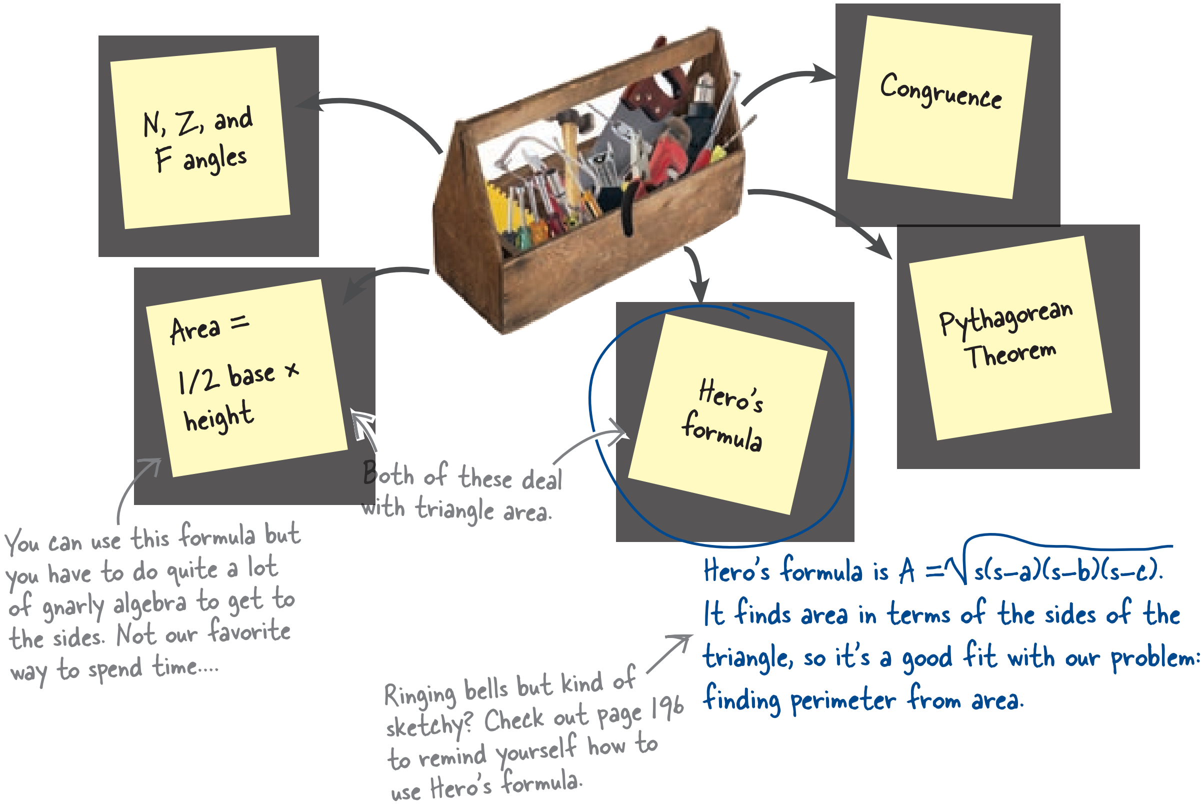
Let's work on the triangle tub next. You need to work backward from area to find side length. Which tool from your Geometry Toolbox is the best one for this job?





Brain Barbell Solution

Let's work on the triangle tub next. You need to work backward from area to find side length. Which tool from your Geometry Toolbox is the best one for this job?



Hero's formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$. It finds area in terms of the sides of the triangle, so it's a good fit with our problem: finding perimeter from area.



Relax

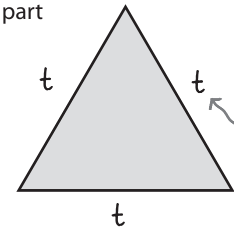
All your toolbox tools are arranged by subject in a downloadable PDF at headfirstlabs.com!

Working smarter, not harder is often about choosing the right tool for the job. Go to *www.headfirstlabs.com/geometry* to grab yourself a handy printable reference sheet of your geometry tools grouped by topic.

~~Pool~~ Hot Tub Puzzle



Use the algebra parts in the pool to find a special version of Hero's formula for your equilateral triangle-shaped hot tub. (You should use each part exactly once.)



Hero's formula already has a letter s in it, so let's call our triangle side "t."

Hero's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ & $s = \frac{a+b+c}{2}$

a, b, and c are sides of a triangle. In our case all the sides are t.

$A = \sqrt{\ldots\ldots\ldots\ldots}$ & $s = \frac{\ldots\ldots}{2}$

substitute for s

(s-t) = ... = ...

substitute for (s–t)

$A = \sqrt{\ldots\ldots\ldots\ldots}$

$A = \sqrt{\ldots\ldots\ldots}$

This is the formula for the area of an equilateral triangle.

A = ...

Use these for the final step.

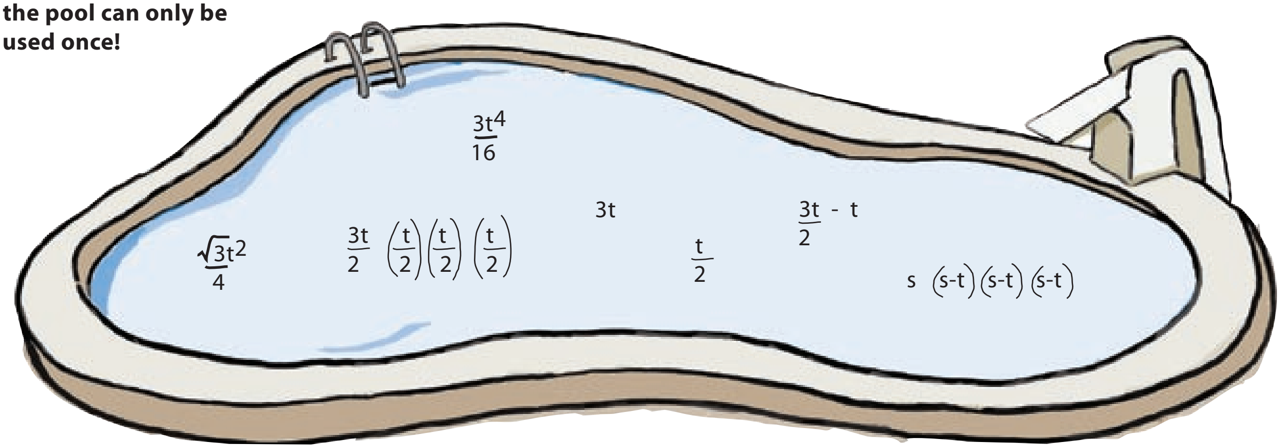
Square roots hints

$\sqrt{16} = 4$

$\sqrt{3} = \sqrt{3}$

$\sqrt{t^4} = t^2$

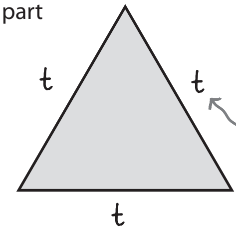
Note: each thing from the pool can only be used once!



Hot Tub ~~Pool~~ Puzzle Solution



Use the algebra parts in the pool to find a special version of Hero's formula for your equilateral triangle-shaped hot tub. (You should use each part exactly once.)



Hero's formula already has a letter s in it, so let's call our triangle side "t."

Hero's formula: $A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$ & $s = \dfrac{a+b+c}{2}$

a, b, and c are sides of a triangle. In our case all the sides are t.

$A = \sqrt{s\,(s-t)(s-t)\,(s-t)}$ & $s = \dfrac{3t}{2}$

substitute for s

$(s-t) = \dfrac{3t}{2} - t = \dfrac{t}{2}$

substitute for (s–t)

$A = \sqrt{\dfrac{3t}{2}\left(\dfrac{t}{2}\right)\left(\dfrac{t}{2}\right)\left(\dfrac{t}{2}\right)}$

This is the formula for the area of EVERY equilateral triangle.

$A = \sqrt{\dfrac{3t^4}{16}}$

$A = \dfrac{\sqrt{3}t^2}{4}$

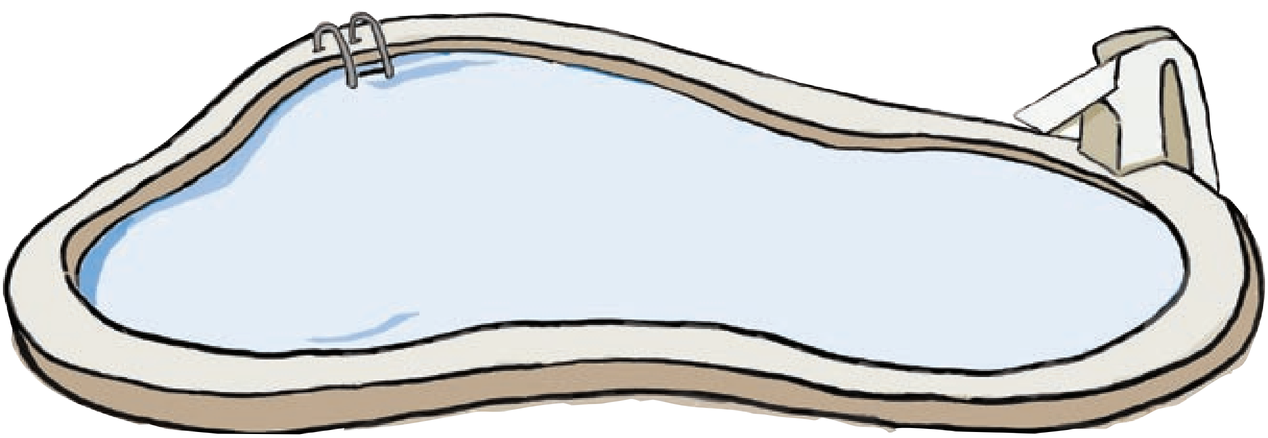
Use these for the final step.

Square roots hints

$\sqrt{16} = 4$

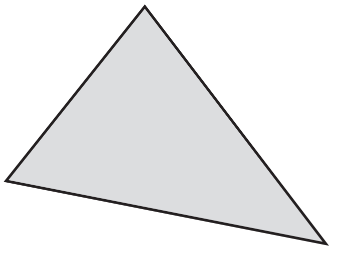
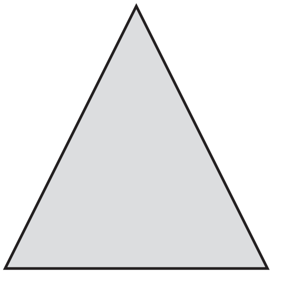
$\sqrt{3} = \sqrt{3}$

$\sqrt{t^4} = t^2$



You've found the formula for the area of an equilateral triangle

Recognizing special shapes is a great shortcut. Being able to spot that a quadrilateral is a square, or that a triangle is an equilateral triangle can save you a lot of work by letting you use extra tools and formulas which apply to that special shape specifically.

Any old triangle		Equilateral triangle
		
Longest must be shorter than other two sides added together	**Sides**	Are equal lengths
Add up to 180°	**Angles**	Are all 60°
1/2 base × height or Hero's formula	**Area**	$= \frac{\sqrt{3}}{4} t^2$

Just try drawing a triangle that breaks this rule!

Sharpen your pencil

Use your new equilateral triangle area formula to find the butt count of the triangular-shaped hot tub.

Sharpen your pencil Solution

Use your new equilateral triangle area formula to find the butt-count of the triangular shaped hot tub.

$$\text{Area} = \frac{\sqrt{3}t^2}{4} \longrightarrow 6 = \frac{\sqrt{3}t^2}{4}$$

$$\frac{24}{\sqrt{3}} = t^2$$

Length of each side → $3.72\text{m} = t$

$$\text{Perimeter} = 3t$$

$$= 3 \times 3.72$$

$$= 11.17\text{m}$$

$$\text{Butts} = \frac{11.17}{0.5} = 22.3 \text{ butts}$$

there are no Dumb Questions

Q: Why bother learning another formula? Won't I be covered if I just know Hero's formula and the "1/2 base × height" version?

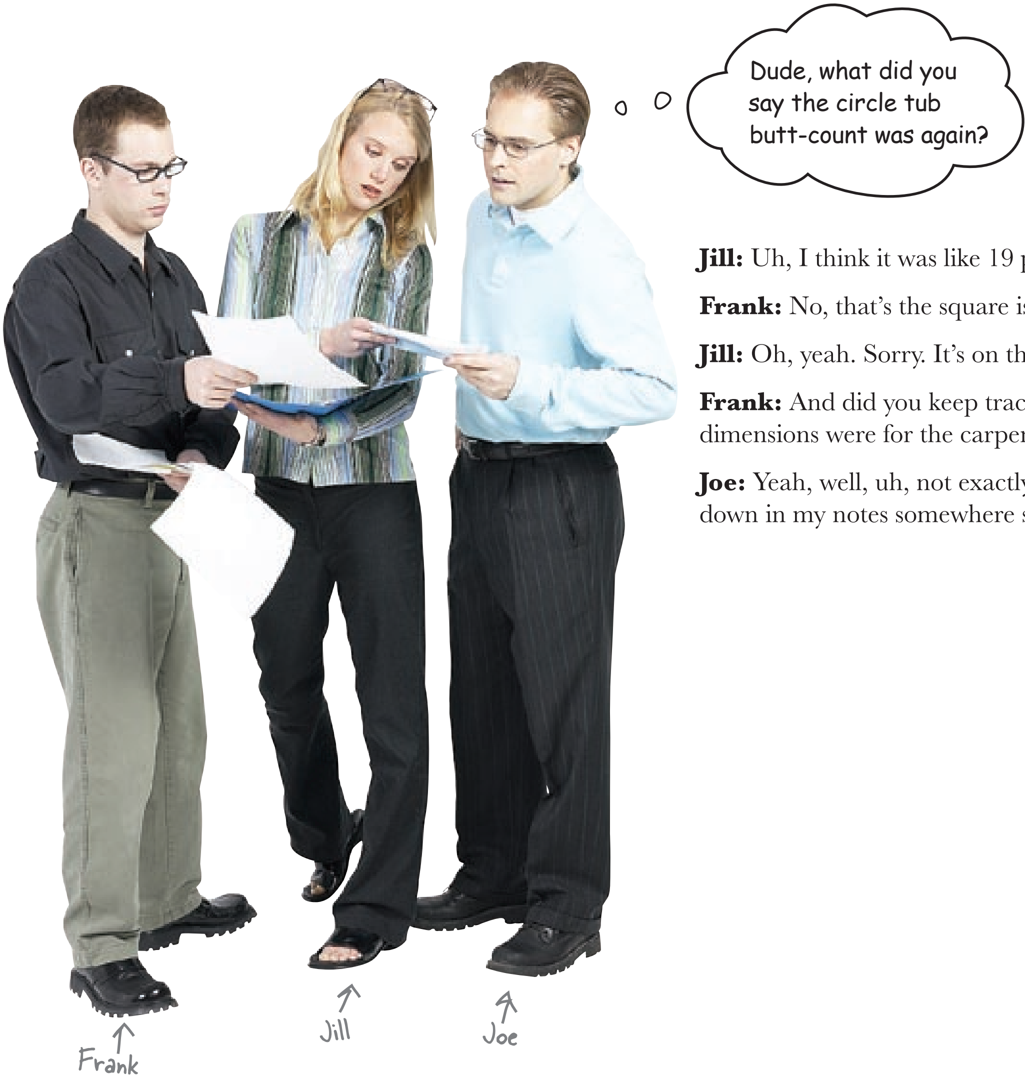
A: Any time you use Hero's formula to work on a equilateral triangle you're gonna wind up doing all the math we just did anyway. If you can learn the equilateral triangle area formula then you won't need to do all that algebra again!

Q: But what if I wanted to use "1/2 base × height." I thought in geometry it didn't matter how you work stuff out, the answer should be the same?

A: That's true, and starting from 1/2 base × height, you'd have ended up with the exact same formula. You just have to do more work to get there. First you'd need to use the Pythagorean Theorem to find the sides in terms of the height, and then you'd need to do some simultaneous equations to put it all together. It would definitely work but there's a lot more to go wrong.

Q: OK, so this formula only works for equilateral triangles? What about triangles that are nearly equilateral?

A: You could use it to approximate area for a triangle with sides of similar length, but it wouldn't be accurate. It's a good way to check your answer in an exam though. Pick the middle side length, work the equilateral triangle area formula and it should come out in the same ball park. If your two answers are 9 and 10, you're all good; if they're 9 and 50, you know you need to check your work.



Jill: Uh, I think it was like 19 point something.

Frank: No, that's the square isn't it?

Jill: Oh, yeah. Sorry. It's on this other piece of paper.

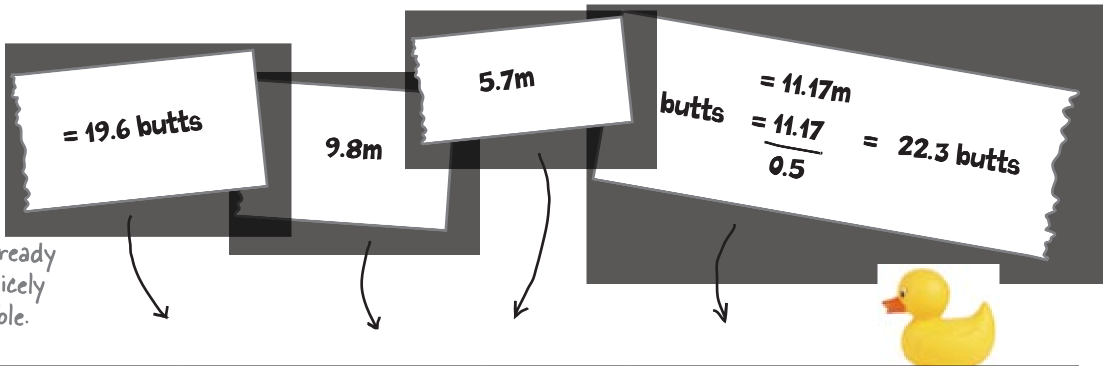
Frank: And did you keep track of what the dimensions were for the carpenters?

Joe: Yeah, well, uh, not exactly, but I did scribble it down in my notes somewhere so I must have it....

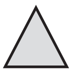
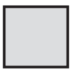
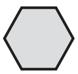
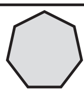
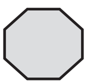
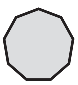
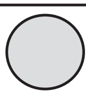
Hmm. This is getting messy fast. Do you think there's a more efficient way to keep track of the calculations for each hot tub?

Keep track of complex comparisons with a table

There's a whole bunch of stuff you're working out to pick the best hot tub. You need to keep track of side length and angles for the carpenters, as well as butt-space to choose the best tub for the band…and you need to know what calculation goes with which tub! A table can help you make sure all that information is right at your fingertips.



Everything you've already worked out can be nicely organized in this table.

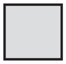
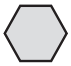
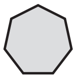
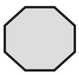
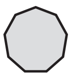
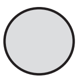
Model	Shape	Polygon	No sides	Perimeter	Butts	Angles	Side length
Tri-tub							
Chill-out-corner							
Hex-it-up							
7th Heaven							
Relaxiv8							
9 Sides							
Magic Circle							



Exercise

Let’s look for some quick answers here!

Based on the work you’ve already done, you should already be able to fill in all the boxes in this table except for the shaded ones.

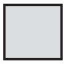
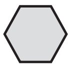
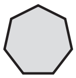
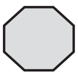
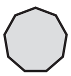
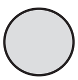
Model	Shape	Polygon Name	No sides	Perimeter	Butts	Angles	Side length
Tri-tub							
Chill-out-corner							
Hex-it-up							
7th Heaven							
Relaxiv8							
9 Sides							
Magic Circle							

These three might make your brain hurt a bit—don’t panic, we’ll talk more about them later.

Exercise Solution

Let's look for some quick answers here!

Based on the work you've already done, you should already be able to fill in all the boxes in this table except for the shaded ones.

Model	Shape	Polygon	No sides	Perimeter	Butts	Angles	Side length
Tri-tub		Equilateral Triangle	3	11.17m	22.3	60°	3.72m
Chill-out-corner		Square	4	9.8m	19.6	90°	2.45m
Hex-it-up		Hexagon	6				
7th Heaven		Septagon	7				
Relaxiv8		Octagon	8				
9 Sides		Nonagon	9				
Magic Circle		Circle	∞	8.67m	17.3	Nearly 180°	Nearly 0

If you need help remembering these, check out the "Nonagon" song by They Might Be Giants on YouTube. Goofy but genius.

That's the infinity symbol!

More on these later in the chapter.

Don't worry if these made your head hurt....

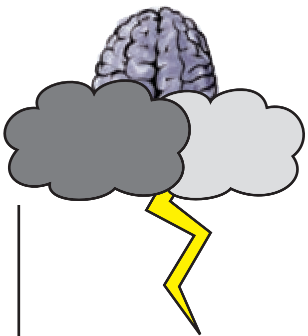
That's a pretty table and all, but it looks like you just ran straight into a **DEAD END**. Triangles, squares, and circles are one thing but helloooo—area of a septagon? We don't have formulas for the areas of any of those other shapes. And if we have to work them out one by one this could take forever. By tomorrow? Not gonna happen.



It's a fair point.

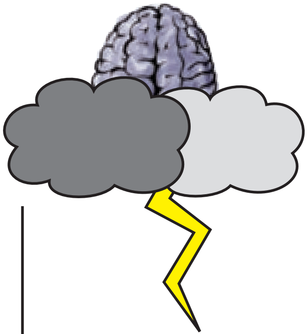
On the surface it looks like most of our table is filled in, but we're still a long way from choosing that ideal hot tub.

Time to change our strategy!



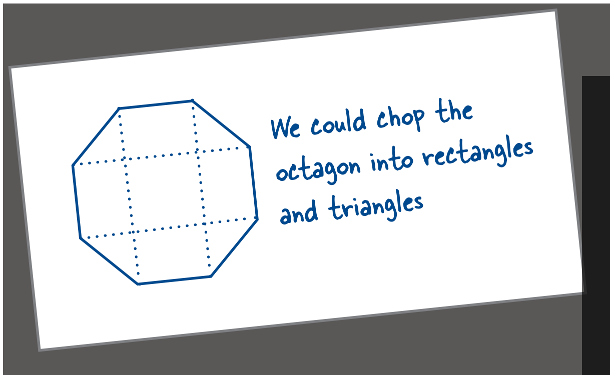
Brain Storm

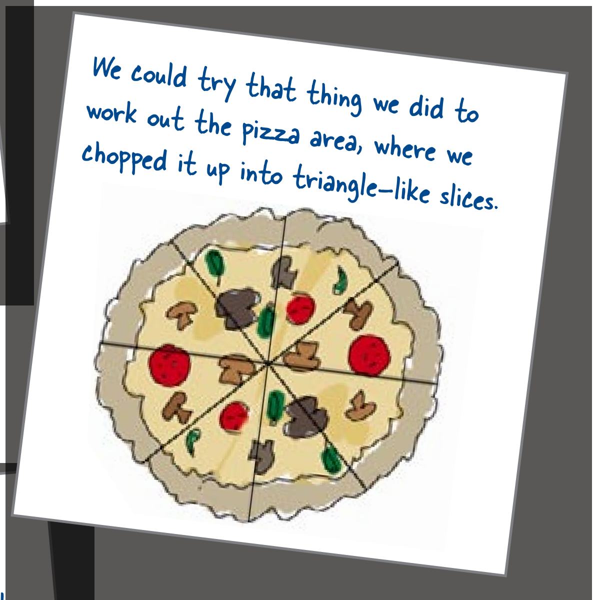
You need to pick a hot tub fast! Write down ANYTHING you can think of that could help you find the area of the 6-, 7-, 8-, and 9-sided tubs. No idea is too crazy. If you run out of space grab some paper. Give yourself at least 5 minutes, and then come and join our brainstorm results over on the next page.…



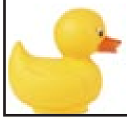
Brain Storm

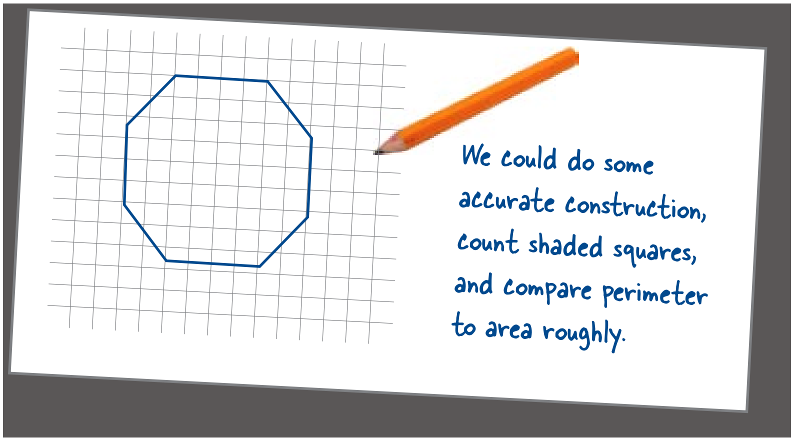
So what might help us find the area of the 6-, 7-, 8-, and 9-sided tubs? Here are the ideas we came up with, but you might have had different ideas, or extra ideas we didn't think of. The more, the merrier!

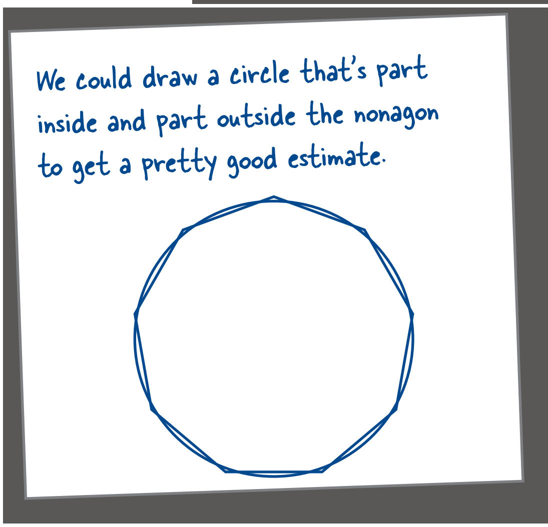


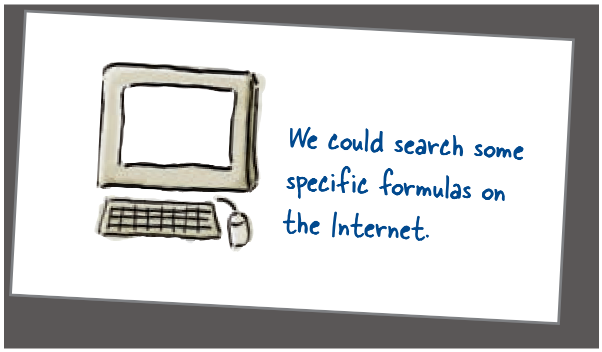










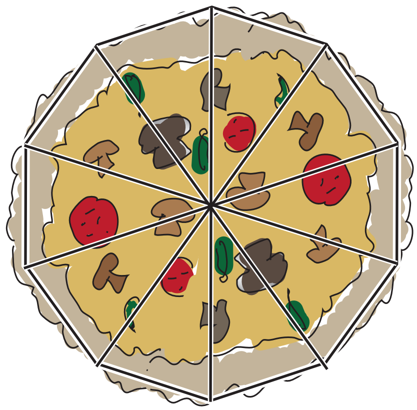


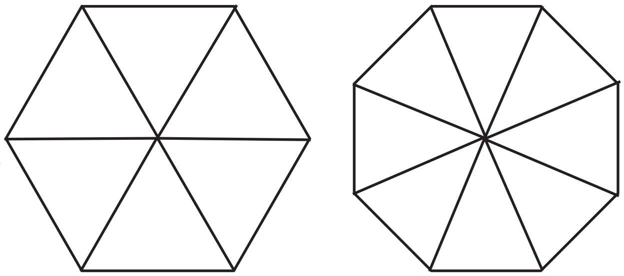
What do YOU want to try?

Chop the polygons into triangles

When you investigated the pizza area formula back in Chapter 5, you created a series of polygons by chopping the pizza into triangular slices.

Any regular polygon can be divided into the same number of congruent triangles as the polygon has sides.





So, if we know the area of one of those triangles, we can easily find the area of the whole polygon.

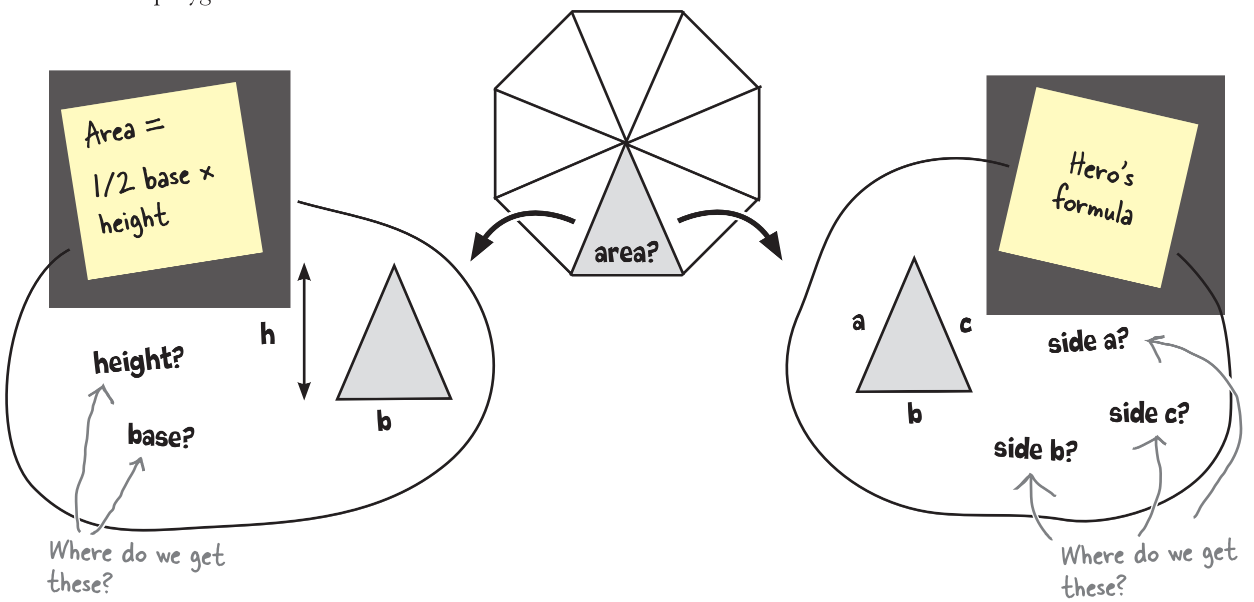


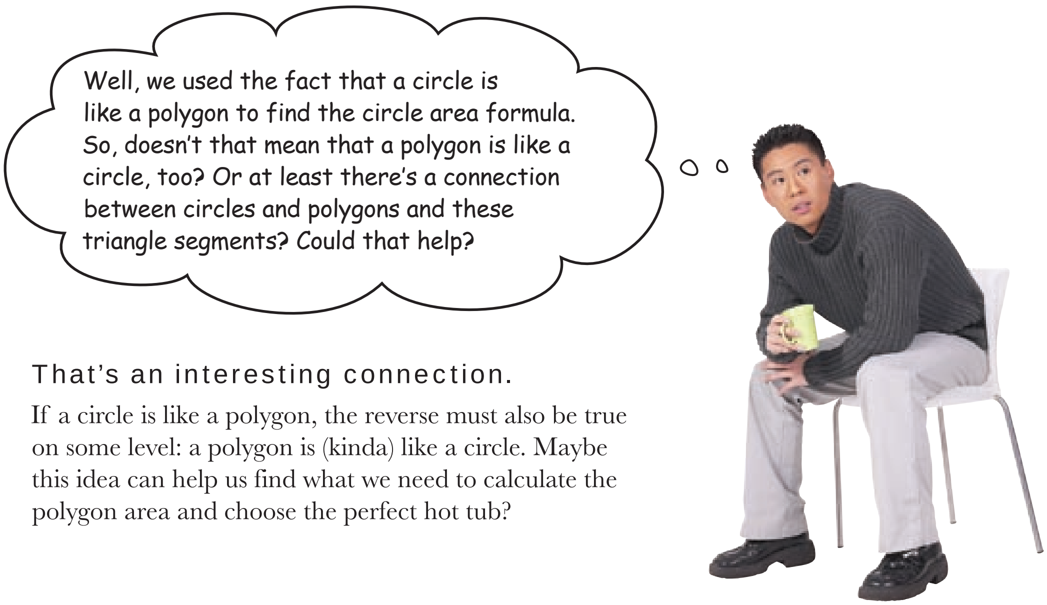
That's true. We don't have enough information.

To use either technique we know for finding triangle area we're going to need to know more about these triangles....

What do we need to know about the polygon triangles?

We know that there are the same number of triangles as sides of the polygon, and we know that all the triangles are congruent so we just need to find the area of one. But what else do we need to know to find the triangle's area, so that we can combine them to find the total polygon area?





That's an interesting connection.

If a circle is like a polygon, the reverse must also be true on some level: a polygon is (kinda) like a circle. Maybe this idea can help us find what we need to calculate the polygon area and choose the perfect hot tub?



Geometry Detective

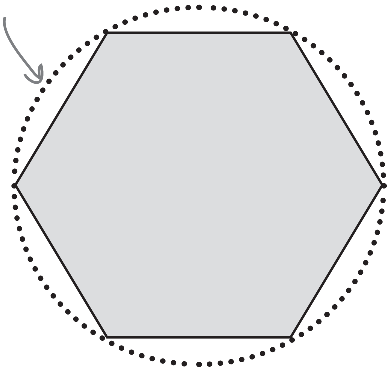
Your mission is to investigate how you could find the missing information needed to use either of the triangle area formulas by using circles fitted to a polygon.

We used these circles to find the triangle centers in Chapter 4.

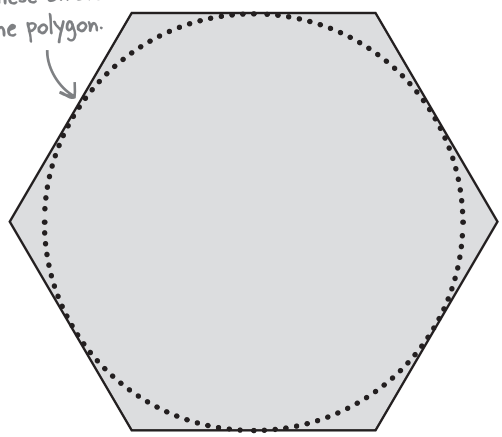
The polygons below are drawn with either their circumscribing circle (around the outside, touching every vertice) or their inscribing circle (within the polygon, touching every side).

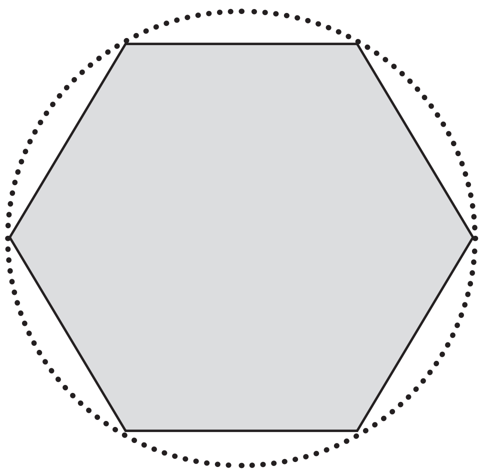
Draw or sketch on the polygons—we've given you some spares so don't be shy!

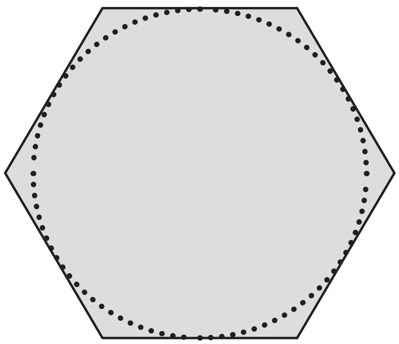
These circles circumscribe the polygon.



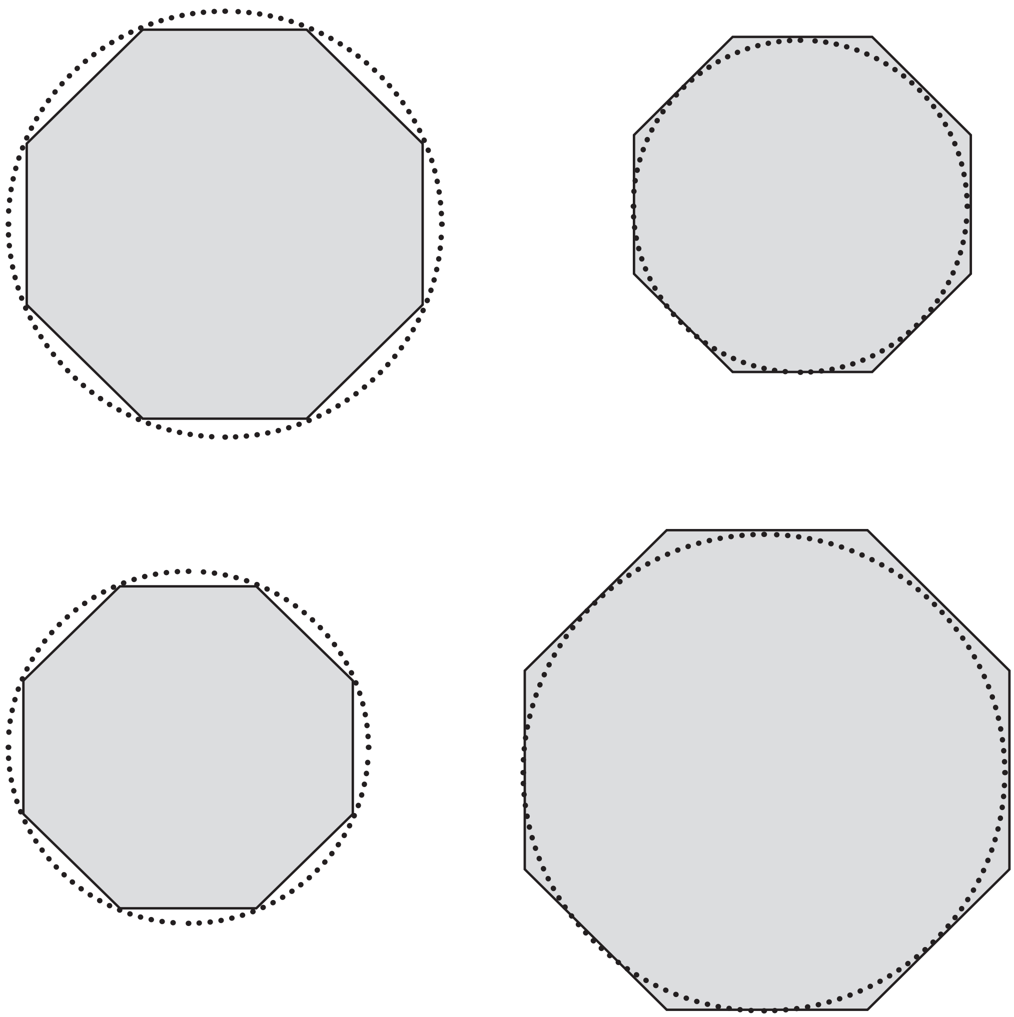
These circles inscribe the polygon.

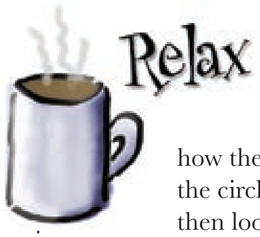






Hint: Start by dividing the polygons into equal triangles. You're looking for base, height, and side length of those.





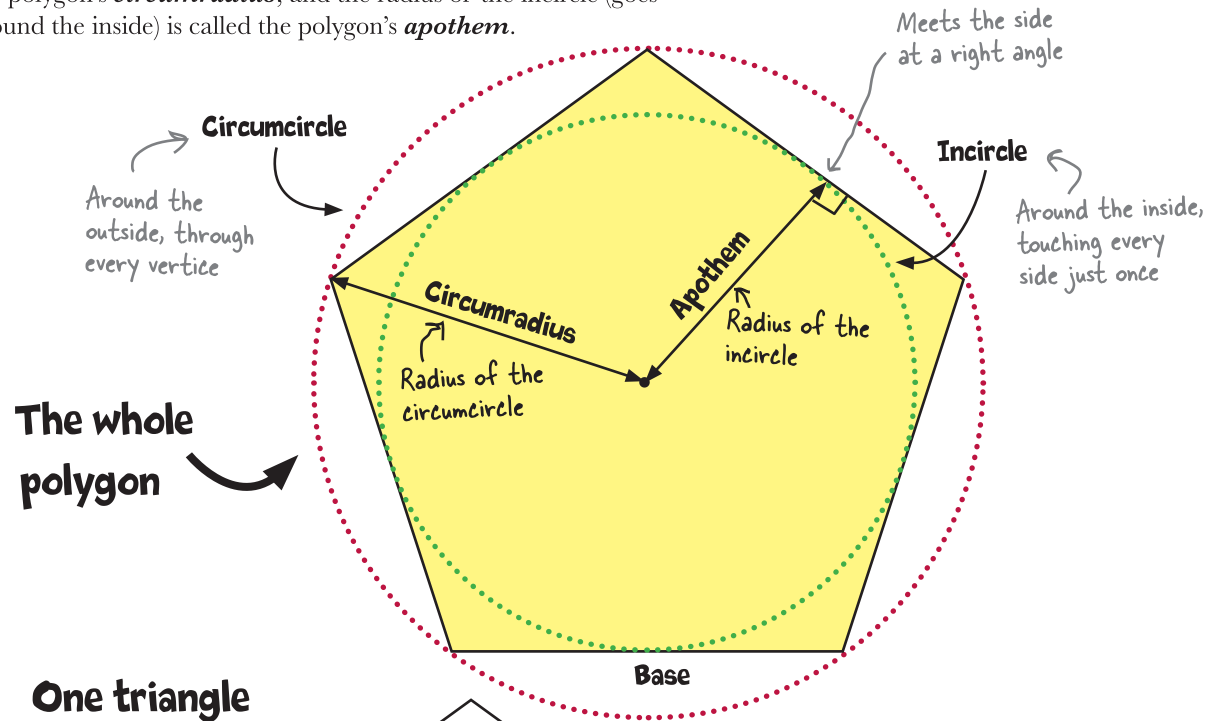
Need a 10-minute time-out?

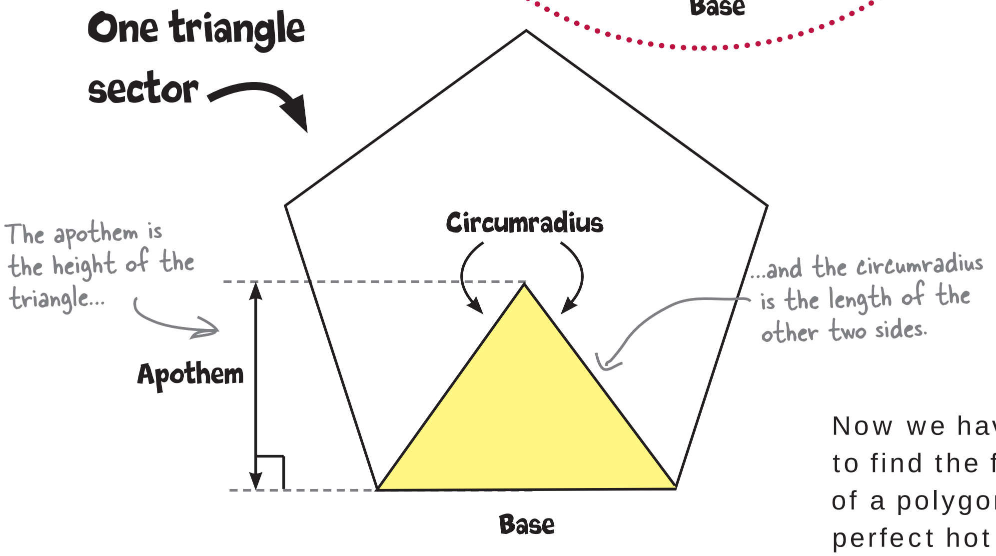
If you're feeling kind of fuzzy about how the polygon triangle properties relate to the circles, try having a 10-minute brain-break, then look again before you flip the page.

The circles give us the properties we need

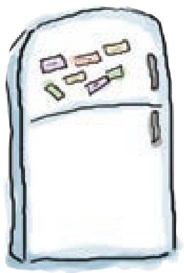
The base of each triangle in the polygon is the same as one side of the polygon, and the height and other two sides of the triangle are radii of the circles inscribing and circumscribing the polygon. The radius of the circumcircle (goes around the outside) of a polygon is the polygon's ***circumradius***, and the radius of the incircle (goes around the inside) is called the polygon's ***apothem***.

Radii is just the plural of radius.





Now we have everything we need to find the formula of the area of a polygon, so we can pick the perfect hot tub. No time to lose!



Polygon Formula Magnets

Rearrange the magnets to find the general formulas for the perimeter and area of our polygon hot tubs. Use each magnet exactly once.

(Hint: it looks a lot like one of the intermediate stages of the circle area formula!)

Think carefully about which area formula you're using.

Area of one triangle 'sector' = [ ] [ ] × [ ]

Combine these.

Total area = [ ] [ ] × [ ] × [ ]

Perimeter = [ ] × [ ]

Total area = [ ] [ ] × [ ]

number of sides

apothem

1/2

side length

1/2

perimeter

apothem

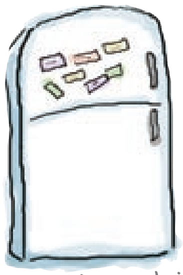
side length

number of sides

1/2

side length

apothem



Polygon Formula Magnets Solution

Rearrange the magnets to find the general formulas for the perimeter and area of our polygon hot tubs. Use each magnet exactly once.

(Hint: it looks a lot like one of the intermediate stages of the circle area formula!)

Using 1/2 base × height

Area of one triangle 'sector' = **1/2** **side length** (Base) × **apothem** (Height)

Combine these.

Total area = **1/2** **side length** × **apothem** × **number of sides**

Perimeter = **side length** × **number of sides**

You can substitute perimeter for side length × sides.

Total area = **1/2** **perimeter** × **apothem**

Doesn't this look like the intermediate formula we had for circle area: 1/2 × circumference × radius?

See page 230 for a reminder if you're fuzzy.

Don't worry if you put the magnets in each equation in a different order—because it's all multiplication the order isn't important.



BULLET POINTS

- The **apothem** joins the center to the middle of a side, bisecting it.
- The apothem meets the side at a **right angle**.
- The **circumradius** joins the center to a vertex.
- The circumradius **bisects** the internal angle.

Polygon area = 1/2 perimeter x apothem

When you add up all the areas of the triangle sectors in a polygon you end up with something pretty neat: 1/2 perimeter × apothem.

Often you'll be given the apothem, or you might have the circumradius and the side length and then you can use the Pythagorean Theorem to find the apothem.

OK, great. So that gives us a formula that we can use for all the hot tubs, whether they've got 6, 7, 8, or 9 sides. That **is** cool.



Joe: Yeah, cool, whatever. Except we still need to find a bunch of apothems to use it!

Frank: Relax, I'm not sure we actually do.

Jill: How come?

Frank: Well, the whole way we found that circle area formula was about adding more and more sides to the polygon until it resembled a circle—yeah?

Joe: Yeah. And?

Frank: So how many butts fit in the circle tub?

Jill: Not many! It's the least we've found. Triangle was the most, square was pretty good.

Frank: Exactly! That's not just a detail—that's a trend!

What do you think the trend is?

Which hot tub are you leaning toward?

More sides = fewer butts

Look again at that intermediate circle area formula compared to the polygon area formula you just created:

Polygon area formula

Area = 1/2 perimeter x apothem

Circle area formula

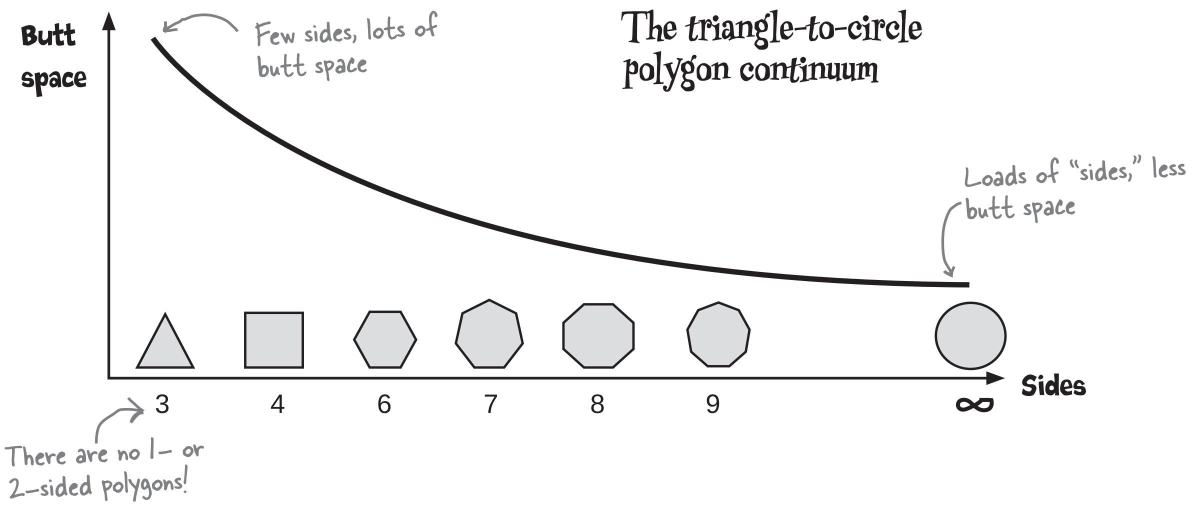
Area = 1/2 circumference x radius

Perimeter ⟷ **Circumference**

Apothem ⟷ **Radius**

A circle really is just another polygon with an infinite number of sides. At the other end of the scale you've got the humble triangle, the lowest polygon of all. In between those two sit every other regular polygon you can imagine.

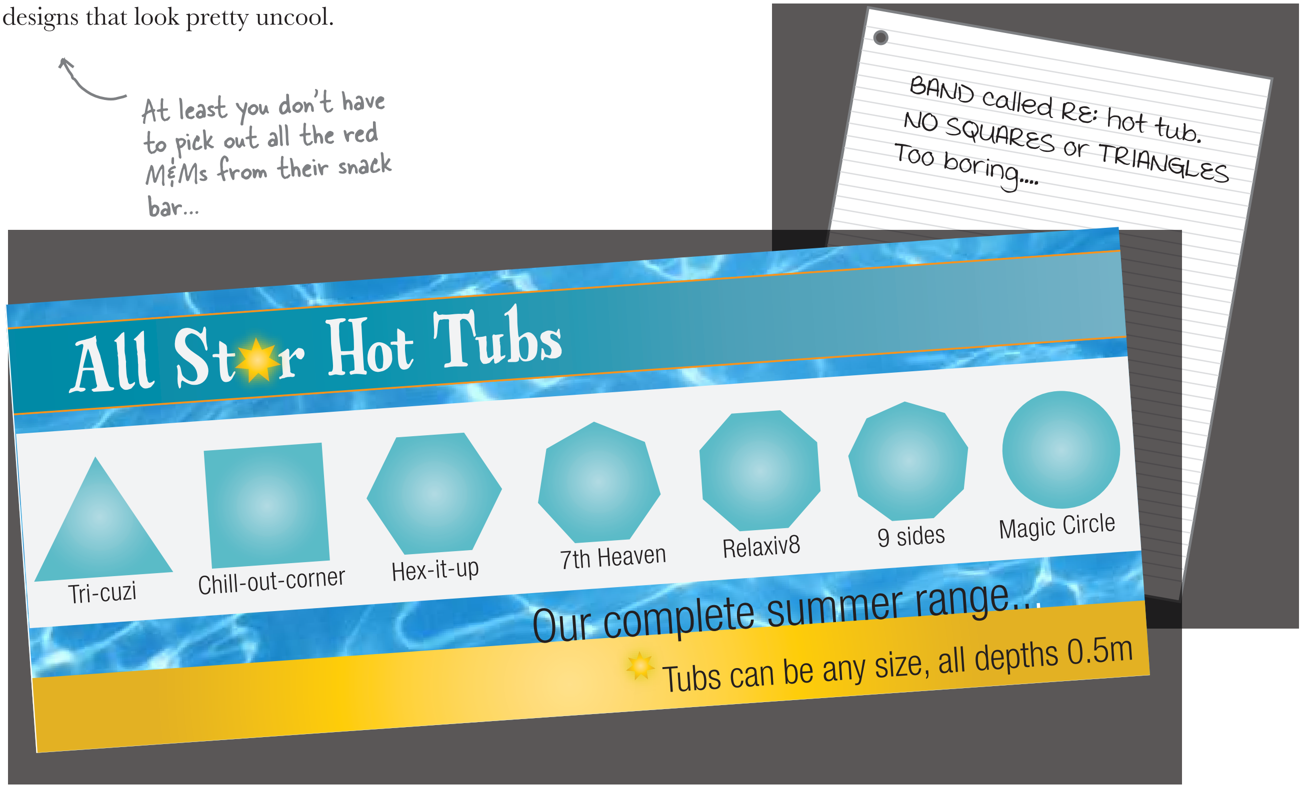
So far, we've found that the most butt-space comes from the hot tub with the fewest sides, and the least butt-space comes from the hot tub with the most sides. It's a trend!



Rock stars—high maintenance?

Uh oh. The band is getting demanding. They've seen the All Star Hot Tubs brochure lying around and think there's a couple of designs that look pretty uncool.

At least you don't have to pick out all the red M&Ms from their snack bar...

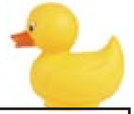




Exercise

Which hot tub do you want to order?

....





Exercise Solution

Which hot tub do you want to order?

The HEX-IT-UP tub with six sides.

there are no Dumb Questions

Q: A circle is just a polygon with infinite sides? Have you gone a bit philosophical or what?

A: Some math geeks get a bit upset about this definition of a circle, but if you could draw enough sides then you'd be approaching a circle. The whole point of infinity is that you never get there, but you get close enough for this to be the basis of computer graphics.

Q: Computer graphics? Polygons? How does that work?

A: To show 3D curvy worlds, all video games rely on triangles, which build into polygons, which fool our eyes into believing we're seeing circles and curves.

Q: Ah! Yeah, when I've zoomed in real close to something in a game sometimes I've seen that it's made out of flat shapes.

A: That's it—exactly.

Q: Circumradius sounds a lot like radius, but in the comparison of the area formulas you compared apothem to radius. That's kind of confusing!

A: Actually a regular polygon's circumradius is sometimes just referred to as its radius. But if you think about it, a circle's apothem and its circumradius can both be equal to its radius (if the apothem is going right to the edge). So when we compared the apothem and the circle's radius, we were comparing an apothem to an apothem!

Q: Does the polygon area formula work for all regular polygons? Even squares and triangles?

A: The polygon area formula works for any regular polygon—whether it's got 3 sides or an infinite number! But you've got some much sharper tools in your tool box for finding square and triangle areas, and they'll always be quicker and easier to use, so it's best to keep this one for regular polygons with five sides or more.

Q: Regular polygons only? This area formula won't work with irregular polygon?

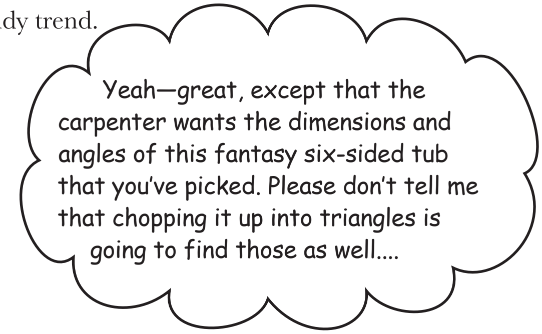
A: Sadly not. Irregular shapes are harder to deal with anyway. When you split them into triangles the triangles aren't congruent. Very high maintenance. Luckily they don't come up in exams much!

Q: Apothem. Let me guess, that means in an algebra version of the formula that would be an "a"? But we're already using "a" for area—why couldn't they have named it something with a different first letter?

A: Good question. We don't have an answer—except that if you remember your formulas as words rather than just letters then it doesn't matter so much. If you're sitting a test and you want to demonstrate that you know the formula, write it out in words and then make it clear which letters you're picking as stand-ins. Whether you use a, A, s, or t for something is less important than what it means. It's also easier to work out whether a formula applies to a problem when you remember your formulas as words and not just letters.

Great tub choice!

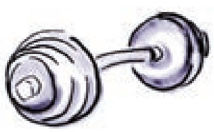
Your pick of the six-sided hot tub has stirred up excitement among all the bands booked for the festival—they think it's gonna be amazing. And you didn't even have to do any more math to choose the hexagon tub, since you already found that handy trend.





Actually, it might.

It's not a bad suggestion at all. We have plenty of triangle-related tools in our toolbox, so when we're faced with a problem we can't already solve, looking for ways to make it about triangles is a great starting point. Let's give it a go.

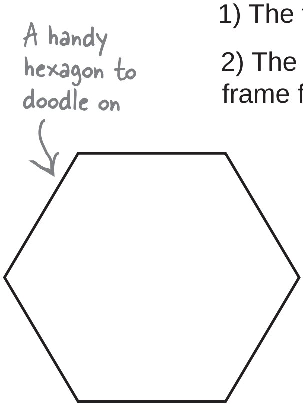


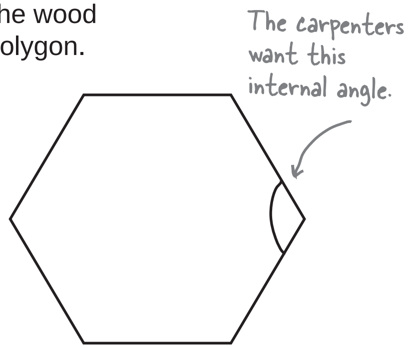
BRAIN BARBELL

We know we can divide the hexagon tub into six congruent triangles. From this, can you find:

1) The total of all the angles of those "sector" triangles.

2) The angle that the carpenters need to build the wood frame for the hot tub: the internal angle of the polygon.





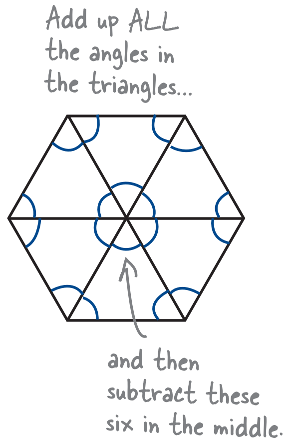


BRAIN BARBELL SOLUTION

We know we can divide the hexagon tub into six congruent triangles. From this, can you find:

1) The total of all the angles of those “sector” triangles.

2) The angle that the carpenters need: the internal angle of the polygon.



1) Angles in a triangle add up to 180°

There are six triangles so the total angles are: $6 \times 180° = 1080°$

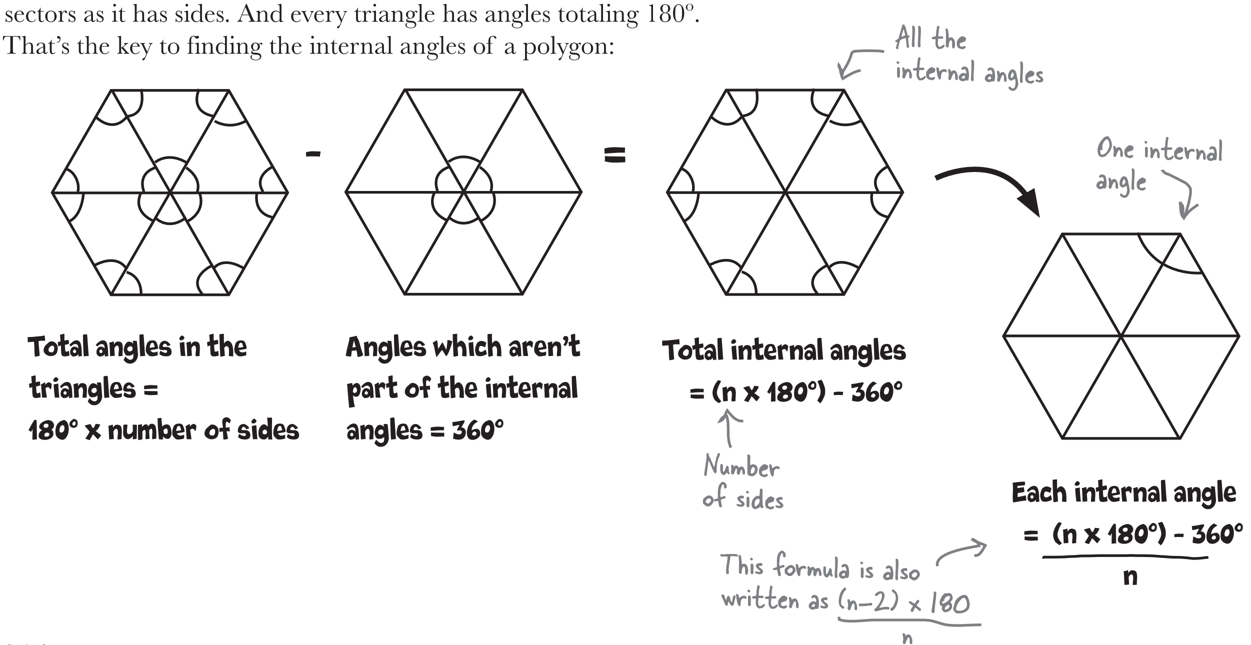
2) The six angles in the middle add up to 360°

So the total of all the internal angles is the total angles minus the six angles in the middle $= 1080° - 360° = 720°$

It's a regular polygon, so the angles are equal $= \frac{720°}{6} = 120°$

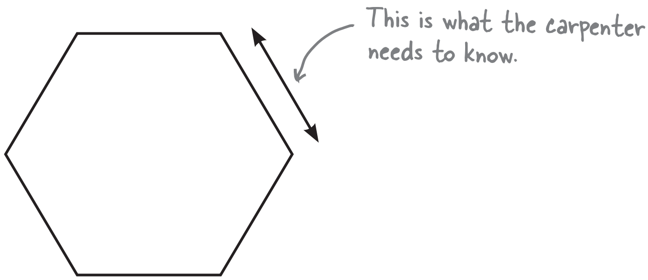
Internal angles of a polygon follow a pattern

A polygon can always be divided into the same number of triangular sectors as it has sides. And every triangle has angles totaling 180°. That’s the key to finding the internal angles of a polygon:



But what about dimensions?

Phew! The only thing left on your to-do list is to tell the carpenters what length each side of the hot tub is.





BRAIN BARBELL

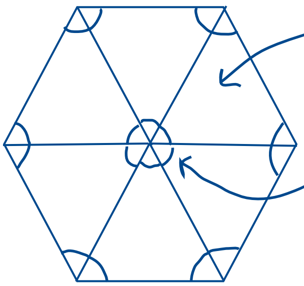
The total area of the hot tub is 6 square meters. Use what you know about the hexagon hot tub angles to find the length of each side of the hot tub. (Hint: it helps if you split it into those sector triangles!)

Hint—look back at page 293 if you're not sure where to start....



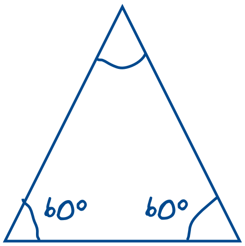
Brain Barbell Solution

The total area of the hot tub is 6 square meters. Use what you know about the hexagon hot tub angles to find the length of each side of the hot tub.



These 6 triangles are the same (congruent). Each one must have an area of 1 square meter.

The angles in the middle add up to 360°, so this angle here must be 60°.



Each internal angle is 120°—and we know the triangle is ISOSCELES because the side lengths are both equal to the circumradius of the polygon, so the other angles must both be 120°/2 = 60°.

An isosceles triangle has two equal sides, and two equal angles.

Each 1 sq meter triangle is EQUILATERAL.

Any triangle with three 60° angles is equilateral.

So we can use the equilateral triangle area formula!

$$\text{Area} = \frac{\sqrt{3}t^2}{4}$$

t is side length.

Area of each triangle is the total polygon area divided by the number of triangles.

$$1m^2 = \frac{\sqrt{3}t^2}{4}$$

$$\frac{4}{\sqrt{3}} = t^2$$

$$2.31 = t^2$$

Use a calculator!

$$1.52m = t$$

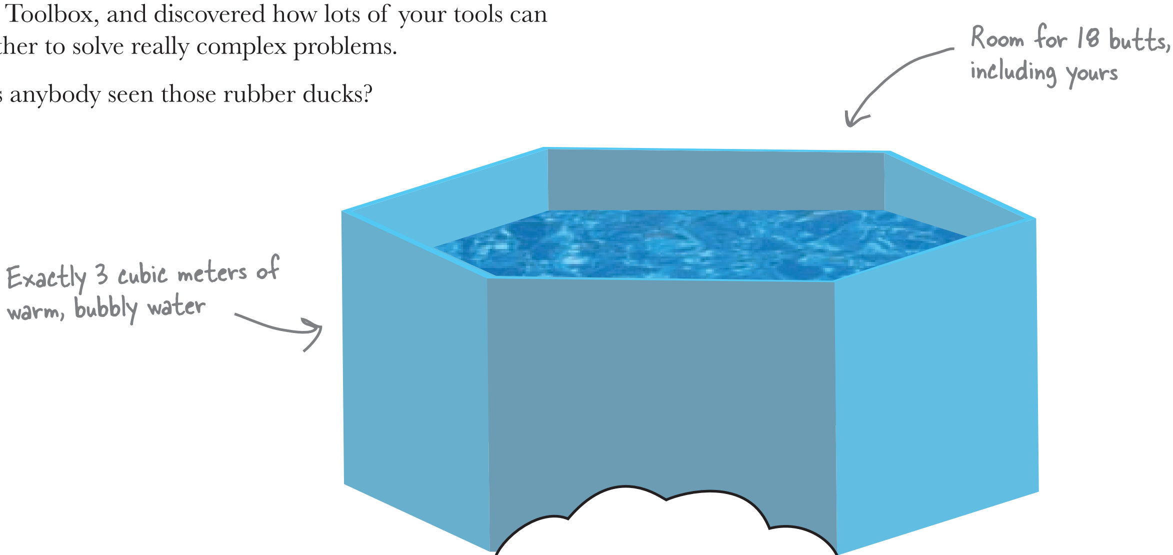
Length of each side.

It's time to relax in the hot tub!

You've earned it. Not only have you picked the perfect hot tub to please both the environmental engineers and the bands, you've even managed to sort out the exact angles and dimensions for the carpenters to get ready to install it immediately!

Along the way you've created a bunch of new tools for your Geometry Toolbox, and discovered how lots of your tools can work together to solve really complex problems.

Now—has anybody seen those rubber ducks?







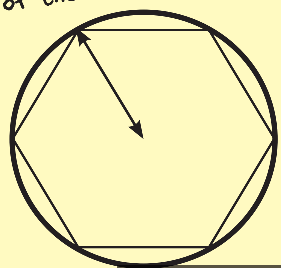
Your Geometry Toolbox

You've got Chapter 7 under your belt and now you've added regular polygons to your toolbox. For a complete list of tool tips in the book, head over to www.headfirstlabs.com/geometry.

Perimeter = Number of sides × Side length

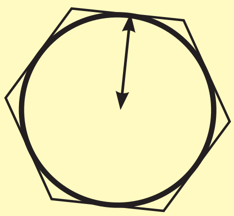
Area = 1/2 × Perimeter × Apothem

Circumradius: radius of the circumcircle



Equilateral triangle area = $\frac{\sqrt{3}\,t^2}{4}$

Apothem: radius of the incircle



Internal angle = $\frac{(n \times 180°) - 360°}{n}$

(n is number of sides)



BULLET POINTS

- The **apothem** joins the center to the middle of a side, bisecting it.
- The apothem meets the side at a **right angle**.
- The **circumradius** joins the center to a vertex.
- The circumradius **bisects** the internal angle.

Leaving town...



It's been great having you here in Geometryville!

We're sad to see you leave, but there's nothing like taking what you've learned and putting it to use. You're just beginning your geometry journey and we've put you in the driver's seat. We're dying to hear how things go, so ***drop us a line*** at the Head First Labs website, **www.headfirstlabs.com/geometry**, and let us know how geometry is paying off for **YOU**! Next stop? *Head First 3D Geometry*!

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